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The Possibility of Meson–Lepton and Dilepton Resonances

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Abstract

Recent hints in experimental data of a $\mu\pi$ resonance at 429 MeV have led us to speculate about the possibility of meson–lepton and dilepton resonances. We show that the proposed $\mu\pi$ resonance at 429 MeV is not in conflict with the muon $g-2$ measurements, and that it could be interpreted as a partner of the excited charged leptons possibly seen at CERN. Moreover, a composite model by Fritzsch and Mandelbaum predicts not only a flavour octet of meson–muon resonances of which this could be one, but also dilepton resonances which were suggested by Ramm as an explanation of the lobe structure in the $\mu\pi$ data. The observed widths and branching ratios of the resonances can be interpreted to suggest that the colour-octet states are considerably smaller than the colour singlets.

1. Introduction

In a series of papers Ramm (1972, 1982, 1985) has found evidence for a neutral $\mu\pi$ resonance $\mathcal{N}_{\mu\pi}$ in the reactions

$$\nu_\mu + \mathrm{N} \rightarrow (\mu^- + \pi^+) + \mathrm{X}, \qquad \bar{\nu}_\mu + \mathrm{N} \rightarrow (\mu^+ + \pi^-) + \mathrm{X},$$

and in $\mathrm{K}_{\mu 3}$ decays. Ramm found that the mass of the resonance is

$$m_{\mathcal{N}_{\mu\pi}} = 0\cdot429 \pm 0\cdot002\ \mathrm{GeV}. \tag{1}$$

More recently Ballagh *et al.* (1984) have searched for a $\mu\pi$ resonance in neutrino reactions at Fermilab and although, perhaps ironically, they did find a peak at 430 MeV, they were eventually led to claim it is not statistically significant. Nevertheless, they pointed out that if there is a $\mu\pi$ resonance at 430 MeV their results are not inconsistent with the Ramm prediction.

In these circumstances it seems desirable to enquire whether such a light $\mu\pi$ resonance is phenomenologically in conflict with other experiments and, indeed, whether there are models of elementary particles which could accommodate such resonances.

Ramm (1985) has further pointed out the existence of a lobe structure associated with the resonance which is also correlated in the various experiments in which the peak appears. He analysed this structure and suggested that it is a result of a second decay channel of the $\mathcal{N}_{\mu\pi}$ which involves a decay to a $\nu\mu$ resonant state.

The experimental situation was summarised by Ramm (1985). The essential points are:

(i) the mass of the $\pi\mu$ resonance is $0\cdot429\pm0\cdot002$ GeV;

(ii) the width of the $\pi\mu$ resonance is less than the resolution (~ 2 MeV);

(iii) there is no observable gap between production and decay of the $\pi\mu$ resonance, so its lifetime is less than 3×10^{-12} s;

(iv) the lobe structure can be simulated if the $\pi\mu$ resonance decays into a μ and a $\mu\nu$ resonance, the latter having a mass of $0\cdot127\pm0\cdot005$ GeV and a width of 5 ± 3 MeV;

(v) the branching ratio of the $\mu(\mu\nu)$ decay compared with the $\pi\mu$ decay is between 2 and 5; and

(vi) the first μ in the $\mu(\mu\nu)$ decay is preferentially emitted in the forward direction.

We may combine (ii) and (iii) into limits on the lifetime of the $\mathcal{N}_{\mu\pi}$ to give

$$3\times10^{-12} > \tau_{\mathcal{N}} > 0\cdot6\times10^{-21}\ \mathrm{s} \tag{2}$$

or, equivalently, on its width

$$10^{-9} < \Gamma_{\mathcal{N}} < 2\ \mathrm{MeV}\,. \tag{3}$$

First we investigate the consistency of the existence of the $\mathcal{N}_{\mu\pi}$ with the muon $g-2$ experiments, since the diagram shown in Fig. 1 below obviously contributes to the muon anomalous magnetic moment. The coupling constant at the $\mu\pi$ vertex is restricted by the width of the $\mathcal{N}$ in equation (3), whereas consistency with the $g-2$ measurement (Bailey *et al.* 1979) requires that

$$\Gamma_{\mathcal{N}} < 1\times10^{-4}\ \mathrm{MeV}\,, \tag{4}$$

which is consistent with and narrows down the range of allowed widths in (3).

