Probabilistic inversion of airborne electromagnetic data for a multidimensional earth

Juerg Hauser
CSIRO ESRE, Australia
Juerg.Hauser@csiro.au

David Annetts
CSIRO ESRE, Australia
David.Annetts@csiro.au

James Gunning
CSIRO ESRE, Australia
James.Gunning@csiro.au

SUMMARY

The inversion of airborne electromagnetic data is inherently non-unique, especially when data uncertainties are taken into account. If one model can be found that fits the data, then it is likely that there are alternative models that fit the data equally well. The probabilistic approach introduced in this work therefore aims at exploring the posterior distribution which is the distribution of models that are in agreement with both the prior information and the data. We quantify the prior information using geostatistics and use a Markov Chain Monte Carlo technique to sample the unknown posterior distribution. Data are predicted taking lateral changes in structure along the flight path into account by employing a 2.5D forward solver. A case study using the Harmony Ni-S deposit in Western Australia shows that our set of samples of the posterior distribution provides a more complete picture of solution space than what can be achieved by non-linear iterative inversion schemes that have previously been employed. Such a picture of the subsurface can ultimately be used to mitigate exploration risk.

Key words: airborne, electromagnetic, probabilistic, inversion, 2.5D.

INTRODUCTION

Imaging the subsurface using airborne electromagnetic data is often based on 1D modelling of the recorded data. That is, a 1D model of resistivity is derived for each sample point along the flight path, potentially using lateral constraints to guide the inversion (e.g. Brodie and Sambridge, 2006, Viezzoli et al., 2008). The commonly-employed deterministic inversion methods address the non-uniqueness of the airborne electromagnetic inverse problem using some form of regularisation (e.g. Constable and Parker et al., 1987, Zhdanov and Tartaras, 2002). Choosing the best values for these regularisation parameters is often an arbitrary process, and their impact on model uncertainty is difficult to estimate. Regularisation parameters are also not easily related to geological information.

The assumption of a 1D earth is considered a valid approximation for a three dimensional geological structure in the centre of large structures such as a floodplain. In the presence of a true 3D geological structure, a 1D approximation is likely to no longer be sufficient for the forward problem. We note that the concept of a moving footprint (e.g. Zhdanov and Tartaras, 2002; Cox et al., 2010) makes it feasible to perform deterministic inversions in 3D.

Deterministic inversions aim at finding a single best fitting model, compromising between the data and the regularisation. They provide little information about model uncertainties. In contrast, Bayesian approaches try to find the ensemble of models that are in agreement with prior information and the data, thus allowing one to estimate model uncertainties by analysing the posterior distribution. They have seen application for the inversion of marine controlled source electromagnetic data (e.g. Gunning et al., 2010; Buland and Kolbjørnsen, 2012), and for airborne electromagnetic data (e.g. Minsley, 2011) using a 1D approximation for the forward problem.

In this work we combine a Markov Chain Monte Carlo algorithm with a 2.5D forward solver. Given a flight path perpendicular to a change in structure, a 2.5D approximation is a clear improvement over a 1D approximation and likely to be sufficiently accurate for conductive long strike length bodies, without requiring the long run times of 3D models. The success of this strategy depends on two capabilities; firstly, rapid forward model computations, and secondly, an efficient Markov Chain Monte Carlo algorithm with a model proposal technique that proposes models that are likely to be accepted while exploring model space sufficiently.

We begin this manuscript by describing our model parameterisation and model proposal techniques. After a synthetic example, the framework is applied to the inversion of airborne electromagnetic data of the Harmony Ni-S deposit in Western Australia. We conclude by emphasising the potential of the proposed technique as a complimentary method to more commonly employed frameworks for the interpretation of airborne electromagnetic data.

METHOD

One way to address the non-uniqueness of the inversion of airborne electromagnetic data is to reduce the number of model parameters and to constrain some of them to known values. In this work, we invert for the probability of different lithologies in each cell of the model, where each lithology has a distinct resistivity that has been chosen a priori. Such a model parameterisation leads to fewer model parameters to be constrained when compared to directly inverting for resistivity values and layer thicknesses.

