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Propagation Effects on the Polarization of Pulsar Radio Emission

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Abstract

The properties of the natural wave modes of a pulsar magnetosphere are derived in a simple way by appealing to the 'low-density limit'. The properties are evaluated explicitly for a general version of the cold plasma model which includes relativistic streaming motions, and it is argued that this model is probably adequate for pulsar magnetospheres. The observed circular polarization in pulsar radio emission could arise as a propagation effect; the conditions under which initially linear polarization could be converted into partially circularly polarized radiation are summarized. The observed 'orthogonal modes' of polarization could be due to components in the two natural modes having slightly different ray paths. The angular separation of the two rays is found to be a strong function of frequency ($\Delta\theta_r \approx 4\times 10^8\, n_e/f^2$), and it is suggested that a study of the frequency dependence of 'orthogonal modes' could provide useful information.

1. Introduction

A pulsar magnetosphere is a diffuse plasma, and it is often assumed that its properties are similar to those of other diffuse astrophysical plasmas, e.g. in leading to Faraday rotation. However, the plasma is thought to be highly relativistic and, at least in the polar-cap zones, to involve relativistic streaming out along the field lines; moreover, the particles are probably in their lowest Landau orbitals (classically, $p_\perp = 0$) due to the short time-scale for them to radiate away any perpendicular energy due to gyroemission. (The gyroradiation is at frequencies much higher than the radio range and is of no interest here.) The dielectric properties of such a plasma can be markedly different from those of more familiar plasmas. In particular, the wave modes can be linearly polarized, rather than circularly polarized, and be a strong function of the angle θ of wave propagation relative to the ambient magnetic field. My purpose in this paper is to derive the properties of these natural modes and to explore some possible implications concerning the interpretation of the observed polarization, notably the circular polarization and the 'orthogonal modes' of polarization.

The properties of the natural modes of a relativistic plasma in a strong magnetic field have been discussed by various authors cited below, and it would be reasonable to conclude from their discussions that the wave properties are very complicated indeed. However, it is possible to give a relatively simple treatment of the wave properties in a particular limit, called the 'low-density limit' here, which should be applicable to pulsar magnetospheres. The low-density limit is defined specifically in Section 2 below. In practice it corresponds to $v_A^2 \gg c^2$, where v_A is the Alfvén speed as usually defined and c is the speed of light. The ratio c^2/v_A^2, which is effectively

the small parameter in which one expands in the low-density limit, is equal to one-half of the ratio of the mass energy density in ionized particles to the magnetic energy density. For example, in say the model of Ruderman and Sutherland (1975) for a pulsar magnetosphere, the matter consists of electrons and positrons with a number density $\sim 10^{10}\ \mathrm{cm}^{-3}$ and Lorentz factor $\gamma \approx 300$ in a region where the magnetic field is $\sim 10^{12}$ G ($\equiv 10^8$ T); with these parameters one has $c^2/v_A^2 \approx 3\times 10^{-17}$, and one should have $c^2/v_A^2 \ll 1$ in any reasonable model for a pulsar magnetosphere.

The parameter c^2/v_A^2 in an electron–positron plasma is just the square of the ratio of the (relativistic) plasma frequency to the (relativistic) gyrofrequency, and more generally the condition $c^2/v_A^2 \ll 1$ corresponds to the plasma frequency being much less than the lower hybrid frequency. In other words, the plasma frequency is in the range of hydromagnetic frequencies. It may well be that the low-density approximation (as defined below) breaks down at frequencies less than a relevant plasma frequency (see Section 3). However, the relevant plasma frequency is well below the frequencies of interest in connection with pulsar radio emission, and consequently any failure of the low-density approximation at very low frequencies is unimportant for the present discussion.

The properties of the natural modes in the low-density limit can be found quite simply for any distribution of particles; this is done in Section 2. In Section 3 it is argued that the cold plasma approximation is adequate for many purposes, and the wave properties derived in Section 2 are found explicitly for a cold plasma model which includes relativistic streaming motions. 'Faraday rotation' is discussed in Section 4.