Having verified that the existence of the N does not conflict with QED, we then inquire whether the N could be the neutral partner of the excited leptons which have been introduced on the basis of constituent models of quarks and leptons (see e.g. Cabibbo *et al.* 1984; de Rujula *et al.* 1984; Choudhury and Joshi 1984). In this case we are able to estimate the $\mathrm{N}\nu\mathrm{Z}^0$ and the $\mathrm{N}\mu^{\mp}\mathrm{W}^{\pm}$ couplings from the production cross section and the decay width respectively, and we find that these couplings are of the characteristic strength expected of weak interactions.

Next we investigate a particular boson–fermion composite model by Fritzsch and Mandelbaum (1981), in which we are able to identify structures that can be identified with the Ramm resonance, and point out the existence of further exotic states in this model which could possibly be identified. In fact it is possible that one of these new exotic states, a dilepton state, has already been seen—it could be the $\mu\nu$ resonance which Ramm (1985) has suggested as an interpretation of the lobe structure observed in the $(\mu\pi)$ mass spectra.

Before closing this Introduction, we would like to comment on the apparent conflict between the existence of low mass charged resonances and their absence in the final particle spectra in e^-e^+ collisions. The former implies that the electromagnetic form factor of such particles is constrained to be 1 at a momentum transfer squared of $q^2 = 0$, whereas the latter tells us that the electromagnetic form factor has become very small when q^2 has a time-like value exceeding the threshold of production of such resonances. Such a situation is contrary to our experience with normal hadrons and leptons but beyond that does not violate anything fundamental. In fact, if the existence of a narrow charged lepton is confirmed, then one has no escape from this unusual situation and any theoretical model must explain it. In this paper, however, we do not consider this point any further.

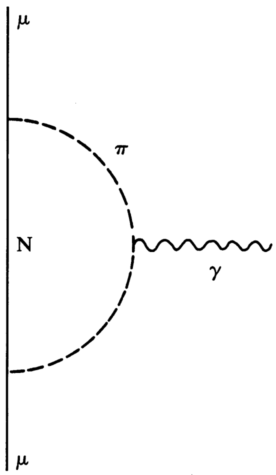

Fig. 1. Diagram through which N contributes to $g-2$ of the muon.

2. Ramm Resonance and $g-2$

Fig. 1 shows the contribution of the Ramm resonance to the muon magnetic moment. In order to calculate the contribution of this diagram to $g_\mu - 2$ we require the $\mathscr{M}\mu\pi$ interaction, which we assume to be a combination of the scalar and pseudoscalar form

$$\mathscr{L}_{\mathscr{N}\mu\pi} = g\{\bar{\psi}_{\mathscr{N}}(a+b\gamma_5)\psi_\mu\,\phi_\pi + \text{h.c.}\}\,. \tag{5}$$

The other obvious choice of interaction is the following combination of vector and pseudovector coupling

$$\mathscr{L}'_{\mathscr{N}\mu\pi} = \frac{g}{m}\{\bar{\psi}_{\mathscr{N}}\,\gamma(\alpha+\beta\gamma_5)\psi_\mu\,\partial_\lambda\,\phi_\pi + \text{h.c.}\}\,, \tag{5'}$$

but we can adapt the equivalence theorem of meson theory (Case 1948) to show that (5′) and (5), for

$$a = \alpha(1-m_\mu/m)\,, \qquad b = \beta(1+m_\mu/m)\,, \tag{6}$$

where m is the mass of the $\mathscr{N}$, give the same contribution to $g_\mu - 2$ to order g^2.

The width of the resonance is given by

$$\Gamma = \frac{g^2}{4\pi}\frac{p_\mu}{m}\{|a|^2(E_\mu+m_\mu)+|b|^2(E_\mu-m_\mu)\}, \tag{7}$$

where E_μ and p_μ are the energy and momentum of the μ in the rest frame of the $\mathcal{N}$. Given an appropriate mixture of coefficients a and b one can estimate the limits on the coupling constant from the limits (3) on the width. For various values of a and b the limits on $g^2/4\pi$ are summarised in Table 1.

