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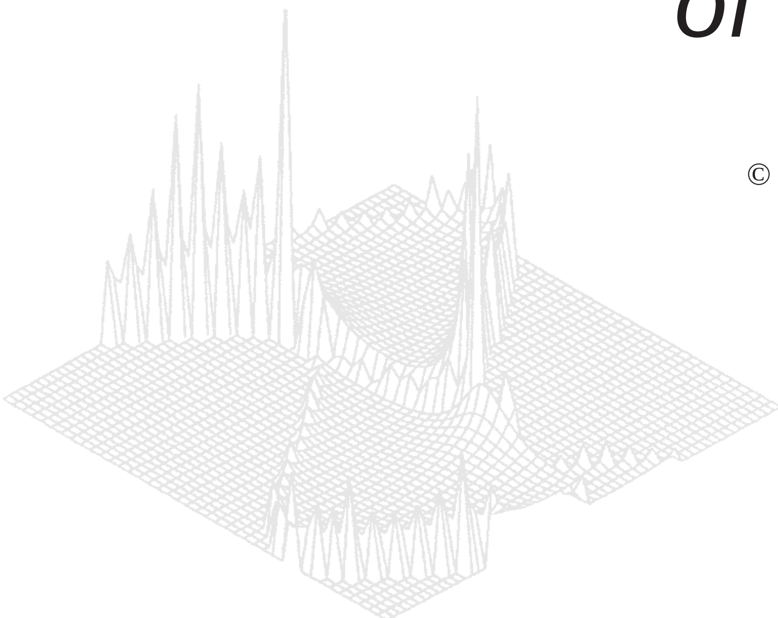

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Non-leptonic B Meson Decays using Perturbative QCD

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Abstract

Using perturbative QCD within the framework of Szczepaniak *et al.* (1990), we analyse different non-leptonic B decays, namely $B \to \pi\pi$, $B \to \pi(K)D$, $B \to DD$, as *heavy-to-light* transitions dominated by tree diagrams and compare our estimates to experimental data and other theoretical model predictions.

1. The Branching Ratio

The two-body non-leptonic decays of the B mesons have been of real interest in recent years, but it is still uncertain whether perturbative QCD (PQCD) methods offer a reliable framework for most of these processes. Assuming factorisation, one can analyse these decays using semi-leptonic amplitudes of exclusive D decays as an input (Deandrea *et al.* 1993). Many theoretical investigations on both non-leptonic and semi-leptonic heavy mesons decays are based on the BSW model (Bauer *et al.* 1987; Wirbel *et al.* 1985). It has become clear that for some processes, for example the rare B decays where the estimates lie below the experimental data, one can significantly increase the numerical values of the branching ratios by using PQCD (Du *et al.* 1997), while for charmed meson decays the $SU(3)$-symmetry breaking effects should be taken into account (Chau and Cheng 1994).

The aim of the present work is to study two-body exclusive modes of the B meson, by employing the *heavy-to-light* PQCD scheme by Szczepaniak *et al.* (1990) for processes considered as short distance events.

We start with the unperturbed effective weak Hamiltonian (Szczepaniak *et al.* 1990):

$$H = \frac{G_F}{\sqrt{2}}\,|V| J_\mu^\dagger J^\mu\,, \tag{1}$$

where V denotes the CKM matrix elements and, by using factorisation at high momentum transfer, one has besides the weak decay matrix element (see Fig. 1):

