Aust. J. Phys., 1993, **46**, 821–37

Nonlinear Aspects of Wave Propagation in a Magnetised Plasma in the Presence of Coriolis Force

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Abstract

Nonlinear modifications in the refractive indices of quasicircular electromagnetic waves, propagating obliquely in a cold homogeneous magnetised plasma in the presence of the Coriolis force of rotation, have been theoretically investigated. Some interesting properties are found to occur, which depend on the gyrofrequency, rotational frequency and the amplitudes of the waves. The characteristic variations of the refractive indices of the left and right circularly polarised waves, for different values of the gyrofrequency and rotational frequency, are shown graphically. The stop-bands, which are located for both the waves in the presence of the Coriolis force, are discussed. The stability criteria of the waves interacting with a plasma are also investigated. The results obtained are more general than those reported previously.

1. Introduction

Nonlinear effects of the interaction of propagating waves with a magnetised plasma including the Coriolis force aspect of rotating plasmas are interesting and important, both in laboratory experiments and for astrophysical plasmas. Chandrasekhar (1953*a*, 1953*b*, 1953*c*) and Lehnert (1954, 1955, 1962, 1971) explored the effects of Coriolis force on propagating waves, particularly in the study of cosmic phenomena. Several authors (Chandrasekhar 1961; Bhatia 1967, 1969*a*, 1969*b*; Bandyopadhaya 1972; Engels and Verheest 1975; Tandon and Bajaj 1966; Das *et al.* 1984) have developed different aspects of the mathematical theory of wave propagation in rotating plasmas. Uberoi and Das (1970) considered the effects of rotation on waves propagating in a plasma in the linearised approximation. They studied the cutoffs and resonances of the waves for various physical situations. Later, Horton *et al.* (1984, 1986) and Paul (1985) observed the effects of nonlinearity in some of these problems of wave–plasma interactions.

Sur *et al.* (1988) investigated the influence of the rotation on the group travel time and the induced magnetic field, and particularly for solar pulses emerging from the sun's interior to its surface. However, they did not consider the influence of the nonlinearity in the refractive indices of the waves at the resonances and cutoffs, or in the shift of wave parameters. It appears that the problems of the nonlinear interaction of circularly polarised waves with a magnetised plasma have not yet been exhaustively studied. However, several such nonlinear aspects have been investigated by Max and Perkins (1972), Steiger and Woods (1972), Max

0004-9506/93/060821$05.00

(1973*a*, 1973*b*), Goldstein and Salu (1973), Lee and Lerche (1978, 1979, 1980), Stenflo (1976, 1980), Chakraborty *et al.* (1986, 1987), Paul *et al.* (1989), Paul (1990) and others. Recently, Paul *et al.* (1992) and Kashyapi *et al.* (1992) considered nonlinear effects on the propagation of a wave in a magnetised rotating plasma, including the variation of the nonlinear refractive indices with the variation of the wave amplitude and rotational frequency. In the present paper the variation of nonlinear refractive indices, particularly at the cutoffs and resonances, and the stability of these waves have been reconsidered. It is found that a wave of power $\simeq 10^{13}$ W m^{-2} becomes unstable in the presence of Coriolis force in the plasma.

2. Dispersion Relations

We assume that the electrons in the plasma are mobile but that the ions are immobile and maintain the macroscopic charge neutrality. The plasma is cold, homogeneous and collisionless. Also, we assume that the electromagnetic wave propagates at an angle θ with the ambient uniform static magnetic field $\boldsymbol{H}_0 = (0, 0, H_0)$ and angular velocity $\boldsymbol{\Omega} = (0, 0, \Omega)$, so that the propagation vector can be taken as $\boldsymbol{k} = (k\sin\theta, 0, k\cos\theta)$. Although rotation generates both the Coriolis force and the centrifugal force, only the Coriolis force is taken into consideration here to avoid mathematical complexity. Since the rotational frequency is small in astrophysical plasmas, the influence of the centrifugal force is negligible. The equivalent magnetic fields of the electrons and ions, due to the presence of the Coriolis force of rotation, are different (Lehnert 1962; Uberoi and Das 1970; Kashyapi *et al.* 1992).

For circularly polarised electromagnetic waves propagating through a magnetised rotating plasma we obtain (Kashyapi *et al.* 1992) the first-order dispersion relations

$$n_{\pm}^2 = 1 - \frac{\omega_{\mathrm{p_e}}^2}{\omega(\omega \pm \pi_{\mathrm{e}})}\,. \tag{1}$$

The nonlinear dispersion relations for the first harmonic part, correct up to third order in the electric field, are

$$n_{\pm}^2 = 1 - \frac{\omega_{\mathrm{p_e}}^2}{\omega(\omega \pm \pi_{\mathrm{e}})} \mp \frac{\alpha_{\mathrm{e}}^2 \omega_{\mathrm{p_e}}^2 \cos^2\theta\, \pi_{\mathrm{e}}(ck_{\mp})}{\omega_{\mathrm{p}}^2(\omega^2 - \pi_{\mathrm{e}}^2)} \left(\frac{ck_+}{\omega - \pi_{\mathrm{e}}} + \frac{ck_-}{\omega + \pi_{\mathrm{e}}}\right)$$
$$+ \frac{\alpha_{\mathrm{e}}^2 \omega_{\mathrm{p_e}}^2 \cos^2\theta}{2\omega_{\mathrm{p}}^2} \frac{ck_+ + ck_-}{(\omega \mp \pi_{\mathrm{e}})} \left(\frac{ck_+}{\omega - \pi_{\mathrm{e}}} + \frac{ck_-}{\omega + \pi_{\mathrm{e}}}\right). \tag{2}$$

Here, $\omega_{\mathrm{p_e}} = [4\pi e^2 n_{\mathrm{e}}^{(0)}/m_{\mathrm{e}}]^{1/2}$, $\pi_{\mathrm{e}} = \Omega_{\mathrm{e}} - 2\Omega$, $\Omega_{\mathrm{e}} = |eH_0/m_{\mathrm{e}}c|$, $\alpha_{\mathrm{e}} = ea/m_{\mathrm{e}}\omega c$, $\omega_{\mathrm{p}}^2 = \omega_{\mathrm{p_e}}^2 - 4\omega^2$; m_{e}, n_{e}, v_{e} and $-e$ are the mass, number density, velocity and charge, respectively, of the electrons; c is the velocity of light, $k_{\pm}$ are the wavenumbers of the left circularly polarised (LCP) and right circularly polarised (RCP) waves respectively, ω is the wave frequency, and a is the amplitude of the wave. Hence $n_{\pm} = k_{\pm}c/\omega$ are the refractive indices for the LCP and RCP waves, respectively.

The relations (2) indicate that the refractive indices ($n_{\pm}^2$) of the LCP and RCP waves depend on the rotational frequency Ω, gyrofrequency Ω_e, the dimensionless wave amplitude α_e, the wave frequency ω, the plasma frequency ω_{p_e}, etc. It is clear that n_+ and n_- depend on α_e. The effects due to the influence of these factors on the refractive indices are discussed below.