Table 1. Limits on coupling constants and the width for various choices of $|a|$ and $|b|$, with $|a|^2+|b|^2 = 1$

The limits are from the data on the resonance (equation 3) and from $\delta\{\frac{1}{2}(g_\mu-2)\}$ given in equation (14). Note that the third choice gives values predicted by a V–A theory

a	b	Resonance data		$\delta\{\frac{1}{2}(g_\mu-2)\}$	
		$(g^2/4\pi)_{\min}$	$(g^2/4\pi)_{\max}$	$(g^2/4\pi)_{\max}$	$\Gamma_{\max}$ (MeV)
1	0	$7\cdot9\times10^{-12}$	$1\cdot6\times10^{-2}$	$8\cdot1\times10^{-7}$	$1\cdot0\times10^{-4}$
0	1	$2\cdot4\times10^{-11}$	$4\cdot9\times10^{-2}$	$9\cdot5\times10^{-7}$	$3\cdot9\times10^{-5}$
0·519	0·855	$1\cdot6\times10^{-11}$	$3\cdot1\times10^{-2}$	$9\cdot5\times10^{-7}$	$6\cdot1\times10^{-5}$

We can now investigate the contribution of Fig. 1 to the electromagnetic vertex $\gamma\mu\mu$. We can write the vertex as

$$\Gamma_\nu = -e\{\gamma_\nu+\lambda_\nu^{(\mathrm{QED})}(p_1,p_2)+\lambda_\nu^{(\mathcal{N})}(p_1,p_2)\}, \tag{8}$$

where $-e\gamma_\nu$ is the vertex at tree diagram level, $-e\lambda_\nu^{(\mathrm{QED})}$ is the QED correction calculated to sixth order (Lautrup *et al.* 1972; Calmet *et al.* 1977; Kinoshita 1979; Kinoshita and Lindquist 1983) and $-e\lambda_\nu^{(\mathcal{N})}$ is the contribution from Fig. 1, given by

$$\lambda_\nu^{(\mathcal{N})}(p_1,p_2) = \frac{g^2}{(4\pi)^2}\{A\gamma_\nu+\tfrac{1}{2}\mathrm{i}\,\sigma_{\nu\rho}\,q^\rho B\}, \tag{9}$$

and where A is a divergent contribution to the charge renormalisation. Further, $\Delta = m_\mu B(p_1,p_2)g^2/(4\pi)^2$ is the contribution to $\frac{1}{2}(g_\mu-2)$, and we find that

$$\Delta = \frac{g^2}{(4\pi)^2}2m_\mu\int_0^1 x\,\mathrm{d}x \times\frac{[-(1-x)\{|a|^2(m+m_\mu)+|b|^2(m-m_\mu)\}+(1-x)^2m_\mu(|b|^2-|a|^2)]}{m^2(1-x)+m_\pi^2\,x-m_\mu^2\,x(1-x)}. \tag{10}$$

In the case $m = m_\mu$ this expression reduces to the value of the charged meson contribution to the nucleon anomalous moment given by Case (1948) and Drell (1948). For our purpose it suffices to consider the limiting case $m_\mu \ll m$ and $m_\pi \ll m$, whereby

$$\Delta \approx -\frac{g^2}{(4\pi)^2}\frac{m_\mu}{m}\left(|a|^2+|b|^2+\tfrac{1}{3}\frac{m_\mu}{m}(|a|^2-|b|^2)\right). \tag{11}$$

Of course, $g-2$ has been the subject of extensive experimental and theoretical investigations, and the close agreement of the experimental value by Bailey *et al.* (1979),

$$\tfrac{1}{2}(g-2)_{\text{exp}} = (1165924\pm8\cdot5)\times10^{-9}\,, \tag{12}$$

and the value from QED (Kinoshita and Lindquist 1983),

$$\tfrac{1}{2}(g-2)_{\text{QED}} = (1165921\pm8\cdot31)\times10^{-9}\,, \tag{13}$$

is one of the triumphs of modern physics, and leaves little room for new contributions, such as that from the N. At the 2σ or approximately 95% confidence level we have

$$\delta\{\tfrac{1}{2}(g-2)\} \leqslant 1\cdot7\times10^{-8}\,. \tag{14}$$

The corresponding upper limits on $g^2/4\pi$ have been shown for various choices of $|a|$ and $|b|$ in Table 1. We note that the limit on the coupling given by $\frac{1}{2}(g_\mu-2)$ is not very sensitive to the choice of $|a|$ and $|b|$, subject to the normalisation $|a|^2+|b|^2 = 1$, because $\frac{1}{3}m_\mu/m$ is a small number.

