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Relativistic Bound States*

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Abstract

The standard results of the quark model rely on nonrelativistic descriptions of the wavefunctions of the quarks in the hadron. However, simple uncertainty principle considerations show that the momenta of the quarks are comparable to their (constituent) masses, so that relativistic dynamics must be used in the description of the hadronic structure. In this paper I describe a number of new and old results which illustrate relativistic effects on the quark structure of hardons, and which employ the method of light cone quantisation to handle relativity.

1. Introduction

One of the early successes of the quark model was the calculation by de Rujula *et al.* (1975) of the magnetic moment of the hadrons in terms of the magnetic moments of the quarks. Each quark is given its Dirac magnetic moment

$$\mu_i = \frac{Q_i e}{2m_i} \tag{1}$$

and the proton magnetic moment, say, is computed from the standard definition

$$\mu_p = \langle p\uparrow \mid \sum_i \mu_i \sigma_i \mid p\uparrow \rangle . \tag{2}$$

Using the well known spin–isospin wavefunction of the proton

$$\mid p\uparrow \rangle = \sqrt{\tfrac{1}{6}} \mid 2u_\uparrow u_\uparrow d_\downarrow - (u_\uparrow u_\downarrow - u_\downarrow u_\uparrow) d_\uparrow \rangle , \tag{3}$$

one obtains the now classical result

$$\mu_{\rm p} = \frac{e}{2M_{\rm N}} \frac{M_{\rm N}}{m_{\rm q}} , \tag{4}$$

assuming that the nonstrange quarks have the same mass $m_{\rm q}$.

* Dedicated to Professor Ian McCarthy on the occasion of his sixtieth birthday.

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Similarly one obtains the result

$$\mu_{\rm n} = \frac{e}{2M_{\rm N}}\left(-\frac{2M_{\rm N}}{3m_{\rm q}}\right). \tag{5}$$

These results of de Rujula *et al.* (1975) represent one of the early and great triumphs of the quark model. The value predicted for the ratio $\mu_{\rm n}/\mu_{\rm p}$, $-\frac{2}{3}$, is very close to the observed value, $-0\cdot685$, and using the experimental value for the proton magnetic moment one obtains the quark mass as $m_{\rm q} = 336$ MeV.

However, in the same paper, de Rujula *et al.* computed the $\Delta - N$ mass splitting from the colour hyperfine interaction, a potential proportional to $\delta(\boldsymbol{r})$. In this way one obtains the result that

$$M_\Delta - M_{\rm N} \propto |\,\psi(0)\,|^2 \propto 1/R_{\rm p}^3\,. \tag{6}$$

In this way the $\Delta - N$ mass difference gives an estimate of the proton radius of $R_{\rm p} \approx 0\cdot5$ fm. Other estimates may be obtained from electron scattering ($R_{\rm p} \approx 0\cdot8$ fm), and nonleptonic decays of the hyperons ($R_{\rm p} \approx 0\cdot4$ fm). A summary of the various estimates of the proton radius parameter was given in the paper of Thomas and McKellar (1984). Through the uncertainty principle they correspond to mean quark momenta of 400, 250 and 500 MeV respectively.

The very paper containing one of the great results of the nonrelativistic quark model contains the information that the quarks are relativistic. Clearly the quark model must be extended to encompass relativistic quarks. This leads one into deep questions—the problem of a pair of relativistic particles interacting through a potential has no unique formulation, as one may have expected because $p \approx m$ implies there is sufficient energy to create pairs. Strictly speaking, the relativistic two-body problem does not exist—one must deal through field theory with an N-body problem, where N is indeterminate. Even the apparently simple case of the electron–positron bound states contain interesting physics when studied closely enough.

In this paper I concentrate on one particular approach to the problem, through light cone quantisation, discussing first the two-body problem and then the dynamical field theory approach.