A Markov Chain Monte Carlo (MCMC) algorithm (Mosegaard and Tarantola, 1995) is a technique to sample the posterior distribution arising from the data-mismatch likelihood and prior distribution (Gelman, et al., 1995). It is a derivative of the Metropolis algorithm (Metropolis et al., 1953) and uses a Markov Chain process to control the sampling of the model space. MCMC algorithms sample the model space at a rate proportional to the posterior probabilities, a process known as importance sampling,

thereby empirically reconstructing the unknown posterior distribution. They achieve this by moving through model space according to the posterior probabilities for these models, essentially performing a guided search and focusing on regions that better fit the prior information and the data. A large acceptance rate and significant model jumps are critical for the success of a MCMC algorithm. The forward model, particularly in more than one dimension, is strongly non-linear and any objective function is likely to be multimodal, unfavourably scaled and complex in shape. If prior information conforms with information in the data, the difficulty of the inverse problem tends to decrease.

We employ SNESIM (Strebelle, 2000), a multiple-point statistics algorithm, to define the prior distribution. Previously Cordua et al. (2012) used SNESIM successfully in their probabilistic inversion of cross-hole ground penetrating radar data. SNESIM is a single normal equation simulation algorithm employing a sequential approach. The multiple-point statistics information is inferred from a training image that is used to build a pattern database. New realisations are generated by simulating values for each node by following a random path through the model visiting all nodes. Each simulated value is obtained by randomly choosing one template from the set of potential templates in the database that observe the conditioning data and values generated previously in the sequence.

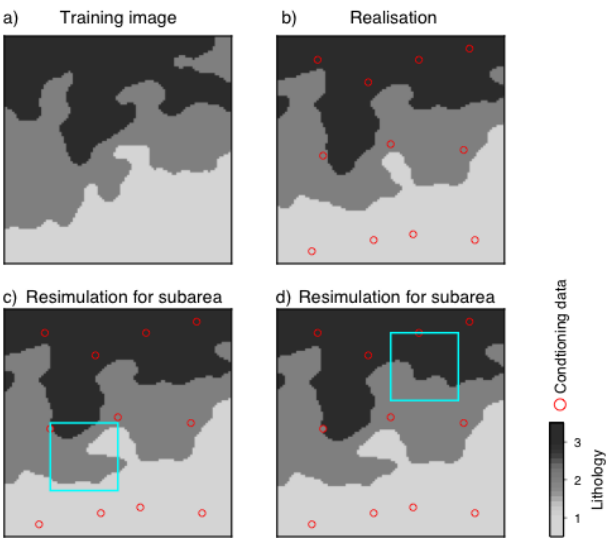

Figure 1. A template database is constructed from a training image a). Using the templates and conditioning data a new realisation b) can be generated. Lithology distributions for different subareas (blue squares) in c) and d) can be regenerated, while being conditional on the current model and the conditioning data.

Figure 1 illustrates how SNESIM can be used to propose new models given a training image and conditioning data. The training image is shown in Figure 1.a and an initial realisation generated using SNESIM, observing the conditioning data, is shown in Figure 1.b. A perturbation of the current model can now be obtained by generating a new realisation only for a subarea of the model while being conditional on the rest of the model and the conditioning data (c.f. Figure 1.c and 1.d). By varying the size of the subarea, Cordua at al. (2012) control the amount of perturbation and thereby which models are accessible from the current model in their MCMC algorithm.

We employ the sampling strategy outlined by Cordua et al. (2012) with the addition of a procedure to preferentially target areas of large misfit along the flight path when choosing a model subarea to perturb. For a simple training image, such model jumps are reversible and do not require probability compensation in the Metropolis Hastings framework. One of the advantages of using geostatistical concepts to describe prior information is that constraints for the prior distribution, for example boreholes or seismic horizons, can easily be taken into account when proposing models in the form of conditioning data.

The forward problem is solved using ArjunAir, a 2.5D frontal solver previously described by Sugeng and Raiche (1997) and Wilson et al. (2006). For a proposed lithological model, the corresponding resistivity distribution is created and interpolated onto a computational grid. To predict the data efficiently, the original serial code was parallelised allowing to currently speed up the forward computations by a factor of 60. In practice, this means that we can solve the inverse problem for a 2.0 km line in less than 48 hours using the MCMC approach developed in this work. This corresponds to about 10000 iterations.