$$\langle L_2|J_\mu|0\rangle = f_2\, q_\mu\,, \tag{2}$$

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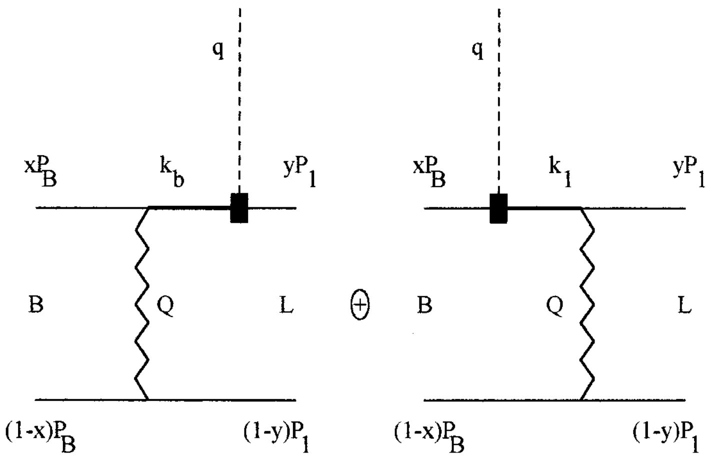

Fig. 1. The contributing diagrams in the current matrix element J_μ.

where $q = P_B - P_1$ and $q^2 = m_2^2 \approx 0$, and the hadronic current between the two remaining states calculated as a trace over spin, flavour and colour indices and integration over momentum fractions (Dariescu and Dariescu 1996*a*)

$$\langle L_1|J_\mu|B\rangle = g_s^2\left\{\mathrm{Tr}\left[\bar{\phi}_1\,\gamma_\mu\,\frac{\gamma\cdot k_b + M}{k_b^2 - M^2}\,\gamma_\alpha\,\phi_B\,\gamma^\alpha\,\frac{\lambda_a\lambda^a}{Q^2}\right]\right.$$
$$\left. + \,\mathrm{Tr}\left[\bar{\phi}_1\,\gamma_\alpha\,\frac{\gamma\cdot k_1 + m}{k_1^2 - m^2}\,\gamma_\mu\,\phi_B\,\gamma^\alpha\,\frac{\lambda_a\lambda^a}{Q^2}\right]\right\}. \tag{3}$$

As in Dariescu and Dariescu (1996*a*, 1996*b*), we shall use for the B meson the wave function ($M = 5280$ MeV and $f_B = 200$ MeV)

$$\phi_B(x) = \frac{f_B}{12}\varphi_B(x)(\gamma\cdot P_B + M)\gamma_5\;,\;\varphi_B(x)$$
$$= N_B\,\frac{x^2(1-x)^2}{[a^2x + (1-x)^2]^2}\,, \tag{4}$$

where the normalised distribution amplitude φ_B contains the parameter $a \in [0\cdot 05 - 0\cdot 1]$ related to the momentum distribution between the pair of quarks. The light pseudoscalar meson of mass m_1 (with $m_1^2 = m^2 \ll M^2$) is described by the wave function

$$\phi_1 = \frac{f_1}{12}\varphi_1(y)(\gamma\cdot P_1 + m)\gamma_5\,, \tag{5}$$

with an asymptotic-type amplitude

$$\varphi_1(y) = y(1-y)\left\{\int_0^1 y(1-y)\,dy\right\}^{-1} \tag{6}$$

or, for the pion, with a Chernyak–Zhitnitsky (1982) one

$$\varphi_{1(CZ)} = y(1-y)(2y-1)^2\left\{\int_0^1 y(1-y)(2y-1)^2\,dy\right\}^{-1} . \tag{7}$$

By introducing the small mass parameter $z = m/M$ (with $z^2 \approx 0$) we still are in the approach of a *heavy-to-light* transition, but we raise the numerical values of the branching ratios which, in the case of neglecting both masses in the final states (Szczepaniak *et al.* 1990), lie much below the experimental data and other theoretical predictions.