2. Light Cone Quantisation

The possibility of quantising a relativistic system using light cone coordinates was introduced by Dirac (1949) in a remarkable paper which, incidently, contains a throw-away line remarking that there is no obvious reason why P and T should be conserved in nature. Dirac identified the basic elements of a quantised theory as the generators P^μ and $M^{\mu\nu}$ of the Poincaré group, and asserted that the aim of any quantised theory is to obtain a representation of the commutation relations which define the Poincaré group:

$$[P^\mu, P^\nu] = 0\,, \tag{7}$$

$$[M^{\mu\nu}, P^\rho] = -g^{\mu\rho}P^\nu + g^{\nu\rho}P^\mu\,, \tag{8}$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -g^{\mu\rho}M^{\nu\sigma} + g^{\nu\rho}M^{\mu\sigma} - g^{\mu\sigma}M^{\rho\nu}m + G^{\nu\sigma}M^{\rho\mu}\,. \tag{9}$$

These commutation relations are taken at equal times. It was Dirac's key insight to separate the role of time as a parameter of the underlying space–time from its role as parametrising the evolution generated by the Hamiltonian operator, and consequently being the parameter held constant in the commutation relations. Dirac therefore investigated alternative quantisation schemes. Before describing the light cone quantisation he introduced, I first review the usual quantisation scheme.

In *instant dynamics* the system is characterised by the coordinates $\boldsymbol{q}_i$ and momenta $\boldsymbol{p}_i$ of the particles at an *instant* $t = \tau_0$. Consider first the case of just one particle. The Poincaré generators divide into two groups:

- *Simple generators* are those that generate transformations which leave the instant $t = \tau_0$ invariant. These comprise the 3-momenta

$$P^i = p^i, \tag{10}$$

and the angular momenta

$$M^{ij} = q^i p^j - q^j p^i. \tag{11}$$

- *Hamiltonian generators* are those which transform the instant $t = \tau_0$. These include the Hamiltonian

$$P^0 = [\boldsymbol{p}^2 + m^2]^{\frac{1}{2}}, \tag{12}$$

and the boost operators

$$M^{0i} = [\boldsymbol{p}^2 + m^2]^{\frac{1}{2}} q^i. \tag{13}$$

It is the simple generators which are additive when one considers a many-particle system, whereas the Hamiltonian generators for the composite system are not additive. If one introduces interactions between the particles, then these interactions modify the Hamiltonian generators in a way which we do not know how to define.

In *light cone dynamics*, introduced by Dirac, instead of concentrating on those generators which preserve the instant $t = \tau_0$, one pays attention to those generators which preserve the *light cone*, $t - z = \lambda_0$. First, recall that the light cone components of any vector A^μ are defined by*

$$A^\pm = \sqrt{\tfrac{1}{2}}[A^0 \pm A^3], \quad A^i = A^i, \quad \text{for } i = 1,2, \tag{14}$$

so that now vectors are represented by the components (A^+, A^-, A^1, A^2), the non-vanishing components of the metric tensor are

$$g_{+-} = g_{-+} = 1, \qquad g_{11} = g_{22} = -1, \tag{15}$$

* Some authors omit the $\sqrt{\frac{1}{2}}$, with consequent alterations to the metric tensor, covariant components, etc.

and the covariant components of the vector are ($A_+ = A^-, A_- = A^+, A_1 = -A^1, A_2 = -A^2$).

In this case the generators again divide into simple and Hamiltonian generators.

- The *simple generators* are now

$$P^i = p^i, \qquad P^- = p^-, \tag{16,17}$$

$$M^{12} = q^1 p^2 - q^2 p^1, \tag{18}$$

$$M^{i-} = q^i p^-, \qquad M^{+-} = q^+ p^-. \tag{19,20}$$

- The *Hamiltonian generators* in the light cone dynamics are

$$P^+ = \frac{p_\perp^2 + m^2}{p^-}, \tag{21}$$

$$M^{i+} = q^i \frac{p_\perp^2 + m^2}{p^-} - q^+ p^i. \tag{22}$$

In this case the construction of many-particle dynamics is just as difficult, but Dirac, and many people since him, have been struck by the analogy between equation (21) for P^+ in the light cone dynamics and the Hamiltonian operator in nonrelativistic quantum mechanics. Moreover, no square roots appear in the generators. This has given hope that there may be a simpler construction of many-particle dynamics in the light cone form than in the instant form—as we see in the next section, this hope is realised in part.