(2a) Variation with Angle of Propagation (θ)

Equation (2) can be written in the form

$$\bar{n}^2 = \tfrac{1}{2}(n_+^2 + n_-^2)$$

$$= 1 - \frac{\omega_{p_e}^2}{\omega^2 - \pi_e^2} + \frac{\alpha_e^2\, \omega_{p_e}^2 \cos^2\theta}{2(\omega_{p_e}^2 - 4\omega^2)} \left(\frac{ck_+}{\omega - \pi_e} + \frac{ck_-}{\omega + \pi_e} \right)^2 . \tag{3}$$

Equation (3) shows that θ, the angle between the direction of wave propagation and the static magnetic field, affects the nonlinear behaviour of the refractive indices. When the wave propagates along the magnetic field (i.e. $\theta = 0$), the refractive indices are maximised, but for wave propagation perpendicular to the magnetic field (i.e. $\theta = \pi/2$), the refractive indices are minimised. Fig. 1 shows the variation of $\bar{n}^2$ for different values of the angle of propagation θ, when $2\Omega/\omega < 1$ and $\Omega_e/\omega < 1$. For Fig. 1 we have assumed that $(\omega_{p_e}/\omega)^2 = 0\cdot6$, $\alpha_e^2 = 0\cdot3$ and $\Omega_e/\omega = 0\cdot2$.

(2b) Variation with Plasma Density (D_v)

The nonlinear refractive indices of the wave depend on the plasma density. From Fig. 1 we see that refractive indices sharply decrease with an increase in the plasma density when $2\Omega/\omega < 1$, $\Omega_e/\omega = 0\cdot2$, $\theta = 30°$ and $\alpha_e^2 = 0\cdot3$.

(2c) Variation with Wave Amplitude

In Fig. 1 the variation of $\bar{n}^2$ with the wave amplitude α_e is also shown. It is seen that an increase of α_e results in an increase of $\bar{n}^2$; this increase is significant when $2\Omega/\omega \approx 1$.

(2d) Analysis of the Stop-band

The wave does not propogate at the cutoff frequencies, where $k_{\pm} = 0$, and resonances occur when $k_{\pm} \to \infty$. Hence the waves exhibit a stop-band, i.e. a band of frequencies for which the waves do not propagate. For frequencies within this band, the wave number is imaginary. Uberoi and Das (1970) derived the linear dispersion relations for the LCP and RCP waves and showed the existence of the cutoffs and resonances for waves in a cold plasma in the presence of a Coriolis force. Later, Sur *et al.* (1989) investigated the propagation of ion–acoustic whistlers in the ionosphere, considering the effect of negative ions, and obtained the expressions for the cutoff and resonance frequencies.

In this section, we examine the effect of nonlinearity on the cutoff and resonance frequencies of obliquely propagating waves in a rotating plasma. These quantities are affected by the Coriolis force; in some cases new stop-bands may be formed. In

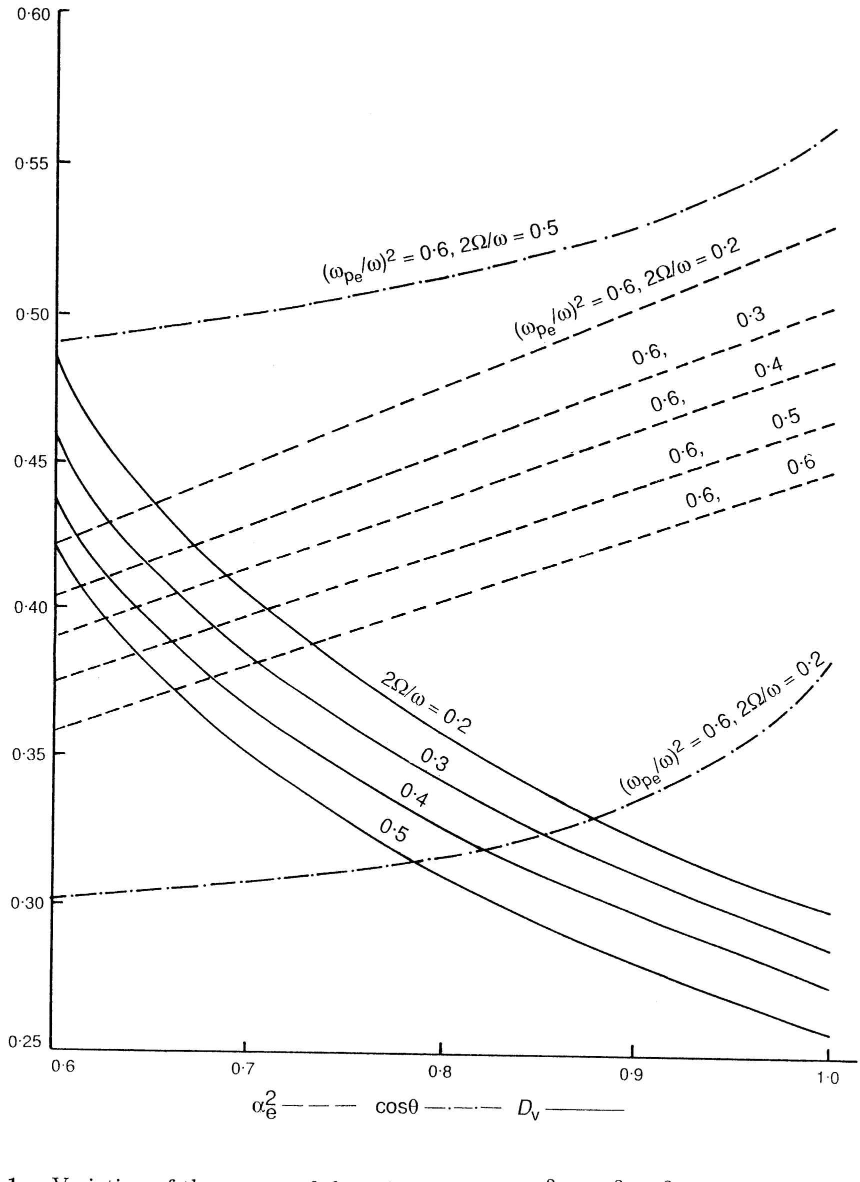

Fig. 1. Variation of the square of the refractive index $\bar{n}^2$ $[=(n_+^2+n_-^2)/2]$ with (i) the square of the amplitude α_c^2 of the EM wave when $\theta = 30°$, (ii) the angle of propagation θ when $\alpha_c^2 = 0 \cdot 3$, and (iii) the plasma density D_v when $\theta = 30°$, $\alpha_c^2 = 0 \cdot 3$. For all cases $\Omega_c/\omega = 0 \cdot 2$, $2\Omega/\omega < 1$.

