Constrained voxel inversion using the Cartesian cut cell method

Robert G. Ellis
Geosoft Inc.
Toronto, ON, Canada
Robert.Ellis@Geosoft.com

Ian N. MacLeod
Geosoft Inc.
Toronto, ON, Canada
Ian.MacLeod@Geosoft.com

SUMMARY

Voxel inversion is a well-established method for constructing a physical property model from geophysical data. However, a limitation on Cartesian voxel inversion is that the voxel earth model is restricted to prism shaped elements which do not conform to geology and can lead to numerical artefacts. Octree and unstructured meshes have been used to overcome this limitation, but both add significantly to either the number of voxel elements, or to the complexity of the representation. We propose an alternative method which maintains much of the simplicity of the regular Cartesian voxel model while allowing very accurate geometric representation of geological surfaces: the Cartesian cut cell (CCC) method.

A significant geologic surface in most voxel inversions is the topography, which is very poorly represented by voxels. There are many other common geologic sub-surfaces including faults, contacts, unconformities, mineralized zones, etc. We demonstrate the necessity of improving the conventional voxel representation to ensure accurate geophysical modelling and how this is achieved with the CCC method. We also show the value of the CCC method in constrained inversion. These examples demonstrate that the CCC method allows accurate representation of geologic surfaces with a minimal extension to the simplicity of a conventional voxel model.

Key words: modelling, inversion, Cartesian Cut Cell, geological surfaces.

INTRODUCTION

Voxel based modelling and inversion has become a familiar tool over the last two decades, due to dramatically reduced computing costs, and as the exploration industry attempts to interpret increasingly complex geophysical data associated with deeper targets. The simplicity of voxel based representations of the earth makes them appealing. They are easily understood and easily programmed, easily rendered, and conform to current computer architectures. However, they have one significant shortcoming: they are not well suited to representing the geological features common in exploration projects because geological features are often described by surfaces. For example, voxel models provide very poor representation of topography; one of the most critical surfaces in geophysical modelling and inversion because it represents a significant contrast in physical properties and is generally relatively close to the geophysical sensor. Voxel models also provide only a coarse approximation to any sort of subsurface contact: faults, ore zones, unconformities, top of basement, etc. Of course, reducing the size of the voxel elements improves their approximation to geology, however, the computational requirements increase as a power of the number of voxels making this brute force approach impractical.

A number of researchers have worked with unstructured or octree meshes to overcome the limitations of Cartesian voxel models, for example, Lelièvre *et al*. (2012), and Haber and Heldmann (2007), amongst others. In this work, we present a Cartesian cut cell (Ingram *et al*., 2003) based alternative which has a number of benefits: simplicity, accuracy, and conformity with existing voxel based methods. Our Cartesian cut cell implementation uses a predominantly Cartesian grid for the majority of the earth model with special treatment applied to a relatively small number of voxels cut by surfaces. The special treatment has been greatly simplified by the development of modern computer geometry libraries, for example, CGAL, and can be added to existing voxel based algorithms. Maintaining a predominantly Cartesian representation allows reuse of existing voxel based algorithms for rendering and model analysis. In the following Sections we demonstrate the importance and advantages of the CCC method for topography representation, for ore body delineation, and in constrained inversion.

METHOD AND RESULTS

Our implementation of the CCC method is most easily understood by considering a simple example. Consider, without loss of generality, the deposit model developed for the San Nicolas deposit in central Mexico (CAMIRO Project 01E01). The ore zone is delineated and represented by a triangulated surface, which we have simplified for display, and shown in Figure 1a. The voxel representation of the triangulated ore zone is shown in Figure 1b and the triangulation and the voxel representations are combined in Figure 1c for comparison. Our CCC method operates by cutting every voxel intersected by the ore surface into two polyhedral sub-volumes; either ore zone or host rock, and assigning the corresponding physical properties. Figure 2 shows a slice through the CCC representation highlighting the sub-volumes formed by cutting with the ore surface. For our geophysical computations, all that remains is to compute the survey response from the polyhedral sub-volumes, which is only a little more involved than computing the response from prisms. It should be noted that our CCC representation provides an exact representation of the triangulated ore zone, independent of the voxel size.