To illustrate, consider the case of three free particles. The appropriate relativistic generalisation of the Jacobi coordinates for the three-particle system are the coordinates $P^-, \boldsymbol{P}_\perp, \eta, \xi, \boldsymbol{Q}_\perp$ and $\boldsymbol{q}_\perp$ which are related to the simple generators $p_{(a)-}$ and $p_{(a)\perp}$, with $a = 1, 2, 3$ labelling the particles, by

$$p^{(1)-} = \xi\eta P^-, \tag{23}$$

$$p^{(2)-} = (1 - \xi)\eta P^-, \tag{24}$$

$$p^{(3)-} = (1 - \eta)P^-, \tag{25}$$

$$\boldsymbol{p}_{(1)\perp} = \boldsymbol{q}_\perp + \xi \boldsymbol{Q}_\perp + \xi\eta \boldsymbol{P}_\perp, \tag{26}$$

$$\boldsymbol{p}_{(2)\perp} = -\boldsymbol{q}_\perp + (1 - \xi)\boldsymbol{Q}_\perp + (1 - \xi)\eta \boldsymbol{P}_\perp, \tag{27}$$

$$\boldsymbol{p}_{(3)\perp} = -\boldsymbol{Q}_\perp + (1 - \eta)\boldsymbol{P}_\perp. \tag{28}$$

In terms of the variables the 'Hamiltonian' P^+ is given by

$$P^+ = \frac{1}{2P^-}(\boldsymbol{P}_\perp{}^2 + M_0^2), \tag{29}$$

where

$$M_0^2 = \frac{\boldsymbol{Q}_\perp^2}{\eta(1-\eta)} + \frac{M_{12}^2}{\eta} + \frac{m_3^2}{1-\eta}, \tag{30}$$

$$M_{12}^2 = \frac{\boldsymbol{p}_\perp{}^2}{\xi(1-\xi)} + \frac{m_1^2}{\xi} + \frac{m_2^2}{1-\xi}. \tag{31}$$

This separates the total momentum $(P^+, P^-, \boldsymbol{P}_\perp)$ from the internal variables which are contained in M_0^2. It is then natural to make the simple ansatz that the wavefunction describing the relative motion is a function only of M_0^2, $\phi(M_0^2)$. In performing integrals over the internal variables the volume element is

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^6} \frac{\mathrm{d}^2 q_\perp \,\mathrm{d}\xi}{2\xi(1-\xi)} \frac{\mathrm{d}^2 Q_\perp \,\mathrm{d}\eta}{2\eta(1-\eta)}. \tag{32}$$

3. Light Cone Wavefunctions and Baryon Electroweak Moments

The calculaton of the anomalous magnetic moments of the hadrons using the light cone method was pioneered by Berestetskii and Terent'ev (1977), and elaborated by Aznauryan and Ter-Isaakyan (1980) and Tupper *et al.* (1988). This work shows how to correct the calculation of de Rujula *et al.* (1975) for the relativistic effects which motivated this discussion. Costella and McKellar (1991) have extended these results to other electroweak moments, in particular the anomalous magnet dipole moments, the electric dipole moments, the anapole moments, and g_A/g_V. An interesting relation is obtained between the relativistic correction to these parameters. In this section I describe these results.