Figs 2 and 3 the stop-bands for the LCP and RCP waves are shown for different values of the rotational frequency $2\Omega/\omega$, gyrofrequency Ω_e/ω, and angle of propagation θ. In these figures the solid curves indicate the variations of the refractive indices of the obliquely propagating LCP (n_+) and RCP (n_-) waves for $\theta = 45°$, and the dashed curves represent the waves propagating along the direction of the magnetic field (i.e. $\theta = 0°$). In Fig. 2a we see that n_+^2 decreases sharply with

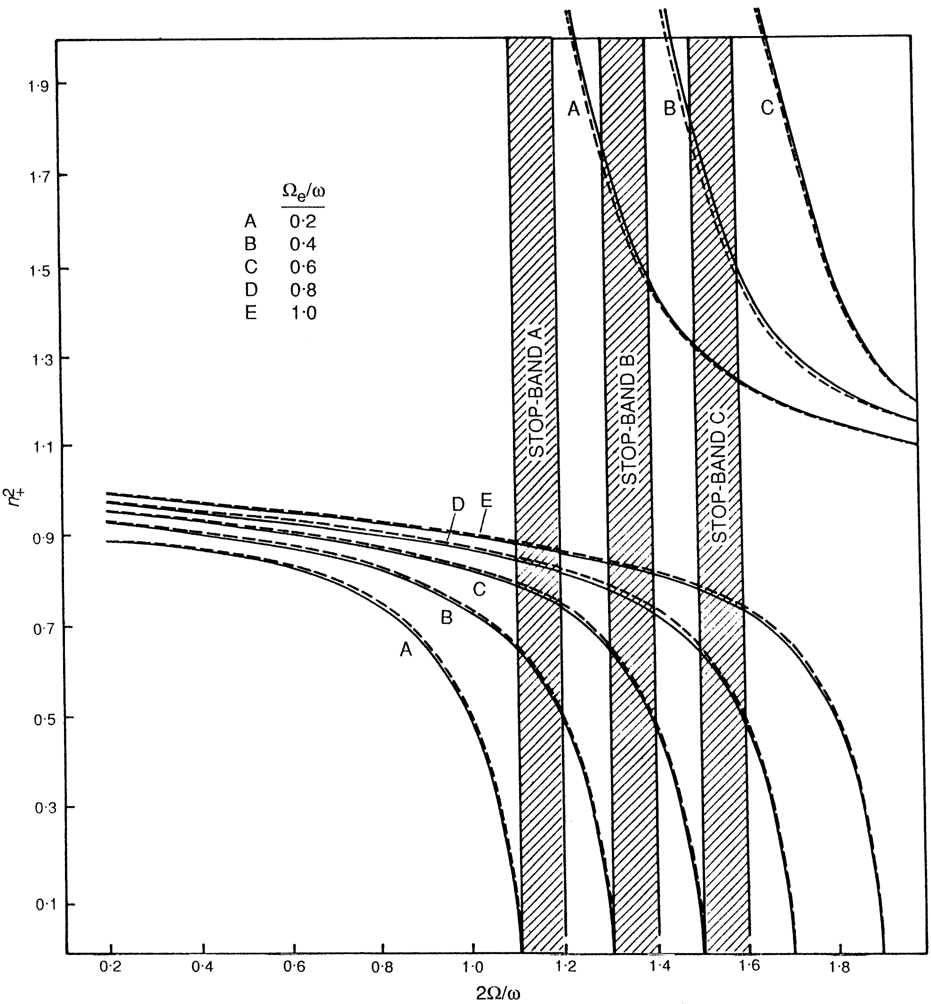

Fig. 2*a*. Variation of the square of the refractive index n^2_+ with the angular frequency of rotation $2\Omega/\omega$ for the LCP wave, for various values of $\Omega_c/\omega \leq 1$ (see curve labels) and with $(\omega_{Pe}/\omega)^2 = 0\cdot 6$, $\alpha_c^2 = 0\cdot 1$. Full curves, $\theta = 45°$; dashed curves, $\theta = 0°$.

an increase in rotational frequency $2\Omega/\omega$. The cutoff point $(k_+ = 0)$ of the LCP wave occurs at $2\Omega/\omega = 1\cdot 1$, when $\Omega_e/\omega = 0\cdot 2$. The resonance $(k_+ \to \infty)$ is found at $2\Omega/\omega = 1\cdot 2$ for the same value of the gyrofrequency. So, for the width of the stop-band (δ) we have the inequality $1\cdot 1 \leq \delta \leq 1\cdot 2$, for $\Omega_e/\omega = 0\cdot 2$. Similarly, $1\cdot 3 \leq \delta \leq 1\cdot 4$, for $\Omega_e/\omega = 0\cdot 4$ and $1\cdot 5 \leq \delta \leq 1\cdot 6$, for $\Omega_e/\omega = 0\cdot 6$.

In Fig. 2*b* the variations of $n^2{}_+$ are shown for different values of Ω_e/ω. Here, we see that $n^2{}_+$ rapidly increases with an increase in Ω_e/ω. For values of $2\Omega/\omega = 1\cdot 4$, $1\cdot 6$ and $1\cdot 8$, the widths of the stop-bands of the LCP waves are given by the inequalities $0\cdot 4 \leq \delta \leq 0\cdot 5$, $0\cdot 6 \leq \delta \leq 0\cdot 7$ and $0\cdot 8 \leq \delta \leq 0\cdot 9$, respectively.