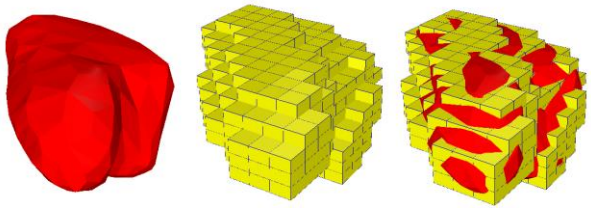

Figure 1(a) The simplified ore zone surface from the San Nicolas deposit model, (b) its voxel representation, (c) the surface embedded in the voxel representation.

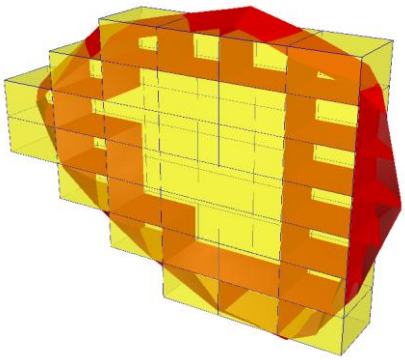

Figure 2. A slice through Figure 1(c) showing the polyhedral sub-volumes (red) resulting from the CCC method, together with the corresponding uncut voxels (transparent yellow).

Cartesian Cut Cell Method and Topography

Any geological surface can be accurately represented by the CCC method and nowhere is the importance of accurate surface representation more essential than in defining the topography. Surprisingly, accurate representation of the topography is important even in areas with very modest variation in topography. To observe this, consider again the San Nicolas deposit, which SRTM data show has very little topography and as can be seen in Figure 3.

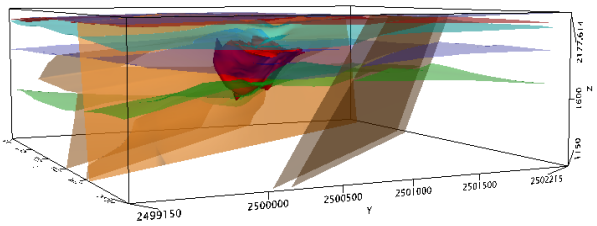

Figure 3. The geological model for the San Nicolas deposit showing the ore zone in red. Volcaniclastic breccia, with susceptibility ~0.005 SI, overlie the ore zone.

In Figure 4a, we see the voxel representation of the topography with the voxels being 20m cubes. The modest topography gives rise to a terraced voxel representation. Next we compute the TMI response from this voxel model with a 30m terrain clearance, Figure 4b. As expected there are significant artefacts in the response, making it difficult to use for geophysical purposes. In Figure 5a, we show the corresponding CCC representation as a wireframe superimposed on the voxel representation. Figure 5b shows the CCC method TMI response, which displays no visible terrain artefacts. Both Figures 4b and 5b are displayed on the same colour stretch and have a range of 25nT. Figure 6 shows the difference between the CCC and voxel responses, and has a range of 10nT, approximately half that of the full response in this case. The surficial volcaniclastic breccia at San Nicolas has a susceptibility of 0.005 SI. The ore zone has susceptibility 0.01 SI.

This example demonstrates that accurate surface representation is required for reliable geophysical modelling, and that the CCC method provides a simple and efficient means of accurately representing geological surfaces.

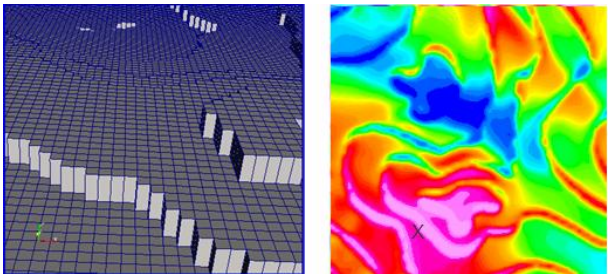

Figure 4(a) A detailed perspective view of the voxel surface approximation in a small but typical part of the terrain at San Nicolas, to illustrate the voxel terracing. Cells are 20m cubes, (b) the voxel TMI response with 25nT colour stretch. The X marks the approximate location of the detailed voxel topography shown in (a).