In the light cone coordinates a convenient choice of Dirac matrices is

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{33,34}$$

$$\gamma^j = \mathrm{i}\,\epsilon^{jk} \begin{pmatrix} \sigma^k & 0 \\ 0 & -\sigma^k \end{pmatrix}, j,k = 1,2, \qquad \gamma^5 = \begin{pmatrix} \sigma^3 & 0 \\ 0 & -\sigma^3 \end{pmatrix}. \tag{35,36}$$

With this representation of the gamma matrices, the positive energy free particle spinor is

$$u(p) = \frac{1}{(2\sqrt{2}\,p_-)^{\frac{1}{2}}} \begin{pmatrix} \phi^\lambda \\ (2p_-)^{-1}(m - \mathrm{i}\,p_\perp \cdot \epsilon_\perp \cdot \sigma_\perp)\phi^\lambda \end{pmatrix}. \tag{37}$$

Here ϕ^λ is a Pauli-type two-dimensional spinor, and $p_\perp \cdot \epsilon_\perp \cdot \sigma_\perp$ is a scalar product of the two-dimensional transverse vectors $p_\perp$ and $\sigma_\perp$ and the two-dimensional alternating tensor $\epsilon_\perp$.

The first step in the straightforward, but complex, task of computing the matrix elements of the quark currents between hadronic states, is to obtain a representation of the hadronic states in terms of the light cone representation of the Dirac spinors. The prescription for these states is that they are obtained by a Melosh (1974) transformation Ω of SU(6) wavefunctions of the type quoted in equation (3). The details are given in Terent'ev and Berestetskii (1976), Terent'ev (1976) and Berestetskii and Terent'ev (1977). Formally, the same matrix elements are obtained by transforming the Dirac operators, e.g.

$$\sigma_\perp^\Omega = \Omega \sigma_\perp \Omega^{-1} . \tag{38}$$

The transformation operator Ω is a product of factors like

$$U = \cos\alpha + \mathrm{i}\,\sigma_\perp \epsilon\, \boldsymbol{n}_\perp \sin\alpha \tag{39}$$

and it implements the mixing of orbital and intrinsic angular momentum characteristic of relativistic effects. I note in passing that Tupper (personal communication 1990) has shown that the Melosh transformation leads to a violation of the Pati–Woo theorem in the calculation of the matrix elements of the weak Hamiltonian between hadronic states, simply because of this mixing. This provides an excellent example of the way in which relativistic effects are able to introduce *qualitative* changes in results. Costella (1990) has given a pedagogical account of the manipulations necessary to obtain the matrix elements.

In principle the procedure is simple. One starts with a sum of quark currents of the form

$$J^\mu = \sum_a j^\mu_{(a)} , \tag{40}$$

$$j^\mu_{(a)} = \overline{q}\left(Q_a \gamma^\mu + \frac{\kappa_a}{2m_a} \sigma^{\mu\nu} k_\nu + D_a \gamma_5 \sigma^{\mu\nu} k_\nu + A_a [\gamma^\mu k^2 - \gamma^\nu k_\nu k^\mu] \right) q \tag{41}$$

(here Q_a, κ_a, D_a, and A_a are the charge, anomalous magnetic moment, electric dipole moment, and anapole moment of the a-quark), transform it by the Melosh transformation and evaluate its matrix elements between hadronic states,

$$\langle H_\mathrm{f} |\, J^\mu \,| H_\mathrm{i} \rangle = \overline{u}_\mathrm{f}\left(Q_h \gamma^\mu + \frac{\kappa_h}{2m_h} \sigma^{\mu\nu} k_\nu + D_h \gamma_5 \sigma^{\mu\nu} k_\nu + A_h [\gamma^\mu k^2 - \gamma^\nu k_\nu k^\mu] \right) u_i , \tag{42}$$

to identify Q_h, κ_h, D_h, and A_h, the charge, anomalous magnetic moment, electric dipole moment, and anapole moment of the hadron h.

In practice the analytic calculations are rather lengthy. When they are done, the results divide into two classes:

- the contribution to the anomalous hadron magnetic moment from the Dirac moments of the quarks, considered by Tupper *et al.* (1988), and
- the contribution of the non-Dirac moments of the quarks to the corresponding moments of the hadrons, considered by Costella (1990) and Costella and McKellar (1991).