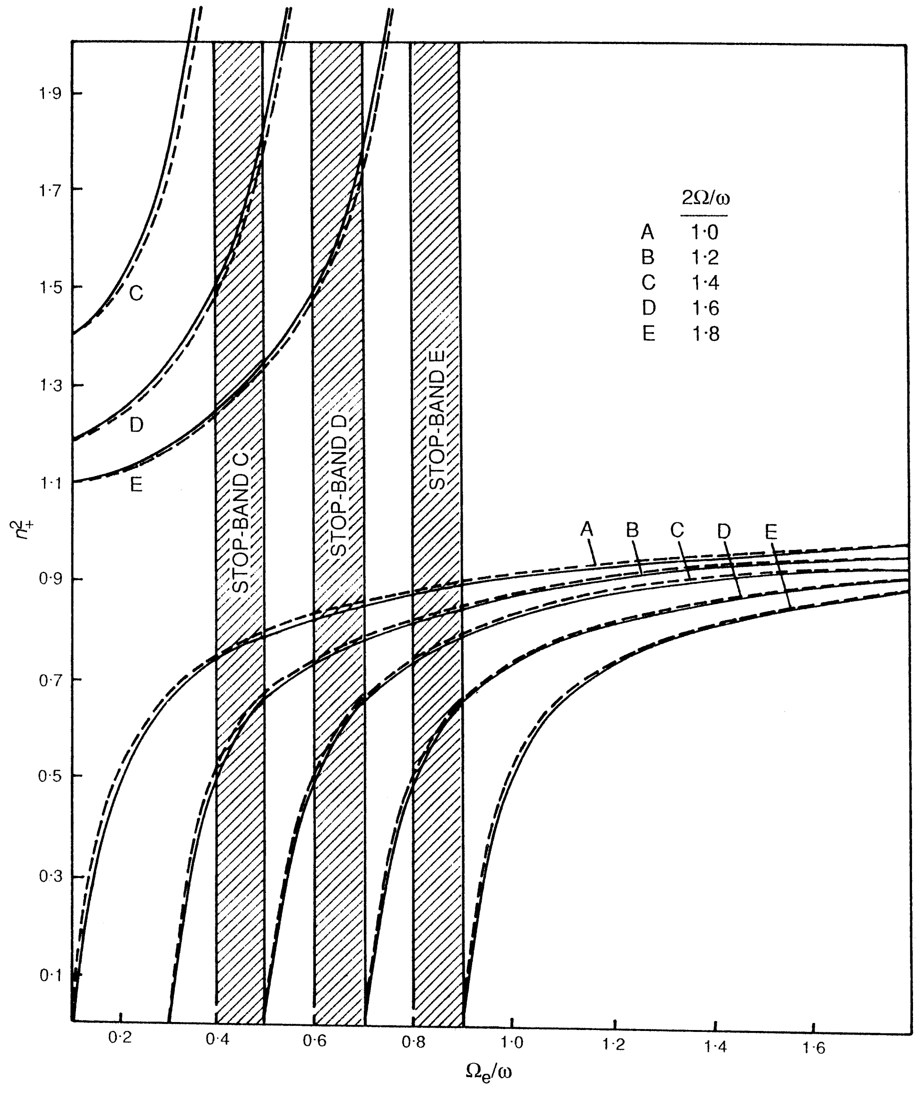

Fig. 2*b*. Variation of the square of the refractive index n_+^2 with the electron-cyclotron frequency Ω_c/ω for the LCP wave, for various values of $2\Omega/\omega \geq 1$ (see curve labels) and with $(\omega_{p_e}/\omega)^2 = 0 \cdot 6$, $\alpha_c^2 = 0 \cdot 1$. Full curves, $\theta = 45°$; dashed curves, $\theta = 0°$.

From Fig. 3*a* it can be observed that n_-^2 increases sharply with increasing rotational frequency $2\Omega/\omega$ for any value of α_e and for $\theta < 90°$. We find that the RCP wave would have a stop-band of width $0 \cdot 2 \leq \delta \leq 0 \cdot 3$, for $\Omega_e/\omega = 1 \cdot 2$, $0 \cdot 4 \leq \delta \leq 0 \cdot 5$, for $\Omega_e/\omega = 1 \cdot 4$, $0 \cdot 6 \leq \delta \leq 0 \cdot 7$, for $\Omega_e/\omega = 1 \cdot 6$, $0 \cdot 8 \leq \delta \leq 0 \cdot 9$, for $\Omega_e/\omega = 1 \cdot 8$. Similarly, Fig. 3*b* shows that the widths of the stop-bands for the RCP waves are $1 \cdot 1 \leq \delta \leq 1 \cdot 2$, $1 \cdot 3 \leq \delta \leq 1 \cdot 4$ and $1 \cdot 5 \leq \delta \leq 1 \cdot 6$, for $2\Omega/\omega = 0 \cdot 2$, $0 \cdot 4$ and $0 \cdot 6$, respectively. In each figure the refractive index changes slightly for obliquely propagating waves, though the nature of its variation remains the

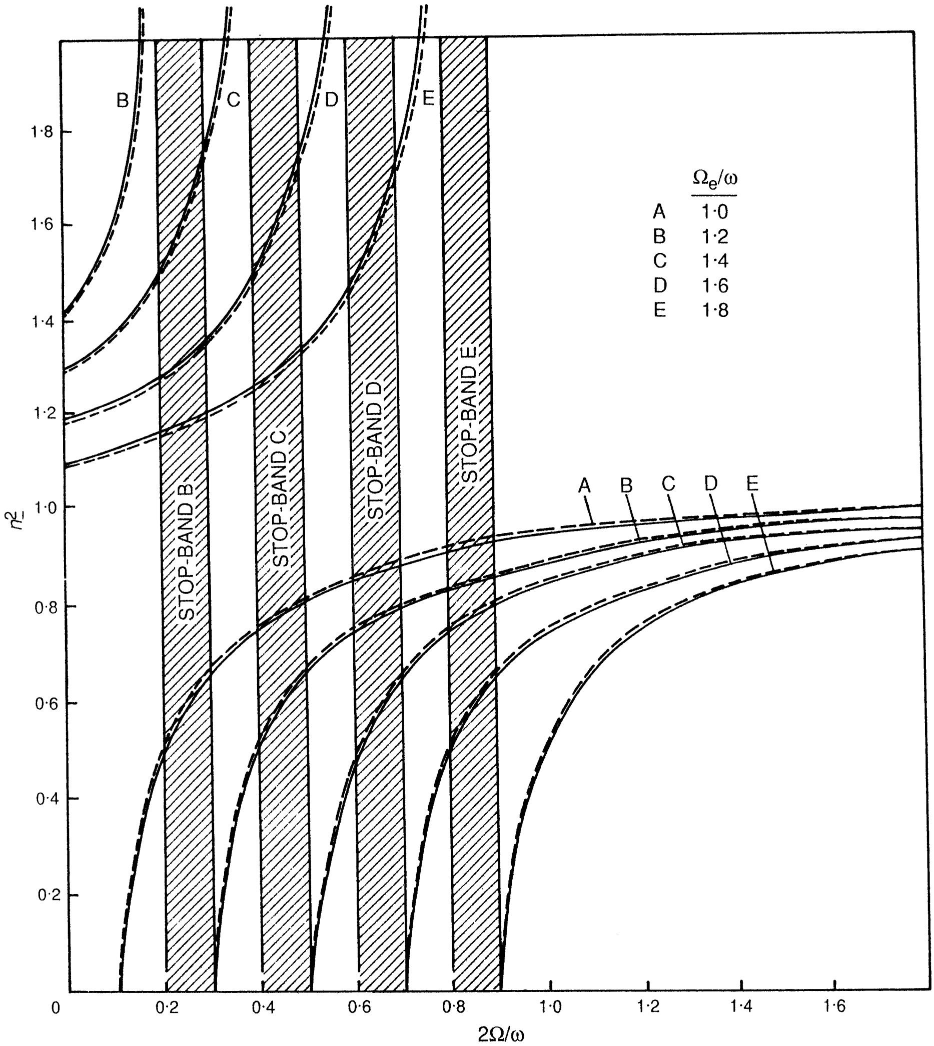

Fig. 3*a*. Variation of the square of the refractive index n_-^2 with the angular frequency of rotation $2\Omega/\omega$ for the RCP wave, for various values of $\Omega_c/\omega \geq 1$ (see curve labels) and with $(\Omega_{p_e}/\omega)^2 = 0\cdot 6$, $\alpha_c^2 = 0\cdot 1$. Full curves, $\theta = 45°$; dashed curves, $\theta = 0°$.

same as for waves propagating along the direction of the magnetic field, but the widths of the stop-band for the LCP and RCP waves decrease slightly. In Figs 1–3 we note that the location of the stop-bands of the LCP and RCP waves changes for different values of $2\Omega/\omega$ and Ω_e/ω, but in each case the width of the stop-band is the same, i.e. $\delta = 0\cdot 1$ (in units of $2\Omega/\omega$).