It is found in Section 3 that the modes are usually linearly polarized, but become significantly elliptical at large distances from the pulsar. This provides a possible interpretation of the observed circularly polarized component in many pulsars as follows: The radiation is initially linearly polarized and separates into components in one or both natural modes. The radiation remains in the natural mode(s) until it reaches a 'polarization limiting region' (PLR) beyond which the plasma density is too low to cause the modes to get significantly out of phase. The escaping radiation is then polarized like the natural mode(s) at PLR. Irrespective of the generation mechanism, partially circularly polarized emission results if the natural modes are significantly circular at PLR. This idea is discussed further in Section 5.

Separation into two natural modes of the ambient plasma is the most plausible available interpretation of the 'orthogonal modes' of polarization observed in many pulsars (Manchester *et al.* 1975; Backer *et al.* 1976). This interpretation requires significant separation in the ray paths for the components in the two modes. The separation between the modes is estimated and discussed in Section 6.

Before proceeding, it is appropriate to summarize the literature on wave properties in a relativistic plasma in a strong magnetic field. Tsytovich and Kaplan (1972) and Kaplan and Tsytovich (1973, p. 230) considered the wave properties in a one-dimensional electron gas with a power-law distribution in the Lorentz factor describing the parallel motion. (By 'one-dimensional' I mean a distribution in which all the particles are in their lowest Landau orbitals; see the Appendix.) Blandford (1975) considered an isotropic distribution which is a constant inside a sphere in momentum space and zero outside the sphere. The case of a cold relativistically streaming electron plasma has been considered by Godfrey *et al.* (1975), by Hardee

and Rose (1976, 1978) who considered only a charge-neutral electron–positron plasma, and by Melrose and Stoneham (1977) who allowed an arbitrary ratio of positrons to electrons and who considered various quantum effects.

2. Wave Properties in the Low-density Limit

The low-density limit may be defined by the condition that the dielectric tensor $\varepsilon_{ij}(\boldsymbol{k},\omega)$ be nearly equal to the unit tensor δ_{ij}. In other words, each of the components of

$$\Delta\varepsilon_{ij}(\boldsymbol{k},\omega) := \varepsilon_{ij}(\boldsymbol{k},\omega) - \delta_{ij}\,, \tag{1}$$

(where the symbols $:=$ and $=:$ define quantities on the left and right respectively) is much less than unity in magnitude in the low-density limit. The wave properties in this limit may be found by a perturbation approach involving an expansion in powers of $\Delta\varepsilon_{ij}$. To lowest order (the zeroth approximation $\Delta\varepsilon_{ij} = 0$) one has a vacuum, and the refractive index $n := kc/\omega$ is equal to unity. In the first-order approximation $\Delta\varepsilon_{ij}(\boldsymbol{k},\omega)$ is taken to be nonzero, and is evaluated at $\boldsymbol{k} = \boldsymbol{\kappa}(\omega/c)$, where

$$\boldsymbol{\kappa} := \boldsymbol{k}/k \tag{2}$$

is the unit vector in the $\boldsymbol{k}$ direction.

The dispersion equation for arbitrary ε_{ij} may be written

$$An^4 - Bn^2 + C = 0\,, \tag{3}$$

with

$$A = \varepsilon_{11}\sin^2\theta + \varepsilon_{33}\cos^2\theta + 2\varepsilon_{13}\sin\theta\cos\theta\,, \tag{4a}$$

$$B = A\varepsilon_{22} + \varepsilon_{11}\varepsilon_{33} - \varepsilon_{13}^2 + (\varepsilon_{12}\sin\theta - \varepsilon_{23}\cos\theta)^2\,, \tag{4b}$$

$$C = \varepsilon_{11}\varepsilon_{22}\varepsilon_{33} + \varepsilon_{11}\varepsilon_{23}^2 + \varepsilon_{33}\varepsilon_{12}^2 - \varepsilon_{22}\varepsilon_{13}^2 + 2\varepsilon_{12}\varepsilon_{23}\varepsilon_{13}\,, \tag{4c}$$

where the magnetic field $\boldsymbol{B} =: B\boldsymbol{b}$ is along the 3-axis, and $\boldsymbol{k}$ is in the 1–3 plane at an angle θ to the magnetic field. Also, in the dispersion equation (3) it is assumed that only the hermitian part of ε_{ij} is retained. In the low-density limit and for the first-order approximation, $\Delta\varepsilon_{ij}$ does not depend on n, and hence equation (3) is a quadratic equation for n^2. Further, only terms up to those quadratic in $\Delta\varepsilon_{ij}$ need to be retained. In this approximation the two solutions for n^2 are (Melrose and Stoneham 1977)