RESULTS

Synthetic example

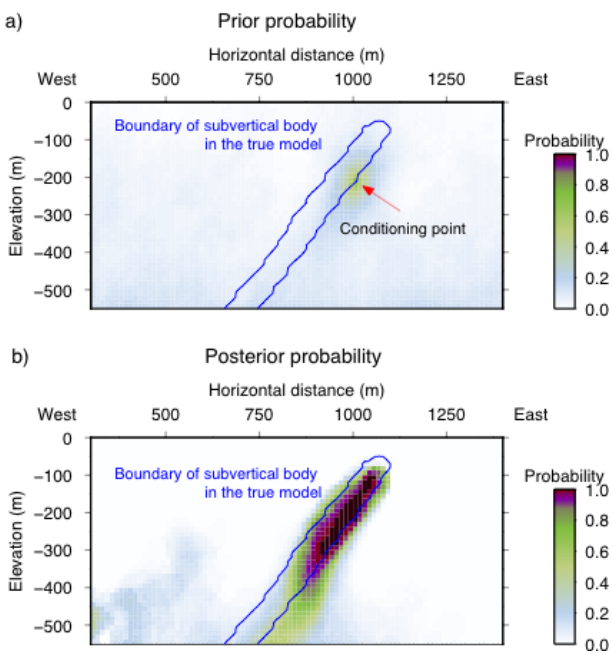

Figure 2. Prior and posterior probability for the lithology of the subvertical body in the synthetic example, averages over 4000 realisations.

In a first synthetic test, we invert for the position of a subvertical body (1 Ωm) embedded in a resistive bedrock (750 Ωm). The airborne electromagnetic system used for this synthetic example is a 25 Hz GEOTEM system with a flight direction from west to east. The data we are trying to fit are the off time channels for the vertical and inline component. The system parameters are given by the ones used to map the Harmony Ni-S deposit, as described in Wolfgram and Golden (2001) and Annetts et al. (2003). The training image consists of a set of westward dipping subvertical bodies with variable

dip angle, width and depth to the top of the subvertical body. Alternatively, a training image could be obtained by forward modelling the formation of subvertical bodies using the range of conditions that are plausible for the deposit. Initially, the subarea where the model is perturbed covers the entire model. To speed up burn-in, we exploit the increased probability of a conductor near the centre of the model by placing a conditioning point in the centre of the model. The influence of this point wanes after burn-in. Such information might be obtained from 1D inversions, qualitative analysis of the data or a simple grid search testing a few positions for the location of a conductor. For the synthetic example we intentionally position the conditioning point on the edge of the true subvertical body.

Figure 2 compares the prior probability for the subvertical body with the posterior probability. There is an increased prior probability near the conditioning data and low probability covering the entire model. The posterior probability shows that the subvertical body's position has been recovered well for the depth range between 100 m and 300m. For depths of more than 400 m the posterior probability for the subvertical body is significantly reduced and at a depth of 500 m we see a low probability for a mis-located subvertical body. This decrease in probability with increasing depth is due to the well-known limited depth of investigation of airborne electromagnetic systems.

Harmony Ni-S deposit

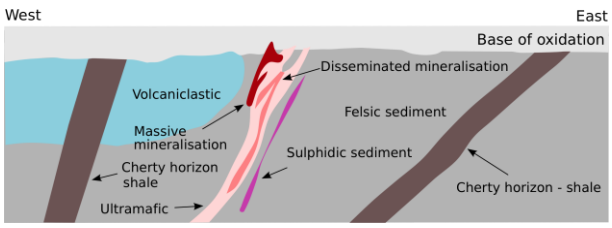

Figure 3. Schematic geological cross-section across the Harmony Ni-S deposit after Wolfgram and Golden (2001); note that the detailed structure of the ultramafic zone is based on drilling information.