From the above discussions it is clear that the characteristic variations of cutoffs and resonances with the rotation introduces new stop-bands due to the nonlinear interaction of the waves and the plasma. Note that different widths of the stop-bands are obtained in comparison to those obtained by Uberoi and Das (1970). In this connection it should be mentioned that the relative locations

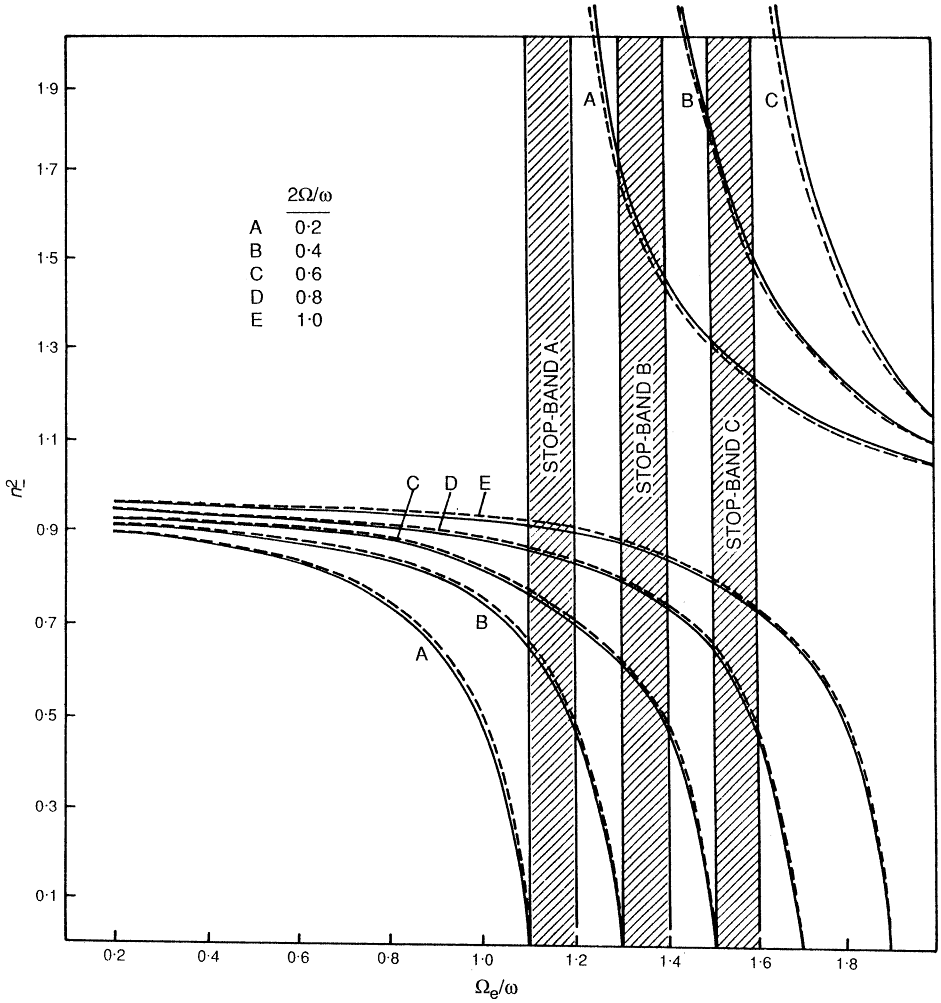

Fig. 3*b*. Variation of the square of the refractive index n_-^2 with the electron-cyclotron frequency Ω_c/ω for the RCP wave, for various values of $2\Omega/\omega \leq 1$ (see curve labels) and with $(\omega_{Pe}/\omega)^2 = 0\cdot6$, $\alpha_c^2 = 0\cdot1$. Full curves, $\theta = 45°$; dashed curves, $\theta = 0°$.

of the cutoffs and resonances may be interchanged for extraordinary waves propagating at right angles to the magnetic field in the presence of Coriolis force in a cold plasma (Uberoi and Das 1970).

3. Nonlinearly Induced Instability of the Wave

We have investigated the instability of the LCP and RCP waves including the effect of Coriolis force by finding the imagninary part of ω or k from (2), because for instability either $\mathrm{Im}\,\omega > 0$ or $\mathrm{Im}\,k > 0$. In temporal evolution problems, complex values of ω are sought from the dispersion relation to locate the instability. In spatial evolution problems, we find the solution of the dispersion relation (2) for k. It is difficult to obtain the solution of (2) for ω, but the solution for k is easily found. We consider the nonlinear dispersion relation (2) for spatial evolution in the following form:

$$\left(\frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega-\pi_e)^2}\right)k_+^2 - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{\omega_p^2(\omega-\pi_e)}\left(\frac{1}{2(\omega+\pi_e)} + \frac{1}{2(\omega-\pi_e)}\right.$$
$$\left. - \frac{\pi_e}{\omega^2-\pi_e^2}\right)k_-\,k_+ + \left[\frac{\alpha_e^2\,\omega_{p_e}^2\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\frac{\pi_e}{(\omega+\pi_e)} - \tfrac{1}{2}\right)c^2\,k_-^2 + \frac{\omega_{p_e}^2}{\omega(\omega+\pi_e)} - 1\right] = 0 \tag{4}$$

and

$$\left(\frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega+\pi_e)^2}\right)k_-^2 - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{\omega_p^2(\omega+\pi_e)}\left(\frac{1}{2(\omega-\pi_e)} + \frac{1}{2(\omega+\pi_e)}\right.$$
$$\left. + \frac{\pi_e}{\omega^2-\pi_e^2}\right)k_+k_- - \left[\frac{\alpha_e^2\,\omega_{p_e}^2\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\frac{\pi_e}{\omega-\pi_e} + \tfrac{1}{2}\right)c^2\,k_+^2 - \frac{\omega_{p_e}^2}{\omega(\omega-\pi_e)} + 1\right] = 0\,. \tag{5}$$

In equation (4) we insert the value of k_- from (1), and obtain for k_+ the quadratic equation

$$A_1k_+^2 - B_1k_+ + C_1 = 0\,. \tag{6}$$

Similarly, using the value of k_+ from (1) in (5), for k_- we obtain

$$A_2k_-^2 - B_2k_- + C_2 = 0\,, \tag{7}$$

where

$$A_1 = \frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega-\pi_e)^2}\,,$$

$$B_1 = -\frac{\alpha_e^2\,\omega_{p_e}^2\,c\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\omega^2 - \frac{\omega\,\omega_{p_e}^2}{\omega-\pi_e}\right)^{1/2},$$

$$C_1 = \frac{\alpha_e^2\,\omega_{p_e}^2\cos^2\theta}{2\omega_p^2(\omega+\pi_e)^2}\,\omega^2\left(1 - \frac{\omega_{p_e}^2}{\omega(\omega-\pi_e)}\right) - 1 + \frac{\omega_{p_e}^2}{\omega(\omega+\pi_e)}\,,$$

$$A_2 = \frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega+\pi_e)^2}\,,$$

$$B_2 = -\frac{\alpha_e^2\,\omega_{p_e}^2\,c\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\omega^2 - \frac{\omega\,\omega_{p_e}^2}{\omega+\pi_e}\right)^{1/2},$$

$$C_2 = -\frac{\alpha_e^2\,\omega_{p_e}^2\cos^2\theta}{2\omega_p^2(\omega-\pi_e)^2}\,\omega^2\left(\frac{\omega_{p_e}^2}{\omega(\omega+\pi_e)} - 1\right) - 1 + \frac{\omega_{p_e}^2}{\omega(\omega-\pi_e)}\,.$$