$$n^2 = 1 + \Delta n_\pm^2\,, \tag{5}$$

$$\Delta n_\pm^2 = \tfrac{1}{2}(a+b) \pm \tfrac{1}{2}\{(a-b)^2 + 4g^2\}^{\frac{1}{2}}\,, \tag{6}$$

with

$$a := \Delta\varepsilon_{11}\cos^2\theta + \Delta\varepsilon_{33}\sin^2\theta - 2\Delta\varepsilon_{13}\sin\theta\cos\theta\,, \tag{7a}$$

$$b := \Delta\varepsilon_{22}\,, \tag{7b}$$

$$g := -\mathrm{i}(\Delta\varepsilon_{12}\cos\theta + \Delta\varepsilon_{23}\sin\theta)\,. \tag{7c}$$

Note that the Onsager relations imply that (when only the hermitian part is retained) $\Delta\varepsilon_{12}$ and $\Delta\varepsilon_{23}$ are imaginary and all other components are real; con-

sequently, all of a, b and g are real. In addition, the axial ratio of the polarization ellipse is given by

$$T = T_{\pm} = 2g/[a-b \mp \{(a-b)^2 + 4g^2\}^{\frac{1}{2}}], \tag{8}$$

where the + and − labellings of the modes are the same as in equation (5). The axial ratio T is defined such that $T = 0$ and $T = \infty$ are linear polarization along $\boldsymbol{\kappa} \times \boldsymbol{b}$ and $\boldsymbol{\kappa} \times (\boldsymbol{\kappa} \times \boldsymbol{b})$ respectively, and $T = +1$ is right-hand circular polarization in a screw sense relative to $\boldsymbol{\kappa}$.

The low-density approximation breaks down near resonances where one or more of the components of $\Delta\varepsilon_{ij}(\boldsymbol{k}, \omega)$ become large. For example, in the cold plasma approximation, terms proportional to $1/(\omega^2 - \Omega_\alpha^2)$ appear which become infinite at the gyrofrequencies, i.e. at $\omega = \Omega_\alpha$. However, it is argued below that most of these resonances should be smeared out when a reasonable spread in particle motions is included and that $|\Delta\varepsilon_{ij}|$ remains less than unity near the gyrofrequencies. The only other case where $\Delta\varepsilon_{ij}$ might be expected to become large is in the limit of very small frequencies. In fact, in the cold plasma approximation, only $\Delta\varepsilon_{33}$ becomes arbitrarily large in the limit $\omega \to 0$, but nevertheless this does imply that the low-density approximation breaks down at very low frequencies.

3. Cold Plasma Approximation

The cold plasma approximation is usually treated using a fluid theory. It may be derived from a more general theory by taking the long wavelength approximation. In the Appendix it is shown that the cold plasma approximation is the long wavelength approximation to a general relativistic quantum theory of plasmas, and that it applies for any nonrelativistic distribution of particles in their Landau orbitals (any nonrelativistic $p_\perp$ distribution in the nonquantum case). The long wavelength approximation corresponds to the wavenumber k times any relevant length (e.g. a gyroradius or a Debye length) being much less than unity.

A general cold plasma model consists of particles of various species, with species α having a charge q_α, rest mass m_α, number density n_α and Lorentz factor γ_α. Actually, the Lorentz factor γ_α refers only to the motion along the magnetic field lines and may be defined by writing

$$p_{\|\alpha} =: \gamma_\alpha \beta_\alpha m_\alpha c, \qquad \text{with} \qquad \gamma_\alpha = (1-\beta_\alpha^2)^{-\frac{1}{2}}. \tag{9}$$

It is convenient to introduce the quantities

$$\omega_{p\alpha} := \left(\frac{4\pi n_\alpha q_\alpha^2}{\gamma_\alpha m_\alpha}\right)^{\frac{1}{2}}, \qquad \Omega_\alpha := \frac{|q_\alpha| B}{\gamma m_\alpha c}, \qquad \varepsilon_\alpha := \frac{q_\alpha}{|q_\alpha|}. \tag{10a, b, c}$$