In the first case the final result is that the *anomalous* magnetic moment of the nucleon is given by

$$\kappa = \int \mathrm{d}\Gamma \, \frac{|\phi(M_0)|^2}{2M_0}[4\tau_3 + (\tfrac{2}{3} - \eta)(2\tau_3 - \tfrac{1}{2})\eta^{-1}]$$

$$\times \frac{2\eta(1-\eta)M_0^2 + \eta m_q M_0 - \frac{1}{2}Q_\perp^2}{Q_\perp^2 + [m_q + (1-\eta)M_0]^2}. \tag{43}$$

In equation (43) the internal wavefunction $\phi(M_0)$ is often taken to be a simple gaussian function with a width α such that α^{-1} is proportional to the radius of the hadron. On simple dimensional grounds we see that, as $m_q \to 0$, α is the only dimensional parameter remaining in the problem, and thus that $\kappa \propto \alpha^{-1} \propto R_h$. Contrast this to the bag model result that for the *total* magnetic moment of the hadron $\mu \propto R_h$. The Drell–Hearn sum rule in fact requires that $\kappa \propto R_h$, giving an argument in favour of the lightcone wavefunction technique over the bag model.

Unfortunately, the numerical results obtained by Tupper *et al.* (1988) show that in the simple gaussian wavefunction model it is not possible simultaneously to obtain a good fit to the anomalous magnetic moments of the octet baryons. Some physics has been left out—a better account of the confinement effects, meson cloud effects and quark anomalous moments have all been proposed as possible cures of the discrepancies, but it would take me too far from my theme to discuss these effects here.

In the second case, the contribution of the quark moments to the corresponding hadron moments, I quote the results for the contribution of the quark electric dipole moments to electric dipole moment of the neutron. I choose the neutron because the electric dipole moment of the neutron has been extensively investigated experimentally and theoretically, as has been reviewed by Ramsey (1990) and He *et al.* (1989). The result is

$$D_n = \tfrac{1}{3}Z_1(4D_\mathrm{d} - D_\mathrm{u}), \tag{44}$$

where

$$Z_1 = \int \mathrm{d}\Gamma \, |\phi(M_0)|^2 \left(1 - \frac{Q_\perp^2}{Q_\perp^2 + [m_\mathrm{q} + (1-\eta)M_0]^2}\right). \tag{45}$$

In the nonrelativistic limit ($m_q \to \infty$), we have $Z_1 \to 1$ giving the standard nonrelativistic result for the contribution of the dipole moment of the quarks to that of the neutron (see e.g. He *et al.* 1989). In the extreme relativistic limit ($m_q \to 0$), we have $Z_1 \to \frac{1}{2}$. For values of m_q and α used by Tupper *et al.* (1988) in their calculation of the hadron magnetic moments, the correction to the nonrelativistic result is of order of a few per cent, indicating that relativistic effects do not play the same large role in this calculation that they played in the magnetic moment calculation. The relativistic correction to the contribution of the quark anomalous moments to the hadron anomalous moments also introduces this same factor Z_1 (Aznauryan and Ter-Isaakyan 1980), which is not surprising when one notes that $\gamma_5 \sigma^{\mu\nu} \propto \epsilon^{\mu\nu\rho\tau}\sigma_{\rho\tau}$.

More interesting is the fact that the same integral appears when one calculates the relativistic correction to the ratio g_A/g_V,

$$g_A/g_V = \tfrac{5}{3}(1-2Z_1). \tag{46}$$

This permits the determination of an *empirical* value of the relativistic correction factor Z_1,

$$Z_1 = \tfrac{1}{2}\left(1 - \frac{3g_A}{5g_V}\right) \approx 0\cdot 12. \tag{47}$$

We obtain the same relation between the relativistic correction to g_A/g_V and that to D_n in the bag model and a number of other models (Costella and McKellar 1990), and conjecture that it may be a general relationship.