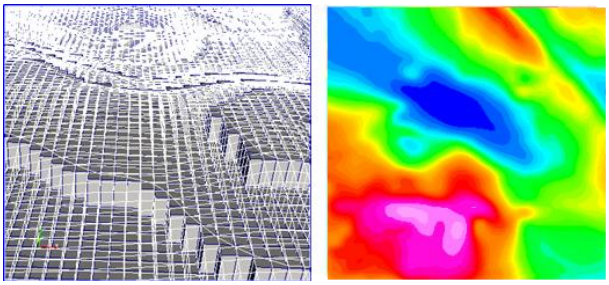

Figure 5(a) The CCC approximation to the terrain at San Nicolas is shown by the wireframe outline with the voxel approximation for reference, (b) the CCC TMI response with same 25nT colour stretch as Figure 4(b).

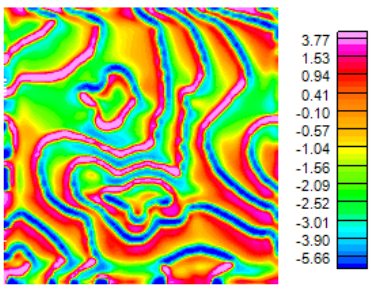

Figure 6. The difference between the CCC (Figure 5b) and voxel (Figure 4b) approximations to the terrain for the TMI response at San Nicolas, in nT.

Cartesian Cut Cell Method and Unconstrained Inversion

We have seen that accurate surface representation in voxel models is required to avoid numerical artefacts in computing

geophysical responses. It is reasonable to expect that accurate surface representation in voxel models is required for accurate inversion. To investigate this we invert TMI data over San Nicolas, first with voxels, and second with the CCC method, using exactly the same parameters for both inversions. The voxel elements are 20m cubes with 30m survey clearance.

Figure 7 shows the result of unconstrained inversion of the TMI data shown in Figure 4b with the voxel approximation to the terrain surface: (a) for a shallow horizontal slice through the model; (b) a deeper slice through the ore zone. Figure 8 shows the corresponding inversion result using the CCC method. Careful comparison of the surface slices in Figures 7a and 8a shows some voxel artefacts when the CCC method is not used, whereas the deeper slice shows no meaningful difference between the voxel and CCC inversions. In fact, the reason that unconstrained voxel inversion has been so successful, in spite of the very poor representation of topography, is that the topography related artefacts appear in the inversion result as surface noise, which is usually ignored in favour of the deeper features normally associated with the inversion target.

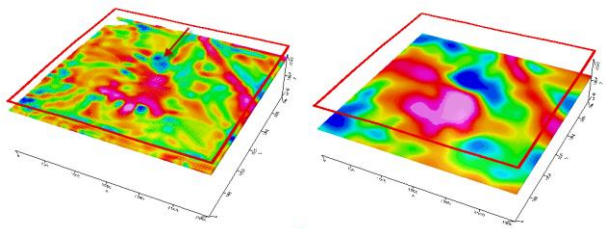

Figure 7. Horizontal slices through the unconstrained voxel inversion result at shallow (100m) and deeper (300m) levels. The arrow indicates an example of surface voxel topography artefacts.

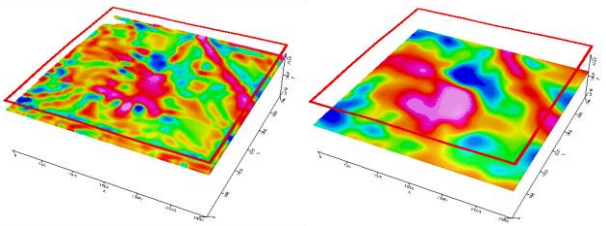

Figure 8. Horizontal slices through the unconstrained CCC inversion result at shallow (100m) and deeper (300m) levels.