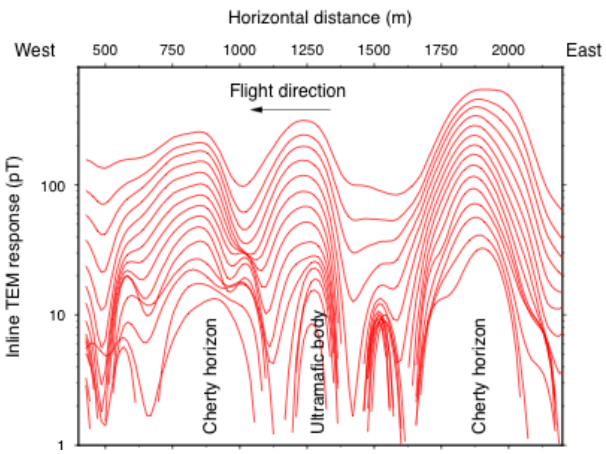

Figure 4. Inline component for the Harmony Ni-S deposit. The data were collected in 1998 using a 25Hz GEOTEM system and are plotted as a function of receiver position.

The Harmony Ni-S deposit in Western Australia, consists of a linear subvertical massive ultramafic body between two conductive cherty horizons (Figure 3). Modelling of the airborne electromagnetic data by Wolfgram and Golden (2001) as well as Annetts et al. (2003) has focused on using a model consisting of three westward dipping conductors with conductivities below 0.25 Ωm, embedded in a 50 Ωm half space, where the lowest resistivity is assigned to the central subvertical massive ultramafic body.

In the initial stages of an exploration program, interest centres on the probability of lithologies with low resistivities at economically minable depths, and detecting targets with mid-range resistivities is of less interest. We therefore attempt to map out the probability for a high conductor (0.1 Ωm) and refer to this lithology in the following as the target lithology.

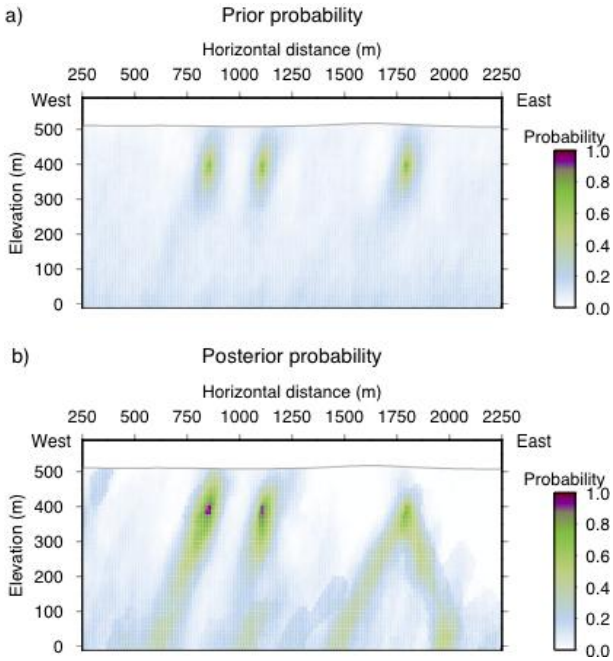

Figure 5. Prior and posterior probability for a low resistivity (0.1 Ωm) target lithology in the Harmony Ni-S deposit, averages over 3500 realisations. The dark grey line is the surface.

Three conditioning points are used to speed up burn-in. The prior distribution shows an increased probability for the target lithology, where one would expect it based on the field data (Figure 4). Figure 5 compares the prior and posterior distribution. The central target and the western-most cherty horizon are well recovered. The eastern-most cherty horizon has not been well recovered in the posterior distribution. This might be due to the choice of a low resistivity of 0.1 Ωm for our target lithology while Wolfgram and Golden (2001) and Annetts et al (2003) used for the eastern most cherty horizon a resistivity of 0.25 Ωm. We also note that the posterior probability for the eastern-most cherty horizon indicates that an eastward dipping structure is as likely as a westward dipping structure. As the depth increases, the probability for the target lithology again decreases.

CONCLUSIONS

The synthetic results and the results for the Harmony Ni-S deposit demonstrate that given information about the expected shape of the anomaly in the form of a training image and very limited conditioning data, our Bayesian framework is able to recover appropriate probabilities for a target lithology. It inverts for the probability of a lithology, where each lithology has a fixed resistivity value, constrained by the spatial connectivity structures implied by the SNESIM categorical model. The results are very different from the smooth resistivity distributions that are commonly obtained if 2D or 3D modelling is performed. Our results should be seen as complimentary to more conventional airborne electromagnetic data inversion products since they provide information about the probability for a presence of a given target lithology. This is potentially useful to mitigate exploration risk.