4. Dynamics and Light Cone Field Theory

Lepage and Brodsky (1980) and others (see the review by Namyslowski 1985) have developed the Feynman rules for the quantisation of fields in the light cone coordinates. The great advantage of the light cone field theory is realised when one compares it with the instant form of field theory. In the instant form, we are familiar with the existence of vacuum fluctuations in which particle–antiparticle pairs are created from the vacuum. This is possible because the 4-momenta in the instant form are conserved at each vertex, and there is no difficulty in satisfying the equation $p_1 + p_2 = 0$. However, the light front form is similar to 'old fashioned' perturbation theory in that (p^-, p^1, p^2) is conserved at the vertices, and p^+ is then determined from the mass shell condition (21). The momentum component p^- is positive, and it is thus impossible to satisfy the equation $p_1^- + p_2^- = 0$, so there can be no vacuum fluctuations into particle–antiparticle pairs. This permits a considerable simplification of the calculations in light cone field theory.

There is however a complication in the theory, which is illustrated by the Dirac equation in light cone coordinates:

$$(\mathrm{i}\gamma^-\partial^- + \mathrm{i}\gamma^+\partial^+ - \mathrm{i}\gamma^1\partial^1 - \mathrm{i}\gamma^2\partial^2 - m)\psi = 0. \tag{48}$$

The light cone evolution is described by $\partial^+ = \partial_- = \partial/\partial x^-$, and thus $\gamma^0\gamma^-\psi$ is not a dynamical component of the Dirac field. Instead it is determined from the dynamical field $\gamma^0\gamma^+\psi$ using the Dirac equation. The existence of these non-dynamical components of the Dirac, Maxwell and gluon fields leads to a proliferation of contact-type interaction terms in the Feynman rules. Here I will not give a complete listing of these, but illustrate the situation by giving the example of electron–electron scattering. In the instant form of QED, with covariant quantisation, there is just one Feynman diagram, illustrated in Fig. 1. Here, the virtual photon 4-momentum, q^μ, is determined by energy–momentum conservation, so that $q^\mu = \ell_i^\mu - \ell_f^\mu$. The final result is the well known amplitude

$$T_{\mathrm{fi}}^{\mathrm{F}} = -e^2\,\frac{\overline{u}(\ell_\mathrm{f})\,\gamma^\mu u(\ell_\mathrm{i})\,\overline{u}(k_\mathrm{f})\,\gamma_\mu\, u(k_\mathrm{i})}{q^2 + \mathrm{i}\epsilon}. \tag{49}$$

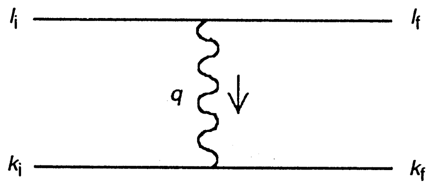

Fig. 1. Feynman graph for electron–electron scattering by exchange of a virtual, covariant, photon.

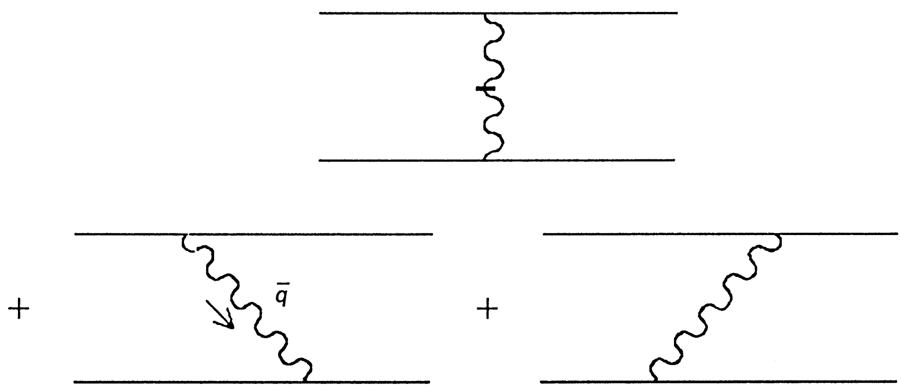

Fig. 2. Graphs for electron–electron scattering in the light cone quantised field theory. The first graph is the contact term, and the second and third represent exchange of transverse photons.