The cold plasma dielectric tensor then leads to (cf. equations 7)

$$a = -\sum_\alpha \frac{\omega_{p\alpha}^2(\cos\theta - n\beta_\alpha)^2}{\omega^2(1-n\beta_\alpha\cos\theta)^2 - \Omega_\alpha^2} - \sum_\alpha \frac{\omega_{p\alpha}^2 \sin^2\theta}{\gamma_\alpha^2 \omega^2 (1-n\beta_\alpha\cos\theta)^2}, \tag{11a}$$

$$b = -\sum_\alpha \frac{\omega_{p\alpha}^2(1-n\beta_\alpha\cos\theta)^2}{\omega^2(1-n\beta_\alpha\cos\theta)^2 - \Omega_\alpha^2}, \tag{11b}$$

$$g = -\sum_\alpha \frac{\varepsilon_\alpha \omega_{p\alpha}^2(\Omega_\alpha/\omega)(\cos\theta - n\beta_\alpha)}{\omega^2(1-n\beta_\alpha\cos\theta)^2 - \Omega_\alpha^2}, \tag{11c}$$

where the sums are over all species of particle, and where n has not yet been set equal to unity. For identical distributions of electrons and positrons, g vanishes; this case has been discussed in detail by Hardee and Rose (1976, 1978). The effect of $g \neq 0$ on the ellipticity of the natural modes has been discussed by Melrose and Stoneham (1977).

The cold plasma approximation breaks down near the gyrofrequencies. However, in a thermal nonrelativistic plasma it is known that the effect of thermal motions smears out the resonance (Pradhan 1957). For particles with a thermal speed V_α, the maximum value of the contribution to k from the resonance for species α is (Stix 1962, p. 196)

$$k_{\max} \approx (\omega_{\mathrm{p}\alpha}^2 \Omega_\alpha / c^2 V_\alpha)^{1/3} . \tag{12}$$

Although the approximation (12) is not directly relevant to the present case, it is reasonable to use it with $V_\alpha \approx c$ to make a rough estimate of how smeared out the resonances are. The resonance occurs at $\omega \approx \Omega_\alpha$, and hence one has

$$(k_{\max} c/\omega)^2 \approx \omega_{\mathrm{p}\alpha}^2/\Omega_\alpha^2 \ll 1 ; \tag{13}$$

that is, the resonance should not cause even a slight bump on the refractive index curves. Consequently, one should ignore the resonances at $\omega \approx \Omega_\alpha$ and join the asymptotic refractive index curve for $\omega \ll \Omega_\alpha$ to that for $\omega \gg \Omega_\alpha$.

The only other regime which could lead to a breakdown in the low-density approximation is for

$$\sum_\alpha \omega_{\mathrm{p}\alpha}^2 \sin^2\theta / \gamma_\alpha^2 \omega^2 (1 - n\beta_\alpha \cos\theta)^2 \lesssim 1 , \tag{14}$$

when the final term in (11a) becomes greater than unity. The condition (14) is similar to that for longitudinal oscillations in the plasma. The longitudinal dispersion relation for parallel propagation ($\cos\theta = 1$) is

$$\sum_\alpha \omega_{\mathrm{p}\alpha}^2 / \gamma_\alpha^2 \omega^2 (1 - n\beta_\alpha)^2 = 1 . \tag{15}$$

For simplicity, suppose there is only one species of particle. Then equation (15) has two classes of solution. One class is for $n\beta_\alpha \ll 1$ and $\omega \approx \omega_{\mathrm{p}\alpha}/\gamma_\alpha$. The other class is for $n\beta_\alpha \approx 1$ and $\omega \gg \omega_{\mathrm{p}\alpha}/\gamma_\alpha$. These are closely analogous to the plasma oscillations of a nonrelativistic plasma. For example, the condition $\omega' = \omega'_{\mathrm{p}\alpha}$ in the rest frame (the primed frame) for species α leads to

$$\gamma_\alpha \omega (1 - n\beta_\alpha) = \omega_{\mathrm{p}\alpha} \tag{16}$$

in the laboratory frame; i.e. for only one species, equation (15) would correspond to an oscillation at the plasma frequency in the rest frame. The cold plasma approximation is inadequate to treat such longitudinal oscillations in a nonrelativistic plasma, and one would expect the contributions of 'thermal' convections to equation (15) to be important in the present case.

