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WAVES THAT APPEAR FROM NOWHERE

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ABSTRACT: Oceanic rogue waves belong to a well-established class of phenomena but their study is hindered due to the great danger that they represent. They exist not only at the surface of the open ocean but they also hit coastal areas as well as appear internally in deeper layers of the ocean. The amplitude of the latter may exceed several times the amplitude of rogue waves at the surface. Surface rogue waves in the deep ocean represent threat even for large ocean liners while rogue waves in shallow waters are dangerous for coastal structures. On the other hand, internal rogue waves are hazardous for submarines. The experimental research of all three types of rogue waves is difficult. The theory provides certain degree of understanding of such waves. Some of the recent achievements in this area of research are reviewed in this article.

Keywords: rogue waves, extreme events, mathematical modelling, nonlinear Schrödinger equation, Gardiner equation

INTRODUCTION

Waves that appear from nowhere are waves that we do not expect. Due to the unexpected nature of these events, they are dangerous for people who experience them. Mathematically, this feature can be represented by solutions of evolution equations that are localised both in time and in space. In addition, the amplitudes of these waves are commonly higher than the amplitude of regular waves. The latter feature increases the danger carried by these waves. There is a large variety of rogue waves in nature. Even if we restrict ourselves to oceanic waves, we have to take into account several types. Firstly, there are surface waves on the interface between water and air (Kharif 2009). Secondly, there are internal waves in stratified media such as ocean water with salinity that varies along the vertical direction (Grimshaw et al. 2010). Shallow-water waves are another type of waves that appear in coastal areas (Soomere 2010). Rogue waves present considerable danger as they are rare, presently unpredictable and can impact with tremendous force.

There are a number of accounts of rogue waves in the media, in research articles (Nikolkina & Didenkulova 2011) and those recorded in scientific measurements. One of these occurred near Sydney on Saturday, 9 January 2016, when a giant rogue wave injured 60 people (see http://strangesounds.org/2016/01/giant-rogue-wave-slams-into-swimmers-sydney-australia-video.html). This was an example of a shallow-water rogue wave that hits the coast line.

Another well-remembered example of a similar event occurred on 'Black Sunday', 6 February 1938, when a set of three consecutive rogue waves hit Bondi beach in Sydney, NSW (see https://bondisurfclub.com/the-club/history/black-sunday/). The size of these waves and the degree of their abruptness were such that many people suffered: some 200 swimmers were swept out into the sea; thirty-five unconscious swimmers were revived on the beach; and five people died. To date, it was the largest rescue operation in Australian history triggered by shallow-water rogue waves. Although extremely large waves at coastal areas are relatively rare, every year a few of them do happen around the world. Their careful study may prevent future potential disasters.

Deep ocean areas represent the most common place of occurrence of rogue waves. These are large, unexpected surface-water waves that can be extremely dangerous, even to large ships such as ocean liners and container ships. One known event is the Draupner rogue wave (Cavaleri et al. 2016) which had a measured height of 25.6 metres. Big waves can break windows on the decks of ships and cause damage and injuries to passengers. One example took place in January 2009 in the Bay of Biscay. A cruise liner was hit by 50-foot waves and had to return to Dover, England (see http://www.dailymail.co.uk/news/article-1129302/Pictured-Storm-tosses-massive-cruise-liner-like-toy-boat.html). Other examples can be found in Nikolkina & Didenkulova (2011).

An additional type of oceanic rogue wave is the so-called internal wave, which appears in stratified media such as ocean waters. The stratification is caused by variations in salinity, pressure, temperature or underwater flows. Internal waves propagate along the layers of stratification and are invisible at the surface. The amplitude of internal rogue waves can be as large as 170 metres (Alford 2015), which is significantly larger than the amplitude of rogue waves at the water surface. Such internal waves may shift submarines to a depth where pressure exceeds the capacity of the hull. An internal wave may have caused the KPI Nanggala 402 submarine disaster on 21 April 21 2021 (see https://www.bbc.com/news/world-asia-56871694). Indonesian Navy officials believe that an internal rogue wave is a more likely explanation for the submarine disaster than other theories put forward after the incident.