Consequently, the expression for the branching ratio (Dariescu and Dariescu 1996a)

$$BR = 7{\cdot}3\,\times\,10^{-15}\,k^2\,\mathcal{I}^2\,, \tag{8}$$

where

$$\begin{aligned}\mathcal{I} = &\int_0^1 \frac{\varphi_B(x)}{1-x}\,dx \int_0^{1-a} \frac{\varphi_1(y)}{(1-y)^2}\,[y(1-2z)+(z-2)]\,dy \\ &+ \int_0^1 \frac{\varphi_B(x)}{(1-x)^2}\,z(1-2x)\,dx \int_0^{1-a}\frac{\varphi_1(y)}{1-y}\,dy\,+\,\mathcal{O}[z^2]\,,\end{aligned} \tag{9}$$

is expressed in terms of three parameters, namely a, z and

$$k = V\,f_1 f_2 \tag{10}$$

containing the decay constants of the final mesons and the CKM matrix elements. In Dariescu and Dariescu (1996a), we kept ourselves at a general level in a graphical analysis of (8), allowing a global order-of-magnitude estimate of the branching ratio, with respect to the three significant parameters k, a, z and we have concluded that the Szczepaniak *et al.* approach (1990) might offer a reliable framework in PQCD. Nevertheless, it is still crucial to improve this global view especially in regard to different types of concrete processes in order to establish the limits of the proposed model. A comparison of the results of the present paper with the existing experimental data and other theoretical predictions will try to answer the following questions:

- how far can we satisfactorily describe not only π or K, but also the D meson, by a light meson wave function
- how the factorable diagrams dominate the processes where they contribute
- the $PQCD$ applicability in processes like $B \to \pi\pi$, since up to now both negative (Huang and Luo 1994) and positive (Li 1995a) conclusions have been drawn
- how satisfactory are the results in the assumption of neglecting the final states interaction (Xing 1995), and

- the necessity of extending our analysis on $B \to \pi(K)\pi$ transitions by including electroweak penguin effects (Dariescu and Dariescu 1996*b*; Carlson and Milana 1995), as for example in $B_s^0 \to \pi(K)K$ (Du and Yang 1995), or the exchange and annihilation contributions, as in $B^0 \to K^- D_s^+$ and $B^+ \to K^0 D^+$.

Before going further and fixing the values of k and z, let us note that by introducing the parameter a, the results obtained generalise the Carlson and Milana (1994) ones based on a strongly peaked wave function for heavy mesons, which for simplicity cast into delta functions. In order to see the evolution of the results with the profile change of the B wave function (Li 1995*a*, 1995*b*), let us compare (see Fig. 2) at a general level our surfaces generated by (8) for the limits of the a range with those obtained by Carlson and Milana (1994) for $\varepsilon_B = (M - m_b/M)$ equal respectively to $0 \cdot 14$ ($m_b = 4500$ MeV) and to $0 \cdot 22$ ($m_b = 4100$ MeV).