In summary, the low-density and cold plasma approximations should be adequate for any discussion of the properties of waves in a pulsar magnetosphere, except at low frequencies, as defined by the condition (14), and except for longitudinal oscillations.

4. 'Faraday Rotation'

For present purposes 'Faraday rotation' is defined to mean the effect of the components in the two natural modes getting out of phase as they propagate. For circularly polarized modes the resulting effect is a rotation of the plane of polarization, for linearly polarized natural modes it is a periodic variation in the axial ratio of the polarization ellipse, and for elliptically polarized modes it is a combination of these two effects.

The rate per unit length at which Faraday rotation occurs is just the difference Δk in the wavenumbers of the two modes. From equation (6) one has

$$\Delta k = (n_+ - n_-)\omega/c = (\omega/2c)\{(a-b)^2 + 4g^2\}^{\frac{1}{2}}. \tag{17}$$

From equations (11a) and (11b), with $n = 1$, one has

$$a-b = \sum_\alpha \frac{\omega_{p\alpha}^2}{\gamma_\alpha^2\,\omega^2(1-\beta_\alpha\cos\theta)^2}\,\frac{\Omega_\alpha^2\sin^2\theta}{\omega^2(1-\beta_\alpha\cos\theta)^2-\Omega_\alpha^2}. \tag{18}$$

Consider the relative contributions of electrons and ions (assuming both to be present) to equation (18). If we have

$$\omega^2(1-\beta_\alpha\cos\theta)^2 \ll \Omega_\alpha^2$$

for all species, the dominant contribution (for $\theta \gtrsim 1/\gamma_\alpha$) comes from the species with the largest value of $\omega_{p\alpha}^2/\gamma_\alpha^2$, that is, of $q_\alpha^2 n_\alpha/\gamma_\alpha^3$. This species is likely to be the electrons (including positrons). On the other hand, given

$$\omega^2(1-\beta_i\cos\theta)^2 \gtrsim \Omega_i^2 \qquad \text{and} \qquad \omega^2(1-\beta_e\cos\theta)^2 \ll \Omega_e^2$$

for ions (i) and electrons (e) respectively then, for $\theta > 1/\gamma_i$ and $\theta > 1/\gamma_e$, the contributions of the ions and electrons to equation (18) are in the ratio

$$\frac{Z_i^2 n_i}{m_i\gamma_i^3}\,\frac{4\Omega_i^2}{\omega^2\theta^4}\,\frac{m_e\gamma_e^3}{n_e} \lesssim \frac{Z_i^2 n_i m_e}{n_e m_i}\left(\frac{\gamma_e}{\gamma_i}\right)^3,$$

with $Z_i := q_i/e$. Again the electrons are dominant under plausible conditions. Hence, under plausible conditions, equation (18) may be approximated by

$$a-b = \omega_{pe}^2\sin^2\theta/\gamma_e^2\,\omega^2(1-\beta_e\cos\theta)^2. \tag{19}$$

Further, comparison between equations (19) and (11c) shows that under most circumstances one has $|a-b| \gg |2g|$ and hence that the modes are nearly linearly polarized, as pointed out by Melrose and Stoneham (1977).

Combining equations (17) and (19) and assuming $(a-b)^2 \gg 4g^2$, one has

$$\Delta k \approx \omega_{pe}^2\sin^2\theta/2c\gamma_e^2\,\omega(1-\beta_e\cos\theta)^2. \tag{20}$$

Inserting numerical values with $f := \omega/2\pi$ in hertz, the approximation (20) becomes

$$\Delta k \approx 4\times10^{-2}\,\frac{n_e}{f\gamma_e^3}\left(\frac{\sin\theta}{4\sin^2(\frac{1}{2}\theta)+1/\gamma_e^2}\right)^2 \ \text{cm}^{-1}. \tag{21}$$