Varieties of oceanic rogue waves include but are not limited to the three basic types that are illustrated above. Rogue waves do exist and require careful analysis in order to understand such phenomena and to develop techniques for preventing disasters. The very first step in this direction is modelling water waves based on partial differential equations. High amplitudes of these waves suggest that the corresponding equations are intrinsßically nonlinear (Osborne 2010). The next step is finding the solutions of these equations that describe the extreme waves. As rogue waves are unexpected, the solutions that describe them must be localised both in time and in space. Below, each type of rogue wave that has been illustrated above is considered separately, starting with the most common, those that occur on the water surface above deep areas of the world oceans.

DEEP-WATER ROGUE WAVES

Deep-water unidirectional surface waves can be described using the nonlinear Schrödinger equation (NLSE) (Osborne 2010; Zakharov 1968). It is written here in dimensionless form,

$$i\frac{\partial\psi}{\partial x}+\frac{1}{2}\frac{\partial^2\psi}{\partial t^2}+|\psi|^2\psi=0$$

where ψ is the normalised wave envelope of the water surface elevation, x is the normalised distance along the water surface and t is the normalised retarded time. For unidirectional propagation, one-dimensional modelling of waves is the simplest and most illustrative way of describing them. The model assumes that the wave and its envelope are propagating the same way. However, we should keep in mind that due to the existence of the second dimension of the water surface, this is not always the case (Chabchoub et al. 2019).

One of the solutions of the NLSE is known as Peregrine breather:

$$\psi=\left[1-4\frac{1+2ix}{1+4x^2+4t^2}\right]\exp(ix)$$

This is a unique rational solution of the NLSE. The modulus of this solution is shown in Figure 1. The main peak of the solution bulges on a homogeneous background and is localised both in time and in space. The amplitude of the main peak is three times the amplitude of the background. As ψ is the envelope of waves, this means that the wave amplitude at the maximum is 3 times the amplitude of surrounding waves. This feature of the solution fits perfectly the definition of rogue waves (Akhmediev et al. 2009). In other words, this solution is a likely candidate for the description of oceanic rogue waves. Experimental rogue waves have been observed in a water tank (Chabchoub et al. 2011).

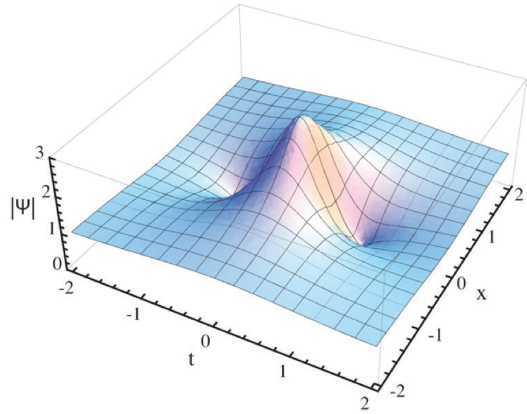

Figure 1: First-order rogue wave solution of the NLSE with the maximal amplitude equal to 3.

Peregrine waves are not the only type that fits the definition of rogue waves. There is an infinite family of higher-order rational solutions with progressively increasing amplitudes 5, 7, 9, 11, … etc. (Akhmediev et al. 2009). These solutions are more complicated, although all of them are rational and all of them can be considered as candidates for even stronger rogue waves than the lowest order one. For example, the second-order rogue wave solution is shown in Figure 2. The main peak here is higher and narrower than in the case of the first-order solution. Thus, it could be more dangerous than the first-order rogue wave. Higher-order rogue waves have also been observed experimentally (Chabchoub et al. 2012a, 2012b).

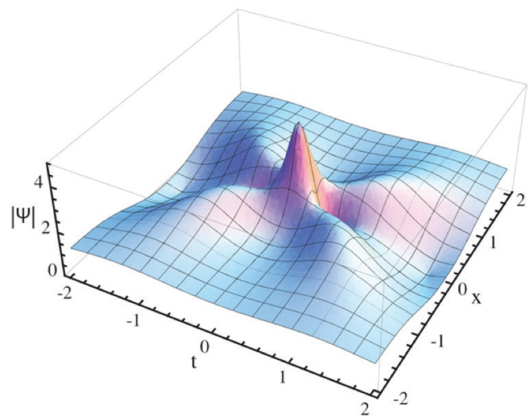

Figure 2: Second-order rogue wave solution of the NLSE with the maximal amplitude equal to 5.