This result is gauge independent because of the conservation of the electron currents,

$$q_\mu \overline{u}(\ell_f)\gamma^\mu u(\ell_i) = q_\mu \overline{u}(k_f)\gamma^\mu u(k_i) = 0 . \tag{50}$$

In the light cone form, the three graphs of Fig. 2 all contribute. The first graph is a contact term, with the value (in the null gauge $A^- = 0$)

$$T_{fi}^{(1)} = e^2 \overline{u}(\ell_f)\gamma_\mu u(\ell_i) \overline{u}(k_f)\gamma_\nu u(k_i) \frac{\eta^\mu \eta^\nu}{(\overline{q}^-)^2} . \tag{51}$$

The 4-momentum $\overline{q}^\mu$ appearing in this equation is defined by conservation of light cone 3-momentum at the vertices,

$$\overline{q}^{-,1,2} = (\ell_i - \ell_f)^{-,1,2} , \tag{52}$$

and by the mass-shell condition for the photon

$$\overline{q}^+ = \overline{q}_\perp^2 / q^- , \tag{53}$$

and η^μ is a unit 4-vector defined by

$$\eta^{+,-,1,2} = (1,0,0,0) . \tag{54}$$

The contact contribution to the e^-e^- scattering amplitude given by equation (51) is analogous to the contribution of the Coulomb interaction in the Coulomb gauge (see e.g. Mandl and Shaw 1984). The analogy is very close as the origin of the Coulomb term and the contact term in the light cone calculation is the fact that one of the components of the Maxwell field—A^0 in the Coulomb gauge, A^+ in the light cone calculation—are not dynamical variables, but are determined from the dynamical fields by the Maxwell equations.

The second and third diagrams in Fig. 2 represent the exchange of transverse photons. They sum to gi'"^ the contribution

$$T_{\rm fi}^{(2)+(3)} = e^2\overline{u}(\ell_{\rm f})\gamma_\mu u(\ell_{\rm i})\overline{u}(k_{\rm f})\gamma_\nu u(k_{\rm i}) \frac{1}{\overline{q}^-q^+ - \overline{q}_\perp^2 + {\rm i}\epsilon} \times \left(-g^{\mu\nu} + \frac{\eta^\mu \overline{q}^\nu + \eta^\nu \overline{q}^\mu}{(\overline{q}^-)^2}\right). \tag{55}$$

It is a straightforward algebraic exercise to show that the sum of all three graphs of Fig. 2 is

$$T_{\rm fi}^{(1)+(2)+(3)} = e^2\overline{u}(\ell_{\rm f})\gamma_\mu u(\ell_{\rm i})\overline{u}(k_{\rm f})\gamma_\nu u(k_{\rm i}) \frac{1}{q^2 + {\rm i}\epsilon} \times \left(-g^{\mu\nu} + \frac{\eta^\mu q^\nu + \eta^\nu q^\mu}{(\overline{q}^-)^2}\right). \tag{56}$$

Now, the use of equation (50) demonstrates that

$$T_{\rm fi}^{\rm F} = T_{\rm fi}^{(1)+(2)+(3)}, \tag{57}$$

and thus that the light cone method and the Feynman method give the same results in this simple case.

This approach to e^-e^- scattering immediately generalises, of course, to e^+e^- scattering and the kernel for the Bethe–Salpeter equation for positronium. It is well known (e.g. Itzykson and Zuber 1980) that the Bethe–Salpeter equation is solved starting from Fock's solution for the momentum space wavefunction of two particles interacting through a Coulomb potential. The light cone treatment suggests that the Bethe–Salpeter equation with the contact interaction of equation (51) as the kernel may be an alternative starting point. I leave you with that thought as an exercise for the reader (and the author).

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