We also show in Table 1, the limits on Γ implied by the requirement that the excellent agreement between theory and experiment for $\frac{1}{2}(g_\mu-2)$ is not upset. We can summarise this Section by stating that our present knowledge of $g_\mu-2$ requires that

$$\Gamma \lesssim 1\times10^{-4}\ \text{MeV}\,. \tag{15}$$

There are now $g-2$ experiments in the planning stage which could impose further constraints on the properties of the $\mu\pi$ resonance, but considerable improvement is necessary before $g-2$ measurements would conflict with Ramm's result quoted in equation (3).

3. Ramm Resonance and Excited Leptons

Composite models have been proposed as one possible generalisation and simplification of the standard model (see e.g. Fritzsch and Mandelbaum 1981), where excited leptons and neutrinos naturally occur in models with composite leptons and quarks. It is then instructive to ask whether the Ramm resonance could be one of these excited neutrinos.

One of the limits on the excited electron mass is provided by the analysis of parity violation in e–d scattering. The ratio of the excited electron mass to the electron mass is at least 10^6. It is of interest to ask whether this ratio, transferred naively to the muon neutrino ν_μ sector (in the absence of a dynamical theory of the masses of the composite leptons), is compatible with the observations. Identifying the Ramm resonance with ν_μ^*, we would then expect $m_{\nu_\mu} \lesssim 400$ eV. This is clearly compatible with the laboratory observations on the ν_μ (Lu *et al.* 1980). A value near the upper limit would be in conflict with the astrophysical limits on the masses of stable neutrinos (Gerstein and Zeldovich 1966; Cowsik and McClelland 1972), a limit that can be evaded, if necessary, if the ν_μ is unstable with an appropriate decay scheme (see e.g. Pakvasa and McKellar 1983; Kolb 1984; Wolfenstein 1984).

In this picture we can write for the ν_μ–ν^*_μ–hadron and μ–ν^*_μ–hadron couplings,

$$\mathscr{L}_{\nu\nu^*} = -\frac{\lambda_1 G_F}{2\sqrt{2}} \bar{\psi}_{\nu^*} \gamma_\mu (1-\gamma_5) \psi_\nu J^\mu_{H,0} + \text{h.c.}, \tag{16}$$

$$\mathscr{L}_{\mu\nu^*} = -\frac{\lambda_2 G_F}{\sqrt{2}} \bar{\psi}_{\nu^*} \gamma_\mu (1-\gamma_5) \psi_\mu J^\mu_{H,+} + \text{h.c.}, \tag{17}$$

exploiting the analogy with the couplings involving ν_μ. Here λ_1 and λ_2 are the ratios of the $\nu\nu^*Z$ to $\nu\nu Z$ coupling and the $\mu\nu^*W$ to $\mu\nu W$ couplings respectively. However, until an extension of the standard $SU(2)\times U(1)$ theory is available we are unable to relate λ_1 and λ_2, although we would expect them to be of the same order.

One can attempt to estimate $|\lambda_1|$ from the production cross section of the ν^*. Ballagh *et al.* (1984) gave estimates of the product rB, where r is the ratio of the ν^* production cross section to the charged current reaction rate and B is the $\nu^* \rightarrow \mu\pi$ branching ratio, for their experiment and Ramm's prediction. Since these values of rB are compatible, we take a weighted mean and estimate that

$$rB = 0{\cdot}0017 \pm 0{\cdot}0010. \tag{18}$$

It is clear that the interaction $\mathscr{L}_{\nu^*\nu}$ predicts that

$$r = |\lambda_1|^2 R_\nu, \tag{19}$$

where R_ν is the ratio of neutral current to charged current events, which is approximately $0{\cdot}3$. We therefore estimate, setting $B \sim 1$, that

$$|\lambda_1| \sim 0{\cdot}075. \tag{20}$$

This suggests that the production process has a coupling constant a little weaker than the weak interaction.

The decay width of the ν^* determines $|\lambda_2|$ through the relation (which follows from the analysis of the previous section)

$$\Gamma(\nu^* \rightarrow \mu\pi) = \frac{|\lambda_2|^2 G_F^2 f_\pi^2}{8\pi} \frac{|p_\mu|}{m_{\nu^*}^2} \{(m_{\nu^*}^2 - m_\mu^2)^2 - m_\pi^2 (m_{\nu^*}^2 + m_\mu^2)\}, \tag{21}$$

where $f_\pi = 0{\cdot}94\, m_\pi$ is the usual pion decay constant. We note in passing that this reduces to the rate given for $\Gamma(L \rightarrow \nu\pi)$ for a heavy lepton L by Tsai (1971) for the appropriate values of the parameters.