It follows from the result (21) that Faraday rotation occurs fastest at $\theta \approx 1/\gamma_e$. The radiation should be generated at $\theta \lesssim 1/\gamma_e$ and then, due to the curvature of the field lines, the ray should be at an angle $\theta \approx 1/\gamma_e$ over a distance $\Delta s \approx \rho_c/\gamma_e$, where ρ_c is the curvature of the field lines. The total amount of Faraday rotation should be

$$\Delta k\, \Delta s \approx 4 \times 10^{-2}\, n_e \rho_c / f \gamma_e^2. \tag{22}$$

For $\rho_c = 10^8$ cm, $f = 3 \times 10^9$ Hz and $n_e = 3 \times 10^7$ cm^{-3} (roughly the number density required to maintain corotation at 3×10^7 cm), the approximation (22) gives

$$\Delta k\, \Delta s \approx 4 \times 10^4 \gamma_e^{-2}.$$

Thus for $\gamma_e \approx 10^2$ the two modes get significantly out of phase.

The opposite limit where the modes are circularly polarized is not directly relevant to the discussion below. In this limit one has, from equations (6) and (11c),

$$\Delta k \approx (1 - 2\zeta_p)(\omega_{pe}^2/\Omega_e c)\{2 \sin^2(\tfrac{1}{2}\theta) - \tfrac{1}{2}\gamma_e^2\}, \tag{23}$$

where only the electronic contribution is retained, $\omega^2(1 - \beta_e \cos\theta)^2 \ll \Omega_e^2$ is assumed, and a fraction ζ_p of the electron–positron gas is assumed to be positrons. Unlike the more familiar case of a nonrelativistic plasma, the result (23) implies that the rotation of the plane of linear polarization would be independent of frequency.

5. Circular Polarization

As pointed out in the Introduction, a possible way in which the observed circular polarization could arise is as a propagation effect. Suppose the emission is linearly polarized and excites only one of the linearly polarized modes. (The emission by relativistic particles is confined to the forward cone $\theta \lesssim \gamma^{-1}$ where the modes should be nearly linearly polarized.) The radiation remains in this mode, even if the properties of the mode change, e.g. from linearly to significantly circularly polarized, provided that Δk times the characteristic length L over which the wave properties change is much greater than unity. In the opposite limit, i.e. for $\Delta k L \ll 1$, the two modes have insufficient path length to get significantly out of phase, and are said to be strongly coupled. The escaping radiation should be polarized like the natural mode at the region $\Delta k L \approx 1$. In other words, the condition $\Delta k L \approx 1$ determines where the PLR is.

Careful consideration of the parameter L is required. In fact, there is a detailed theory (mode coupling theory) whose purpose can be summarized as the evaluation of L, or equivalently of $\Delta k L$ or the 'coupling ratio' $Q := 1/\Delta k L$. No detailed calculations have been performed to determine L in the present case. The following comments indicate that a detailed calculation is warranted.

Mode coupling occurs only when the properties of the natural modes change along the ray path. The important changes are in the polarization of the natural modes. For linearly polarized modes, the most important changes would be twists in the magnetic field lines. (A 'twisted' field line, as opposed to a 'bent' one, is not confined to a plane.) Twists may not be important well inside the light cylinder. On the other hand, coupling should be most effective when the polarization begins to change due to the axial ratios $T_\pm$ (see equation 8) becoming

significantly different from 0 or ∞. It follows from, say, the expressions (20) and (23) for Δk that one has

$$|g/(a-b)| \approx (1-2\zeta_p)\gamma_e^2\,\omega\theta^4/8\Omega_e\,, \tag{24}$$

which increases very rapidly with radius, e.g. as r^7 for $\Omega_e \propto r^{-3}$ and $\theta \propto r$. When $|g/(a-b)|^2$ becomes nonnegligible (with 'nonnegligible' to be determined by mode coupling theory) the rapid change in the polarization will lead to mode coupling.

The important qualitative point to note is that one expects the PLR to occur just where the natural modes start to become significantly circular. Thus, even for a linearly polarized emission mechanism, significant circular polarization is to be expected, provided the PLR occurs well inside the light cylinder.