For the LCP wave we now substitute $k_+ = k_{r_+} + \mathrm{i}k_{i_+}$ in equation (6), and for the RCP wave we write $k_- = k_{r_-} + \mathrm{i}k_{i_-}$ in equation (7). Then the imaginary and real parts are

$$k_{r_+} = -\frac{\alpha_e^2\,\omega_{p_e}^2\,c\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\frac{\omega^2(\omega-\pi_e) - \omega\,\omega_{p_e}^2}{\omega-\pi_e}\right)^{1/2}$$

$$\times\left(\frac{2c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{\omega_p^2(\omega-\pi_e)^2}\right)^{-1}, \tag{8}$$

$$k_{i_+} = \pm\left(\frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega-\pi_e)^2}\right)^{-1}\left\{\left(\frac{c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{2\omega_p^2(\omega-\pi_e)^2}\right)\right.$$

$$\times\left[\frac{\alpha_e^2\,\omega_{p_e}^2\cos^2\theta}{2\omega_p^2(\omega+\pi_e)^2}\,\omega\left(\frac{\omega^2-\pi_e\,\omega-\omega_{p_e}^2}{\omega-\pi_e}\right) - 1 + \frac{\omega_{p_e}^2}{\omega(\omega+\pi_e)}\right]$$

$$\left. - \frac{\alpha_e^4\,\omega_{p_e}^4\,c^2\cos^4\theta}{4\omega_p^4(\omega^2-\pi_e^2)^2}\,\omega\left(\frac{\omega^2-\pi_e\,\omega-\omega_{p_e}^2}{\omega-\pi_e}\right)\right\}^{1/2}, \tag{9}$$

$$k_{r_-} = -\frac{\alpha_e^2\,\omega_{p_e}^2\,c\cos^2\theta}{\omega_p^2(\omega^2-\pi_e^2)}\left(\frac{\omega^2(\omega+\pi_e) - \omega\,\omega_{p_e}^2}{\omega+\pi_e}\right)^{1/2}$$

$$\times\left(\frac{2c^2}{\omega^2} - \frac{\alpha_e^2\,\omega_{p_e}^2\,c^2\cos^2\theta}{\omega_p^2(\omega+\pi_e)^2}\right)^{1/2}, \tag{10}$$

$$k_{\mathrm{i}_-} = \pm\left(\frac{c^2}{\omega^2} - \frac{\alpha_{\mathrm{e}}^2\,\omega_{\mathrm{p_e}}^2\,c^2\cos^2\theta}{2\omega_{\mathrm{p}}^2(\omega+\pi_{\mathrm{e}})^2}\right)^{-1}\left\{\left(\frac{c^2}{\omega^2} - \frac{\alpha_{\mathrm{e}}^2\,\omega_{\mathrm{p_e}}^2\,c^2\cos^2\theta}{2\omega_{\mathrm{p}}^2(\omega+\pi_{\mathrm{e}})^2}\right)\right.$$

$$\times\left[\frac{\alpha_{\mathrm{e}}^2\,\omega_{\mathrm{p_e}}^2\cos^2\theta}{2\omega_{\mathrm{p}}^2(\omega-\pi_{\mathrm{e}})^2}\,\omega\left(\frac{\omega^2+\pi_{\mathrm{e}}\,\omega-\omega_{\mathrm{p_e}}^2}{\omega+\pi_{\mathrm{e}}}\right) - 1 + \frac{\omega_{\mathrm{p_e}}^2}{\omega(\omega-\pi_{\mathrm{e}})}\right]$$

$$\left. - \frac{\alpha_{\mathrm{e}}^4\,\omega_{\mathrm{p_e}}^4\,c^2\cos^4\theta}{4\omega_{\mathrm{p}}^4(\omega^2-\pi_{\mathrm{e}}^2)^2}\,\omega\left(\frac{\omega^2+\pi_{\mathrm{e}}\,\omega-\omega_{\mathrm{p_e}}^2}{\omega+\pi_{\mathrm{e}}}\right)\right\}^{1/2}. \tag{11}$$

Relations (8)–(11) show that the amplitude-dependent terms cause the instability of the wave, because if $\alpha_{\mathrm{e}} = 0$, then from (8) and (10) it is seen that $k_{\mathrm{r}_+} = 0 = k_{\mathrm{r}_-}$. But relations (9) and (11) show that

$$k_{\mathrm{i}_+} = \pm\,\mathrm{i}\left(1 - \frac{\omega_{\mathrm{p_e}}^2}{\omega(\omega+\pi_{\mathrm{e}})}\right)^{1/2}, \tag{12}$$

$$k_{\mathrm{i}_-} = \pm\,\mathrm{i}\left(1 - \frac{\omega_{\mathrm{p_e}}^2}{\omega(\omega-\pi_{\mathrm{e}})}\right)^{1/2}; \tag{13}$$

hence k_+ and k_- become real, which indicates that the waves are neither amplified nor attenuated. These results also follow from the dispersion relation (1) of the linearised approximation. Evidently, the LCP and RCP waves become unstable due to nonlinear interaction with the plasma (i.e. k_{i_+} is real) when the dimensionless wave amplitude satisfies the condition

$$\alpha_{\mathrm{e}}^2 > \frac{[1\pm(\pi_{\mathrm{e}}/\omega)-(\omega_{\mathrm{p_e}}/\omega)^2]\,[1\pm(\pi_{\mathrm{e}}/\omega)]\,[1\mp(\pi_{\mathrm{e}}/\omega)]^2}{(\pi_{\mathrm{e}}/\omega)\,[(\omega_{\mathrm{p_e}}/\omega)^2\mp 2]\cos^2\theta}. \tag{14}$$

This is the limiting value of the amplitudes of the LCP and RCP waves for instability in a rotating magnetised plasma. This value of α_{e} may be less than or greater than unity for any value of the rotational frequency and gyrofrequency. Moreover, expression (14) determines the actual limit of intensity of the LCP and RCP waves for occurrence of the instability at different values of the magnetic field. The limiting value of the power of unstable waves is obtained from the relation (Chakraborty 1977) (1 erg $\equiv 10^{-7}$ J)

$$P = 6{\cdot}7\times10^{-4}\,\alpha_{\mathrm{e}}^2\,\omega^2\ \mathrm{erg\,cm^{-2}\,s^{-1}}. \tag{15}$$