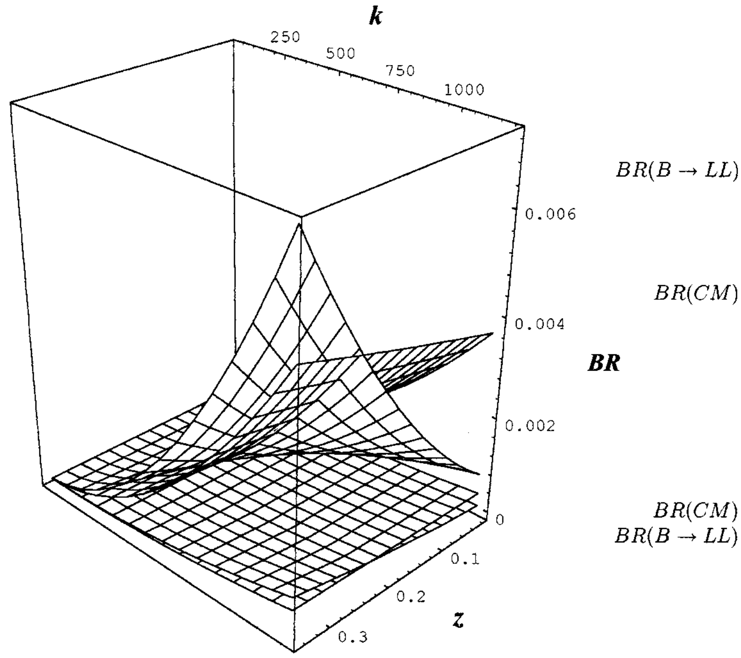

Fig. 2. Our $BR(B \to LL)$ values compared to the more stable ones drawn in the peaking approximation and denoted by $BR(CM)$. The lower surfaces correspond to $a = 0 \cdot 1$ and respectively to $\varepsilon_B = 0 \cdot 22$, while the upper ones corrrespond to $a = 0 \cdot 05$ and $\varepsilon_B = 0 \cdot 14$.

Inspecting the lower surfaces, corresponding to $a = 0 \cdot 1$ and $\varepsilon_B = 0 \cdot 22$, and the upper ones ($a = 0 \cdot 05$ and $\varepsilon_B = 0 \cdot 14$) we notice rather stable BR values on the Carlson–Milana surfaces, almost z-independent for a given k. On the contrary, even though we have neglected z^2 in the expression For the form factor, our result is strongly z-dependent, especially on the $a = 0 \cdot 05$ surface, accommodating a wide range of processes. In spite of the drastic differences, we point out the value $z \approx 0 \cdot 2$, where the branching ratio obtained is model independent. This should correspond to the mass in the final state $m_1 \approx 1000$ MeV, and can be tested in $B \to K$ decays, even in the radiative ones if we assume that PQCD and factorisation are applicable to penguin diagrams (Dariescu and Dariescu 1996*b*).

2. Concrete Non-leptonic Decays

Let us turn now to concrete processes by fixing the pair $\{k, z\}$ for different B meson non-leptonic decays described by tree diagrams. Also, we neglect final state interactions and describe all final mesons by the light wave functions (6) or (7). Although it has been stated that factorable diagrams dominate in any process where they contribute, there is still the question of whether other types of diagrams should be taken into account (Gronau *et al.* 1995).

(a) $B \to \pi\pi$ *Decays*. Working only with tree diagrams and neglecting the colour-suppressed and penguin contributions, one is able to test now the applicability of the model presented above in the case of $B^+ \to \pi^+\pi^0$ and $B^0 \to \pi^+\pi^-$ having $\{k, z\} = \{124, 0\cdot 027\}$. Employing the pion wave function (7), we increase the branching ratio values to $BR \in [3\cdot 17 \times 10^{-6}, 3\cdot 8 \times 10^{-5}]$, which perfectly accommodate the experimental result $0\cdot 84 \times 10^{-5}$ (Wurthwein 1999). A comparison with other theoretical estimates, namely $BR(B^0 \to \pi^+\pi^-) = 3\cdot 5 \times 10^{-6}$ (Carlson and Milana 1994), $1\cdot 8 \times 10^{-5}$ (Abreu *et al.* 1995), $1\cdot 43 \times 10^{-5}$ for CKM(ρ, η) $= (-0\cdot 12, 0\cdot 34)$ (Palmer and Wu 1995) picks up respectively $a = 0\cdot 097$, $0\cdot 062$ and $0\cdot 066$.

(b) $B \to K\pi$ *Decays*. We shall not take into account the processes with very small values for k and z since the estimates lie below both theoretical and experimental data. Anyway, it is well known that in $B \to K\pi$ decays the penguin diagrams play a significant role leading to large contributions in the branching ratio value. The situation is much under control for $B_s^0 \to \pi^+K^-$ (Du and Yang 1995) with $\{k, z\} = \{153, 0\cdot 092\}$ where the BR significantly increases from $3\cdot 64 \times 10^{-6}$ $(a = 0\cdot 1)$ to $2\cdot 71 \times 10^{-5}$ $(a = 0\cdot 05)$, very close to the upper limit $2\cdot 6 \times 10^{-5}$ (Abreu *et al.* 1995). Here the theoretical estimates cover a wide range, from $[1\cdot 1 - 1\cdot 8] \times 10^{-5}$ (Carlson and Milana 1994) to $4\cdot 95 \times 10^{-6}$ (for $N_c = \infty$) (Du and Yang 1995), corresponding to values of a from $[0\cdot 068 - 0\cdot 058]$ to $0\cdot 09$.