A possible alternative cause of circular polarization is gyromagnetic absorption (I. Lerche, personal communication). The idea is that in the region where the wave frequency equals the Doppler shifted gyrofrequency, i.e. where the equality $\omega(1-\beta_e\cos\theta) = \Omega_e$ $(= eB/\gamma_e mc)$ obtains, radiation of one handedness will be preferentially absorbed (provided that the numbers of electrons and positrons are not equal there). Such absorption would occur further from the neutron star than where the PLR is assumed to occur in the above discussion.

6. 'Orthogonal Modes'

The suggestion that the observation of 'orthogonal modes' of polarization in pulsars can be explained by splitting of the emitted signal into two natural modes requires that the two ray paths be significantly different. For waves in an anisotropic medium, the ray direction and the wave-normal direction $\boldsymbol{\kappa}$ are different. It can be shown that the angle between the two rays (for $n \approx 1$) is given by

$$\Delta\theta_r = \frac{\partial(n_+ - n_-)}{\partial\theta} = \frac{c}{\omega}\frac{\partial\Delta k}{\partial\theta}. \tag{25}$$

Using the result (21), it follows that the greatest angle between the rays occurs at $\theta \approx 1/\gamma_e$. It is plausible that the angle between the escaping rays will be of the order of this maximum angle. (A detailed ray tracing would be required to substantiate, or disprove, this suggestion.) Assuming this to be the case, the angle between the escaping rays would be

$$\Delta\theta_r \approx 4(\omega_{pe}^2/\omega^2)\gamma_e \approx 4\times10^8\, n_e/f^2 \quad \text{rad}. \tag{26}$$

With $n_e = 3\times10^7\ \text{cm}^{-3}$ and $f = 10^9$ Hz, the result (26) implies a separation of 10^{-2} rad, which is a significant separation ($\sim\frac{1}{2}°$) in the application to pulsars.

An obvious feature of the result (26) is that the separation (in angle) of the rays depends only on the ambient electron density and the frequency. If all frequencies came from the same region, one would expect the observed properties of 'orthogonal modes' to change rapidly as a function of frequency. However, from the change in the pulse separation with frequency in pulsars with double or multiple pulses, it seems that lower frequencies must come from greater heights. In view of this, n_e at the point of emission probably decreases with decreasing f, and n_e/f^2 may not decrease with increasing frequency as might otherwise be expected.

A systematic study of the variation of 'orthogonal modes' could provide useful information, especially if any significant trend is found. The foregoing discussion indicates that any systematic variation in the occurrence of 'orthogonal modes' with frequency should correlate with the pulse separation in double-pulsed pulsars.

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Appendix

The dielectric tensor for an electron plasma in the long wavelength limit was calculated by Svetozarova and Tsytovich (1962) and by Melrose (1974). In units with $\hbar = c = 1$ and with

$$\varepsilon_n := (m^2 + p_z^2 + 2neB)^{\frac{1}{2}} \tag{A1}$$

and

$$f_n^{\mathrm{S}} := n^+(\varepsilon_n) + n^-(\varepsilon_n), \qquad f_n^{\mathrm{D}} := n^+(\varepsilon_n) - n^-(\varepsilon_n), \tag{A2a, b}$$

where n^+ and n^- are the occupation numbers of electrons and positrons respectively, the result given by Melrose (1974) for the polarization tensor α_{ij} takes the form ($\varepsilon_{ij} =: \delta_{ij} + 4\pi\alpha_{ij}/\omega^2$)

$$\alpha_{11} = \alpha_{22} = \sum_{n=0}^{\infty} \frac{e^3 B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi} \left\{ (f_{n+1}^{\mathrm{S}} - f_n^{\mathrm{S}}) \frac{\varepsilon_{n+1} - \varepsilon_n}{\omega^2 - (\varepsilon_{n+1} - \varepsilon_n)^2} \left(1 - \frac{m^2 + p_z^2}{\varepsilon_n \varepsilon_{n+1}} \right) \right.$$
$$\left. + (f_{n+1}^{\mathrm{S}} + f_n^{\mathrm{S}}) \frac{\varepsilon_{n+1} + \varepsilon_n}{\omega^2 - (\varepsilon_{n+1} + \varepsilon_n)^2} \left(1 + \frac{m^2 + p_z^2}{\varepsilon_n \varepsilon_{n+1}} \right) \right\}, \tag{A3a}$$