INTERNAL ROGUE WAVES

The term ‘internal rogue wave’ has been coined by Grimshaw et al. (2010). It was also suggested in Grimshaw et al. (2010) that in certain approximations internal waves could be modelled using the Gardner equation (GE):

$$\psi_x + \psi\psi_t + \psi^2\psi_t + \psi_{ttt} = 0$$

The meaning of variables in the GE is similar to those in the case of the NLSE. The function ψ here describes the vertical displacement of water particles, x is the distance along the stratification, while t is the retarded time.

There are a number of other ways to model mathematically nonlinear waves in layered media (Apel et al. 2007; Grimshaw et al. 2007; Zheng 2002). These techniques vary in complexity. The convenience of using the GE is in dealing with a single partial differential equation as in the above case of deep water waves. Then the solutions of the GE localised both in x and t would describe rogue waves.

Following Bokaeeyan et al. (2019), we assume that the rogue wave solutions of this equation are its rational solutions, as in the case of the NLSE. However, in contrast to the NLSE case, this is not the first-order solution. We give here the third-order solution that exemplifies the internal rogue wave:

$$\psi_3 = 72\frac{G_3}{D_3} - 1$$

where

$$G_3 = t^{10} + 90t^8 + 7560t^5x + 5400t^4\left(x^2 - 18\right) + 259200t^3x$$
$$-32400t^2\left(2x^2 + 27\right) + 43200tx\left(x^2 + 54\right) + 194400\left(5x^2 + 27\right)$$

and

$$D_3 = t^{12} + 36t^{10} + 120t^9x + 4860t^8 + 2160t^6\left(x^2 + 234\right)$$
$$-233280t^5x + 97200t^4\left(2x^2 + 45\right) - 86400t^3x\left(x^2 - 108\right) -$$
$$-3499200t^2\left(x^2 - 27\right) + 777600tx\left(2x^2 - 135\right) +$$
$$+129600\left(4x^4 + 594x^2 + 729\right)$$

This solution is illustrated in Figure 3. It also has a high-amplitude central peak at the origin equal to 3. However, the background of this solution is not homogeneous, as in the case of the NLSE rogue wave solution. There are tails with lower amplitude that extend to infinity. Thus, excitation of such rogue waves requires specific initial conditions. More details can be found in Bokaeeyan (2019).

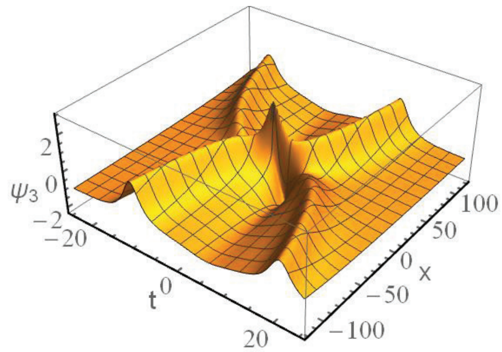

Figure 3: Third-order rogue wave solution of the GE with the maximal amplitude equal to 3.

SHALLOW-WATER ROGUE WAVES

There are various approaches to shallow-water rogue waves. Here, the work of Ankiewicz et al. (2019) is followed, based on solutions of the complex Korteveg-de Vries (KdV) equation.

$$q_x + 6qq_t + q_{ttt} = 0$$

The KdV equation is well known in the theory of shallow water waves (Korteweg & De Vries 1895). However, the function q in this equation is commonly considered to be a real variable that is responsible for water level elevation. This equation was the main tool for developing the theory of solitons, or solitary waves (Zabusky & Kruskal 1965). The real KdV equation does not have rogue wave solutions and it is assumed that the function q is complex (Ankiewicz et al. 2019). When this is case, the KdV equation acquires rogue wave solutions. They are also rational solutions, with the main peak localised both in space and time. However, this is not the first-order solution. Only higher-order rational solutions start to have rogue wave features, for example, the third-order rogue wave solution of the complex KdV has the form:

$$q_3 = 48\frac{N_3}{F_3^2} - 1$$

where

$$N_3 = T^{10} - 10T^9\left(X - i\right) + 45T^8\left(X - i\right)^2 - 120T^7\left(X - i\right)^3 +$$
$$+210T^6\left(X - i\right)^4 - 36T^5\left(7X^5 - 35iX^4 - 70X^3 + 70iX^2 + 35X + 5i\right)$$
$$+30T^4\left(7X^6 - 42iX^5 - 105X^4 + 140iX^3 + 185X^2 + 110Xi + 45\right) -$$
$$-120T^3\left(X^7 - 7iX^6 - 21X^5 + 35iX^4 + 115X^3 + 15iX^2 + 125X - 15i\right) +$$
$$+45T^2\left(X - i\right)^2\left(X^6 - 6iX^5 - 15X^4 + 20iX^3 + 335X^2 + 410iX + 15\right) -$$
$$-10T(X^9 - 9iX^8 - 36X^7 + 84iX^6 + 1086X^5 - 1830iX^4 - 740X^3 +$$
$$-4380iX^2 - 135X + 135i) + X^{10} - 10iX^9 - 45X^8 + 120iX^7 + 2619X^6 -$$
$$-7020iX^5 - 16250X^4 - 13000iX^3 - 17475X^2 - 5850iX - 2025$$

and

$$F_3 = T^6 - 6T^5(X-i) + 15T^4(X-i)^2 -$$
$$-20T^3X(X^2-3iX-5) + 15T^2(X-i)(X^3-3iX^2-11X-3i)$$
$$-6T(X^5-5iX^4-30X^3+40iX^2+5X-15i) + X^6 - 6iX^5 -$$
$$-55X^4+120iX^3-245X^2-450iX-45$$

The modulus of the complex q-function for this solution is shown in Figure 4. It has a large peak at the centre equal to 49. The peak is localised both in x and t directions which is the characteristic of a rogue wave. The basic background of the solution is 1 while there are tails on this background extending to infinity. More details can be found in Ankiewicz et al. (2019).

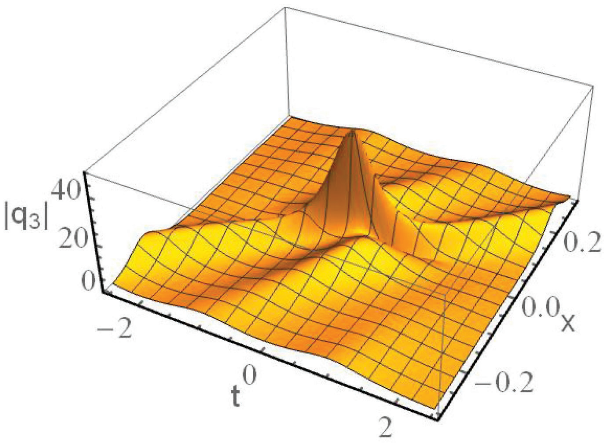

Figure 4: The modulus of the third-order rogue wave solution of the complex KdV equation.

CONCLUSIONS

Taking into account the large variety of rogue waves that exist in nature, there is no universal way of describing them. Only three types of rogue waves and the simplest ways of modelling them that provide us with major features of the phenomenon have been considered here. Clearly, the models can be improved in many ways. Even if we restrict ourselves to oceanic surface rogue waves, their modelling can be made more accurate extending the NLSE to its modified versions. There are several extensions known today (Dysthe 1979; Sedletsky 2003; Slunyaev 2005). Correspondingly, the solutions of the NLSE also have to be modified in order to obtain more accurate descriptions. More realistic models must take into account the two-dimensional feature of oceanic wave propagation. This has been done, for example, by Onorato et al. (2006) and Chabchoub et al. (2019).

Mathematical modelling of internal rogue waves and shallow water rogue waves can also be improved. Ideas presented in this review article are only the beginning of research efforts in this direction. Developing these ideas is of practical interest. Understanding and timely predicting the appearance of rogue waves may save lives and reduce the damages that they cause. Continuing this research will also contribute to understanding similar waves in optics and other branches of science.

Conflict of interest

The author declares no conflict of interest.

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