In the following, the critical values of the dimensionless amplitude/power of the unstable electromagnetic waves are estimated for different values of $\Omega_{\mathrm{e}}/\omega$, $(\omega_{\mathrm{p_e}}/\omega)^2$ and $2\Omega/\omega$, and these are furnished in Tables 1 and 2. In a cold plasma having an electron density $\simeq 10^{12}$ cm^{-3}, the powers of the unstable electromagnetic waves are (i) $5{\cdot}1\times10^{13}$ W m^{-2}, (ii) $6{\cdot}7\times10^{13}$ W m^{-2}, and (iii) $9{\cdot}95\times10^{13}$ W m^{-2}

Table 1. Limiting value of the power of the unstable wave

$(\omega_{p_e}/\omega)^2$	θ	π_c/ω	$2\Omega/\omega$	Ω_c/ω	α_c	P (10^{13} W m^{-2})
0·10	0°	0·45	0·70	0·25	0·87	5·1
		0·40	0·60	0·20	0·95	6·2
		0·35	0·50	0·15	1·00	6·7
0·20	0°	0·37	0·65	0·28	1·01	6·72
		0·36	0·62	0·26	1·02	6·95
		0·35	0·57	0·22	1·03	7·20
0·25	0°	0·34	0·63	0·29	1·04	7·23
		0·30	0·64	0·34	1·11	8·24
		0·28	0·68	0·38	1·20	9·95

Table 2. Limiting value of the static magnetic field for the unstable wave

$(\omega_{p_e}/\omega)^2$	θ	π_c/ω	$2\Omega/\omega$	Ω_c/ω	α_c	H_0 (10^4 G)
0·1	0°	0·35	0·50	0·15	1·00	1·0
		0·36	0·57	0·22	1·02	1·7
		0·34	0·63	0·29	1·04	2·3

when the wave frequency, plasma frequency and gyrofrequency satisfy the following conditions: (i) $\pi_e/\omega = 0{\cdot}45$, (ii) $\pi_e/\omega = 0{\cdot}35$, (iii) $\pi_e/\omega = 0{\cdot}28$, and (i) $(\omega_{p_e}/\omega)^2 = 0{\cdot}1$, (ii) $(\omega_{p_e}/\omega)^2 = 0{\cdot}20$, (iii) $(\omega_{p_e}/\omega)^2 = 0{\cdot}25$, respectively. Similar analysis yields the limiting value of the static magnetic field (H_0) for which a particular electromagnetic (EM) wave will be unstable in a rotating plasma. We see that an EM wave having power $\simeq 10^{13}$ W m^{-2} and frequency $\simeq 10^{10}$ Hz becomes unstable in a plasma having density $\simeq 10^{12}$ cm^{-3} and rotational frequency 10^8 rad s^{-1} when the static magnetic field is greater than 10^2 G. For these values of the field parameters in the plasma, the gyrofrequency Ω_e is less than the wave frequency and greater than the angular frequency of rotation (i.e. $\Omega < \Omega_e < \omega$).

Paul (1990) considered the case of $\Omega_e > \omega$ in a non-rotating relativistic plasma and obtained the limiting value of the powers of electromagnetic (EM) waves in a laser-induced plasma. However, comparing the results of Paul (1990) with ours we see that the limiting value of the power of the unstable wave is much higher in our case. This may be due to the effect of the Coriolis force in the plasma, or to the low value of the static magnetic field when $\Omega_e < \omega$. Here the effect of relativistic variation of mass of the plasma particles is not considered. To get an idea of the role of the Coriolis force of rotation on the stability of the wave, we have plotted the attenuation of the LCP and RCP waves in Fig. 4. It is seen that the attenuation of the LCP wave decreases with an increase in the value of $2\Omega/\omega$, but for the RCP wave the attenuation increases sharply with an increase in the rotational frequency. In an astrophysical plasma the condition for stability of the wave in the presence of Coriolis force may be satisfied. Then it may become easy to explain some astrophysical phenomena, e.g. mass loss, certain explosions, heating processes, etc.

Parker (1965), Dessler (1967), Barnes (1969) and others have suggested that a non-thermal mechanism is the source of heating of the plasma in the solar corona. The dissipation of magnetohydrodynamic (MHD) waves may also be the

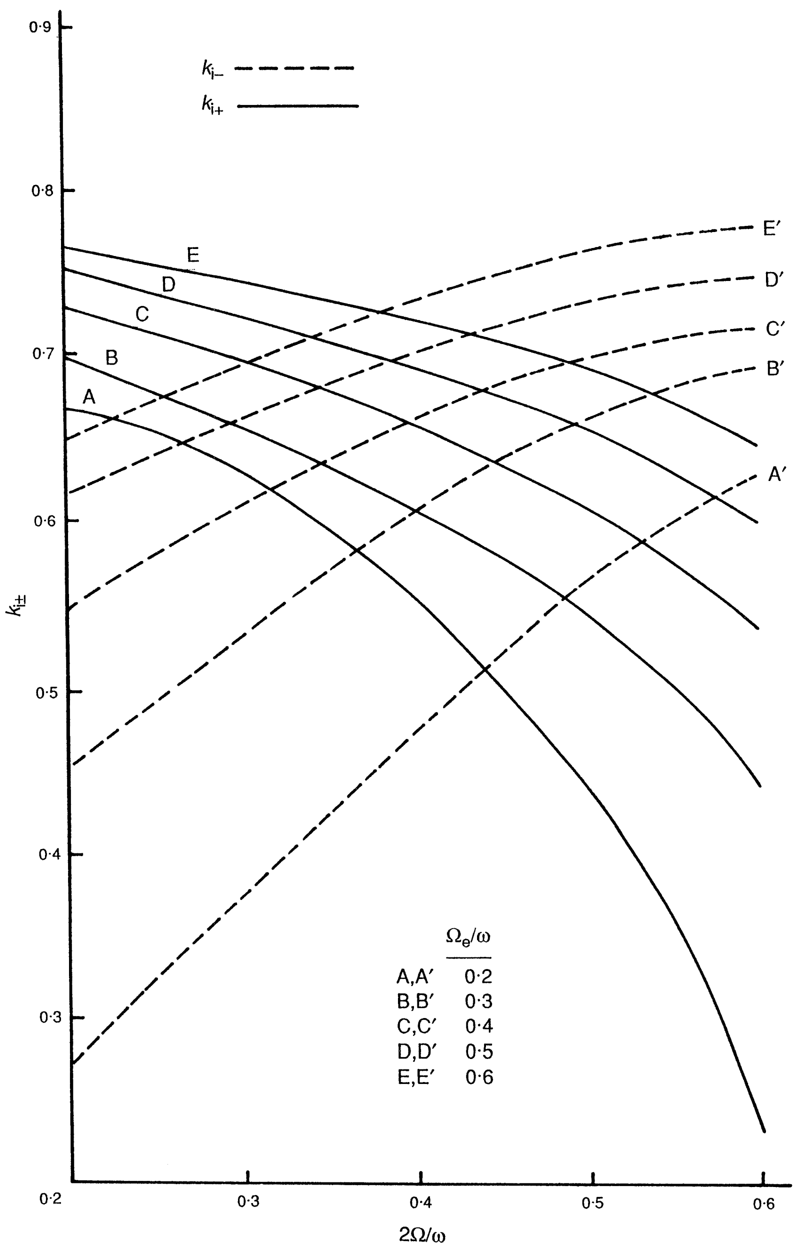

Fig. 4. Variation of the attenuation $k_{i\pm}$ of the LCP and RCP waves with the angular frequency of rotation $2\Omega/\omega$, for various values of $\Omega_c/\omega < 1$ (see labels) and with $(\omega_{pe}/\omega)^2 = 0 \cdot 55$, $\omega = 5 \times 10^6$ rad s^{-1}, $\alpha_c^2 = 0 \cdot 1$, $\theta = 45°$.

source of heating of astrophysical plasmas (Sturrock 1966). MHD waves may be dissipated in a collision-dominated plasma due to electrical resistivity, viscosity or the formation of shock waves (Schatzman and Suffrin 1967; Jordan 1968), but Barnes (1967, 1968) showed that the damping of collisionless hydromagnetic waves is the cause of heating of the solar corona. Later, Bandyopadhaya (1972) suggested that astrophysical objects may be heated by damping of MHD waves inside rotating stellar bodies.