Cartesian Cut Cell Method and Constrained Inversion

We have demonstrated that unconstrained inversion is rather tolerant of voxel approximation induced artefacts; however, the situation is very different for constrained inversion where geological surfaces are involved. As an example, consider a constrained inversion of the San Nicolas TMI data. For illustrative purposes, the constraint will be that the surficial volcaniclastic breccia has a susceptibility of 0.005 SI. Leaving all other inputs to the inversion the same as the unconstrained inversion gives the results shown in Figure 9, which show (a) the deeper slice through the inversion for the voxel representation and (b) for the CCC representation. In contrast to the unconstrained inversion, the constrained inversions show a significant difference in the deeper sections of the model between the voxel and CCC methods. The reason for this is that enforcing the volcaniclastic breccia constraint forces the artefacts shown in Figure 6 into the inversion and the inversion cannot compensate by creating near surface noise in the model, due to the constraint. In turn, this means that the inversion is forced to attempt to correct for the surface voxel induced predicted response artefacts by introducing deeper structures imitating the terrain geometry into the model. In general, a higher data misfit must also be allowed in the voxel inversion since the deeper parts of the model cannot adequately reproduce the voxel induced predicted response artefacts. Figure 9b shows that the CCC method, which allows accurate representation of geologic surfaces, correctly resolves the ore zone.

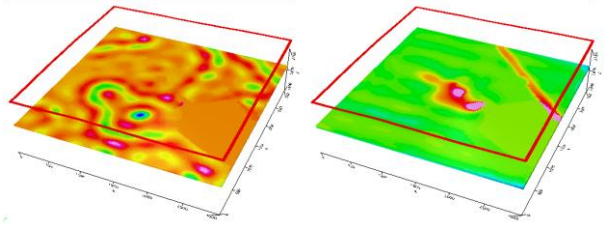

Figure 9a Horizontal slices through the constrained voxel inversion (left) and the constrained CCC inversion result (right) at the deeper level (300m).

CONCLUSIONS

Representing geological surfaces, for example, topography, ore boundaries, faults, unconformities, etc. is challenging for voxel models, unless the voxel sizes are made impracticably small. It is not unrealistic to expect topography voxel artefacts of order 10% of the total response, as was shown using the TMI response from the San Nicolas deposit model, even though it has very little topography. The artefacts will be larger where modest or severe topography is present. Similar effects occur for geological sub-surfaces, particularly when drilling constraints are imposed. To overcome these issues we presented a Cartesian cut cell method which is an accurate but minimal extension of the traditional Cartesian voxel representation. It is minimal in the sense that it leaves the majority of the representation Cartesian and requires special treatment only where geological surfaces cut the voxels. Modern computer geometry algorithms make voxel cutting straightforward. The advantage of a minimal extension is that most display and post processing algorithms can be easily extended to handle CCC representations. Further, since only the surface cutting voxels need special treatment, the CCC method is computationally efficient.

We demonstrated that the CCC method produces artefact free topographic responses and that it improves inversion results over those obtained with a simple voxel representation. In particular, we observed that voxel artefacts when representing geological surfaces are more problematic when using constraints than with unconstrained inversion. This is because unconstrained inversion allows great flexibility in the inversion result and can accommodate the surface artefacts while still producing a minimum structure model. However, when constraints strictly impose the voxel representation of a geological surface, the inversion is seriously degraded unless a better surface representation is used.

We conclude that constrained inversion necessitates accurate representation of the constraint surfaces and that the Cartesian

cut cell method provides an effective and efficient surface representation, and should do so for all geophysical applications.

REFERENCES

CAMIRO Project 01E01, 2004, Detection of Mineral Deposits by Airborne Gravity Gradiometry, Mira Geosciences Ltd., Ontario Geological Survey MRD Release.

CGAL, Computational Geometry Algorithms Library, http://www.cgal.org

Haber, E., and Heldmann S., 2007, An octree multigrid method for quasi-static Maxwells equations with highly discontinuous coefficients: Journal of Computational Physics, 223, no. 2, 783–796.

Ingram, D.M., Causon, D.M., Mingham, C.G., 2003, Developments in Cartesian cut cell methods: Mathematics and Computers in Simulation, 61, 561-572.

Lelièvre, P.G., Carter-McAuslan A., Farquharson C., and Hurich C., 2012. Unified geophysical and geological 3D Earth models: The Leading Edge, 31, 322-328.