For the synthetic example, the amount of random noise added to the data was known. In the case of the Harmony data, no quantitative measure of noise on the data is available. While the recovered probabilities confirm the earlier modelling and the geological model, the lack of reliable noise estimate makes it nearly impossible to robustly quantify the fit to the data and thus the quality of a model. A misfit between an observation and a Bayesian posterior prediction can be due to incorrect prior information, overly parsimonious modelling, inaccurate forward physics, the observation being affected by unexpected noise processes, or several of these. Clearly, in the case of noise affecting measurements, attempting to fit a given observation would lead to an incorrect image of the subsurface. Ultimately, understanding the full noise process is critical for the success of a probabilistic inversion strategy. We note that inference of the statistical properties of the noise process that informs the Bayesian likelihood is notoriously difficult: this noise process is a complex mixture of instrumental errors, external noise, and forward modelling errors.

At this stage it is still necessary to choose a target resistivity a-priori within an order of magnitude of the true value. Further, the likelihood function we attempt to maximize is based on an L^2 norm of the data. Future work will focus on finding more appropriate definitions of the fit to the data than a simple least square measure between observed and predicted data, a better understanding of data noise, and ultimately inverting for the target resistivity.

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REFERENCES

Annetts, D., Sugeng, F. and Raiche, A, 2003, Modelling and matching the airborne EM response of Harmony and Maggie Hays, 15th International Geophysical Conference and Exhibition, Adelaide.

Brodie, R. and Sambridge, M., 2006, A holistic approach to inversion of frequency-domain airborne EM data, Geophysics, 71(6) , G301–G312.

Buland, A. and Kolbjørnsen, O., 2012, Bayesian inversion of CSEM and magnetotelluric data, Geophysics, 77, E33–E42.

Constable, S. C., Parker, R. L. and Constable, C. G., 1987, Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data, Geophysics, 52(3), 289–300.

Cordua, K. S., Hansen, T. M. and Mosegaard, K., 2012, Monte Carlo full-waveform inversion of crosshole GPR data using multiple-point geostatistical a priori information, Geophysics, 77, H19-H31.

Cox, L. H., Wilson, G. A. and Zhdanov, M. S., 2010, 3D inversion of airborne electromagnetic data using a moving footprint, Exploration Geophysics, 41(4) 250-259.

Gelman, A., Carlin, J. B., Stern, H. S., and Rubin, D. B., 1995, Bayesian data analysis, Chapman and Hall.

Gunning, J., Glinsky, M. E. and Hedditch, J., 2010, Resolution and uncertainty in 1D CSEM inversion: A Bayesian approach and open-source implementation, Geophysics, 75(6), F151–171.

Minsley, B., 2011, A trans-dimensional Bayesian Markov chainMonte Carlo algorithm for model assessment using frequency-domain electromagnetic data, Geophysical Journal International, 187(1), 252-272.

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H. and Teller, E., 1953, Equation of State Calculations by Fast Computing Machines, The Journal of Chemical Physics, 21(6), 1087–1092.

Mosegaard, K and Tarantola, A. 1995, Monte Carlo sampling of solutions to inverse problems, Journal of Geophysical Research, 100(7), 12431–12447.

Strebelle, S., 2002, Conditional simulation of complex geological structures using multiple-point statistics, Mathematical Geology, 34, 1–21.

Sugeng, F. and Raiche, A., 1997, ArjunaAir, Modelling program developed for the sponsors of Australian Minerals Industries Research Association Project P223C.

Viezzoli, A., Christiansen, A. V., Auken, E., and Sørensen, K., 2008, Quasi-3D modeling of airborne TEM data by spatially constrained inversion, Geophysics, 73(3), F105–F113.

Wilson, G. A., Raiche, A. P. and Sugeng, F., 2006, 2.5D inversion of airborne electromagnetic data, Exploration Geophysics, 37, 363–371.

Wolfgram, P. and Golden, H., 2001, Airborne EM applied to sulphide nickel - examples and analysis, Exploration Geophysics, 32, 136–140.

Zhdanov, M. S. and Tartaras, E., 2002, Three-dimensional inversion of multi transmitter electromagnetic data based on the localized quasi-linear approximation, Geophysical Journal International, 148(3), 506–519.