This then implies, with the limits (3) and (15), that

$$20 \lesssim |\lambda_2| \lesssim 6000, \tag{22}$$

so that the decay process of the resonance is stronger than the normal weak interaction.

It thus appears that it is quite plausible to regard the Ramm resonance as an excited neutrino, analogous to the excited charged leptons possibly seen in the anomalous Z^0 decays. Both its production rate and decay rate are consistent with this, if we postulate that structure effects induce deviations of λ_1 and λ_2 from unity.

4. Composite Model for the Ramm Resonance

Fritzsch and Mandelbaum (1981) have proposed a number of quark and lepton composite models, in which a hypercolour interaction in SU(n) binds the constituents. We consider the model in which the preons consisting of the fermions α and β and bosons X and Y have the properties listed in Table 2.

Table 2. Properties of the quark constituents α, β, X and Y in the Fritzsch–Mandelbaum constituent model of quarks and leptons

Particle	Spin	Charge	Colour	Hypercolour
α	$\frac{1}{2}$	$-\frac{1}{2}$	3	n
β	$\frac{1}{2}$	$+\frac{1}{2}$	3	n
X	0	$-\frac{1}{6}$	3	$\bar{n}$
Y	0	$+\frac{1}{2}$	$\overline{3}$	$\bar{n}$

It should be noted that, in this model, the preons carry colour as well as hypercolour. The leptons are colour singlet states of these preons. It is therefore possible that there is a residual colour interaction between the leptons. Indeed, since the weak interactions proceed by exchange of bound states of the fermionic preons which are identified with the weak vector bosons, these residual strong interactions manifest themselves as the weak interactions. At energies of the scale of the hypercolour interactions, one would expect the colour interactions between leptons to become strong. In this sense the Fritzsch–Mandelbaum model implies a 'non-standard' unification of strong and electroweak interactions, but it is not our purpose to follow this through here.

The 'usual' quarks and leptons have the structure

$$\nu_\mu = (\bar{\alpha}\overline{\mathrm{Y}})_1\,, \quad \mu = (\bar{\beta}\overline{\mathrm{Y}})_1\,, \quad \mathrm{u} = (\bar{\alpha}\overline{\mathrm{X}})_3\,, \quad \mathrm{d} = (\bar{\beta}\overline{\mathrm{X}})_3\,, \tag{23}$$

where the subscripts refer to the colour representation. In this picture there will also be the colour-octet leptons

$$\nu_8 = (\bar{\alpha}\overline{\mathrm{Y}})_8\,, \qquad \mu_8 = (\bar{\beta}\overline{\mathrm{Y}})_8 \tag{24}$$

which, because of colour confinement, will not exist as free states. However, by analogy with the baryonia states (Chan and Høgaasen 1977; Anderson and Joshi 1979; Ellis *et al.* 1980), we would expect the overall singlet states

$$\mathscr{D}_{\nu\mu} = (\nu_{\mu 8}\,\mu_8)_1\,, \qquad \mathscr{M}_{\pi^+\mu} = \big((\bar{\mathrm{u}}\mathrm{d})_8\,\mu_8\big)_1\,, \qquad \text{etc.}\,, \tag{25, 26}$$

to be physically accessible states. The $\pi\mu$ resonances occur naturally in this model, and we then suggest the tentative identification of the $\mathscr{M}_{\pi^+\mu}$ with the Ramm resonance.

These states should be considered as *six preon* bound states (hyper-QCD-baryonia). In the symmetry limit we have an exact chiral symmetry and all fermions are massless. Mass is then generated by a chiral symmetry breaking mechanism (as introduced by Fritzsch 1983, 1984) and fermions develop a small mass. In all symmetry breaking scenarios, chiral symmetry remains exact to fairly low energies to ensure low masses for leptons and quarks as well as *multi-preon* systems. Consequently, the low mass of the Ramm resonance cannot be taken as an argument against this identification, and we proceed to investigate its consequences.