It is well known that huge amounts of ionised and non-ionised gases are emitted from (i) the nuclei of galaxies (Rougoor and Oort 1960; Woltjer 1965; Oort 1971; Bandyopadhaya 1974; Bandyopadhaya *et al.* 1974), (ii) the sun (Zheleznyakov 1964; Harrision 1986; Low 1986; Kahler 1987; Hundhansen 1988), (iii) Be-stars (Underhill 1960; Weymann 1963; Strittmatter *et al.* 1970), (iv) supergiants (Deutsch 1966; Morton 1967*a*, *b*, 1969), and (v) Wolf–Rayet stars (Basu and Bandyopadhaya 1971), as well as many other hot astrophysical bodies. Various authors have suggested different mechanisms to explain the mass loss phenomenon (Rubra and Cowling 1959; Strittmatter *et al.* 1970; Basu and Bandyopadhaya 1971; Paul 1977), but no theory has been found to be quite conclusive. However, several authors (Kippenhahn 1970; Strittmatter *et al.* 1970) believe that rotational motion makes an important contribution to the ejection of mass from stars and other astrophysical objects. A part of the rotational energy is transformed into turbulent energy by some agency (e.g. magnetic field), which in turn dissipates as heat, thereby heating the overlying gas and, as a result, mass is ejected from these objects.

4. Concluding Remarks

Nonlinear interaction of waves and plasmas gives rise to self-action effects, e.g. self-focusing, self-trapping, wavenumber shift/frequency shift, precessional rotation, etc. The effect of a shift of the wavenumber and frequency is expected to occur when the field intensity is not too high or too low. When the intensity is very high the refractive index increases locally to a considerable value and the nonlinear effects considered above are destroyed by the consequent bending of the ray-direction. In the very-low-intensity limit, the linearised solution holds good and all the nonlinear effects vanish automatically. Kaw and Dawson (1970) showed that the threshold power of an EM wave for the occurrence of self-action phenomena in a dense plasma (electron density $\simeq 10^{18}$ cm^{-3}) is 10^{23} W m^{-2}.

A dc magnetic field is generated by the circular motion of the plasma electrons in the plasma (the inverse Faraday effect, IFE). Deschamps *et al.* (1970) experimentally observed that a uniform magnetisation of the order of 10^{-2} G is generated in a plasma by a pulsed microwave signal having frequency 3000 MHz and a repetition frequency of 10 Hz. In the present investigation the power of the EM wave is below the threshold power obtained by Kaw and Dawson (1970), so our results will have physical significance for laboratory experiments and space plasmas. As the induced magnetisation observed by Deschamps *et al.* (1970) is very low in comparison with the static magnetic field used in numerical calculations, the IFE is neglected in the present analysis.

The instability of the EM wave discussed here is not due to self-focusing or filamentation; it is a modulational instability. Paul (1990) showed that, due to self-action effects, EM waves having power much below that for the occurrence of nonlinear phenomena are unstable in a strongly magnetised, relativistic dense plasma ($H_0 = 10^8$ G, $P = 10^{15}$–10^{19} W m^{-2}, $N_0 = 10^{20}$ cm^{-3}). Here the limiting power of the unstable wave is assumed to be 10^{13} W m^{-2}, $H_0 = 10^4$ G, the plasma density $N_0 = 10^{12}\,\mathrm{cm}^{-3}$ and rotational velocity $5\times10^8\,\mathrm{rad\,s}^{-1}$. Astrophysical bodies such as magnetic stars, pulsars, the Crab nebula, etc., have rotational frequencies very close to their gyrofrequencies, so for these systems the modulational instability of the wave will be very significant. It is well known that hot stars, including the sun, emit different types of waves having different frequencies and powers. Moreover, such objects have magnetic fields of different strengths. Not all the waves emitted from a star will be modulationally unstable: only if the conditions for the power, frequency and gyrofrequency causing the instability are satisfied, will a given wave be unstable.

In recent years, several authors have studied the effects of nonlinearity on wave propagation in relativistic plasmas (Lerche 1967, 1969; Kaw and Dawson 1970; Winkles and Eldridge 1972; Stenflo *et al.* 1983; Kennel and Pellat 1976; Shih 1978; Stenflo and Tsinstadze 1979; Hora 1981; Tsytovich and Stenflo 1983; Shukla *et al.* 1986). It has been found that relativistic mass correction effects for electron and ion motions are significant in the shift of wave parameters (Sluijter and Montgomery 1965; Tidman and Stainer 1965; Boyd 1967; Das 1968, 1971; Chandra 1974), precessional rotation (Arons and Max 1974; Lai and Wonnacott 1976; Chakraborty 1977; Khan and Chakraborty 1979; Bhattacharyya and Chakraborty 1979, 1982; Bhattacharyya 1983; Chakraborty *et al.* 1984; Chakraborty and Paul 1984), the IFE (Steiger and Woods 1972), solitons and shocks (Das and Paul 1985; Nejoh 1987, 1988; Das *et al.* 1988; Roy Chowdhury *et al.* 1988, 1989*a,b*, 1990; Salauddin 1990; Chakraborty *et al.* 1992) and other nonlinear phenomena (Das and Sihi 1977; Shih 1978) in plasma. It has been observed that an EM wave will not penetrate a dense plasma unless the velocity of the electrons is relativistic, and it becomes imperative to consider the mass variation of the electron. During solar bursts and in the evolution of stars and pulsar radiation, ionised particles are ejected from these astrophysical bodies with very high velocities (close to the velocity of light: Zheleznyakov 1964; Kaplan and Tsytovich 1979; Snow 1979; Sreenivason 1979; ter Haar and Tsytovich 1981; Low 1986; Harrision 1986; Kahler 1987). It is therefore expected that more interesting results will be obtained from analysis of the stability of waves in a rotating plasma, if relativistic effects are taken into account. This would provide further ideas for understanding the peculiar and unknown characteristics of waves in astrophysical plasmas as well as in laboratory experiments.

Acknowledgment

The authors are grateful to the referee for useful comments and suggestions which helped in bringing the paper to its present version.

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Manuscript received 1 December 1992, accepted 24 August 1993