(c) $B \to \pi(K)D$ *Decays*. We split these processes into two categories:

(i) The $B \to D$ transitions characterised by $z = 0\cdot 35$ and large k values. We include here $B^- \to K^-D^0$, $B^0_{(s)} \to K^+D^-_{(s)}$, with $k = 253$ and neglecting the final state interactions (Zheng 1995) we obtain $BR \in [3\cdot 4 \times 10^{-5}, 3\cdot 2 \times 10^{-4}]$. Also $B^0_{(s)} \to \pi^+D^-_{(s)}$ and $B^- \to \pi^-D^0$ with $k = 936$ are good candidates since their $BR \in [4\cdot 7 \times 10^{-4}, 4\cdot 4 \times 10^{-3}]$ is in good agreement with the theoretical prediction $3\cdot 2 \times 10^{-3}$ (Carlson and Milana 1994; Li 1995*b*) corresponding to $a = 0\cdot 055$ and with the data $BR[B \to D\pi] = (3 \pm 0\cdot 4) \times 10^{-3}$ requiring $a \in [0\cdot 054 - 0\cdot 059]$.

(ii) The small k and z transitions, for example $\{k, z\} = \{42, 0\cdot 092\}$ in $B_s^0 \to K^-D^+$ or $\{k, z\} = \{34, 0\cdot 027\}$ in $B^0 \to \pi^-D^+$, $B^+ \to \pi^0D^+$, lead to branching ratios whose values lie below 3×10^{-6}. Keeping the same very light final state but increasing k, for example in $B^+ \to \pi^0D_s^+$ or $B^0 \to \pi^-D_s^+$, where $\{k, z\} = \{124, 0\cdot 027\}$, we obtain BR in the range $BR \in [3\cdot 17 \times 10^{-6}, 3\cdot 8 \times 10^{-5}]$.

(d) $B \to DD$ *Decays*. We come now to processes characterised by large values of both z and k, namely $\{k, z\} = \{1130, 0\cdot 35\}$. There are a lot of such decays, for example $B^+ \to D_s^+\bar{D}^0$ and $B^0_{(s)} \to D_s^+D^-_{(s)}$, which offer, even with the assumption of considering D as a light meson described by equations (5) and (6), valuable confirmation for the model employed in our analysis. Thus, by comparing our branching ratio values $BR \in [6{\cdot}8 \times 10^{-4}, 6{\cdot}4 \times 10^{-3}]$ to the Carlson and Milana (1994) prediction $BR(B \to DD_s) = 7{\cdot}7 \times 10^{-3}$ we note that the lower limit $0{\cdot}05$ is a good choice for a, being also in the middle of the range $a \in [0{\cdot}041, 0{\cdot}058]$ given by the comparison with the data $BR(B^0 \to D^- D_s^+) = (8 \pm 4) \times 10^{-3}$ (see Review of Particle Properties 1994).

3. Conclusions

Within the framework of Szczepaniak *et al.* (1990), we have developed a systematic study of the non-leptonic B meson decays described by factorable diagrams and determined the value of a for which our results match other theoretical predictions and experimental data. In general, in the $B \to \pi(K)\pi$ decays, working with very small z values, a wide range of results arises when a runs from $0{\cdot}1$ to $0{\cdot}05$. On the other hand, the favoured $B \to D$ decays ($z \approx 0{\cdot}35$), characterised by $k = 936$ (in $B^- \to \pi^- D^0$) or by $k = 1130$ (in $B \to DD$), impose the value for the a parameter close to the lower limit of its range.

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