$$\alpha_{12} = -\alpha_{21} = \sum_{n=0}^{\infty} \frac{-\mathrm{i}e^3B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi} \left\{ (f_{n+1}^{\mathrm{D}} - f_n^{\mathrm{D}}) \frac{\omega}{\omega^2 - (\varepsilon_{n+1} - \varepsilon_n)^2} \left(1 - \frac{m^2 + p_z^2}{\varepsilon_n \varepsilon_{n+1}}\right) \right.$$

$$\left. + (f_{n+1}^{\mathrm{D}} - f_n^{\mathrm{D}}) \frac{\omega}{\omega^2 - (\varepsilon_{n+1} + \varepsilon_n)^2} \left(1 + \frac{m^2 + p_z^2}{\varepsilon_n \varepsilon_{n+1}}\right) \right\}, \tag{A3b}$$

$$\alpha_{33} = \sum_{n=0}^{\infty} \frac{4e^3B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi} \left\{ f_{n+1}^{\mathrm{S}} \frac{\varepsilon_{n+1}}{\omega^2 - 4\varepsilon_{n+1}^2} \left(1 - \frac{p_z}{\varepsilon_{n+1}^2}\right) + f_n^{\mathrm{S}} \frac{\varepsilon_n}{\omega^2 - 4\varepsilon_n^2} \left(1 - \frac{p_z^2}{\varepsilon_n}\right) \right\}, \tag{A3c}$$

with $\alpha_{ij} = 0$ otherwise. For $n \ll B_{\mathrm{c}}/B$, where $B_{\mathrm{c}} := m_{\mathrm{e}}^2/e = 4{\cdot}4 \times 10^{13}$ G is the 'critical' magnetic field, the approximation

$$\varepsilon_n = \varepsilon_0 + n\Omega, \qquad \Omega := eB/\varepsilon_0 \tag{A4a, b}$$

applies. Inserting equations (A4a) and (A4b) in (A3a)–(A3c) gives

$$\alpha_{11} = \alpha_{22} = -\frac{e^3B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi\varepsilon_0} \frac{\omega^2}{\omega^2 - \Omega^2} \left(f_0^{\mathrm{S}} + 2 \sum_{n=1}^{\infty} f_n^{\mathrm{S}}\right), \tag{A5a}$$

$$\alpha_{12} = -\alpha_{21} = \frac{\mathrm{i}e^3B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi\varepsilon_0} \frac{\omega\Omega}{\omega^2 - \Omega^2} \left(f_0^{\mathrm{D}} + 2 \sum_{n=1}^{\infty} f_n^{\mathrm{D}}\right), \tag{A5b}$$

$$\alpha_{33} = -\frac{e^3B}{2\pi} \int \frac{\mathrm{d}p_z}{2\pi\varepsilon_0^3} m^2 \left(f_0^{\mathrm{S}} + 2 \sum_{n=1}^{\infty} f_n^{\mathrm{S}}\right), \tag{A5c}$$

where intrinsically quantum corrections (of order ω/m or ω/ε_0) have been neglected. Irrespective of whether the distribution of Landau orbitals is degenerate or nondegenerate, one may define a one-dimensional distribution

$$F(p_z) := \frac{eB}{(2\pi)^2} \left(f_0 + 2 \sum_{n=1}^{\infty} f_n\right), \tag{A6}$$

which is the classical one-dimensional distribution in the classical limit. (The state $n = 0$ is the ground state and corresponds to a specific spin helicity, whereas the states with $n > 1$ are doubly degenerate due to the two spin states.) The cold plasma result follows for $F(p_z) = n_{\mathrm{e}}\,\delta(p_z)$, and the result used in equations (7a)–(7c) (cf. Melrose and Stoneham 1977) can then be derived by a Lorentz transformation (Melrose 1973).

Note that the distribution need not be one-dimensional ($f_n = 0$ for $n > 0$) to lead to the cold plasma dielectric tensor. Any distribution of Landau orbitals up to $n \approx B_{\mathrm{c}}/B$ leads to the cold plasma result in the long wavelength limit. The condition $n < B_{\mathrm{c}}/B$ corresponds classically to $p_\perp < m_{\mathrm{e}}\,c$, that is, the cold plasma dielectric tensor applies for any nonrelativistic distribution of perpendicular momentum.

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