We note two features of this identification. In spite of the results obtained in the previous section, we would not expect the decay interaction

$$\mathscr{M}_{\pi^+\mu} \rightarrow \mu\pi^+ \tag{27}$$

to be a weak interaction, as it does not involve any change of flavour, but instead a rearrangement of the colour couplings. One would therefore anticipate almost a strong interaction decay rate for (27), again by analogy with baryonia. The amplitude for (27) will however be further reduced if the spatial overlap of the $(u\bar{d})_8$ state with the $(u\bar{d})_1$ state is significantly less than unity, a point to which we return below. For the moment we write the invariant amplitude for (27) as

$$M = G_{\mathscr{M}\pi\mu}\, \bar{u}_\mu \gamma_5 u_{\mathscr{M}}\,. \tag{28}$$

(Since the decay proceeds by strong interactions we do not introduce any parity violating coupling.) We estimate $G_{\mathscr{M}\pi\mu}$ in equation (39) below.

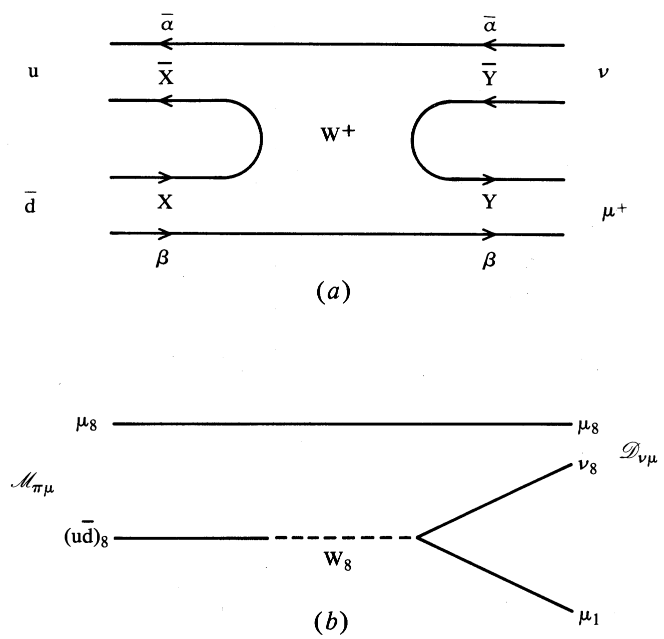

Fig. 2. (*a*) Subquark diagrams for the weak decay $u\bar{d} \rightarrow W^+ \rightarrow \nu\mu^+$. Each state can either be a colour singlet or a colour octet. (*b*) The decay $\mathscr{M}_{\pi^+\mu^-} \rightarrow \mu^+\mathscr{D}_{\nu\mu^-}$.

Secondly, we emphasise that these colour octet–octet states have a very rich spectrum. For example there will be a flavour octet of states analogous to the $\mathscr{M}_{\pi^+\mu}$;

i.e. the $\mathcal{M}_{\mathrm{K}^+\mu}$, $\mathcal{M}_{\eta\mu}$ etc. These states will obey a Gell-Mann–Okubo mass relation

$$m^2(\mathcal{M}_{\eta\mu}) = \tfrac{4}{3}\, m^2(\mathcal{M}_{\mathrm{K}^0\mu}) - m^2(\mathcal{M}_{\pi^0\mu})\,. \tag{29}$$

In addition, we expect the dilepton state $\mathscr{D}_{\nu\mu}$ of equation (25), which is especially interesting. Just as

$$(\mathrm{u}\bar{\mathrm{d}})_1 \rightarrow \mu\bar{\nu}_\mu\,, \tag{30}$$

we would expect

$$(\mathrm{u}\bar{\mathrm{d}})_8 \rightarrow \mu_8^+\,\nu_\mu \rightarrow \mu^+\,\nu_{\mu_8}\,. \tag{31a, b}$$

These decays proceed through the subquark diagrams of Fig. 2. Thus, we expect the decays

$$\mathcal{M}_{\pi^+\mu^-} \rightarrow \nu_\mu\, \mathscr{D}_{\mu^+\mu^-} \quad \llcorner\!\!\rightarrow \mu^+\mu^-\,, \qquad \mathcal{M}_{\pi^+\mu^-} \rightarrow \mu^+\, \mathscr{D}_{\nu\mu} \quad \llcorner\!\!\rightarrow \nu\mu^-\,. \tag{32a, b}$$

In other words, in this Fritzsch–Mandelbaum model *the decay of the $\pi^+\mu$ resonance into a dilepton resonance is an immediate consequence of the model*, as long as it is kinematically permitted. This is precisely the interpretation of the lobe structure which Ramm (1985) proposed, and we see that in this particular model such a phenomenon is to be expected.

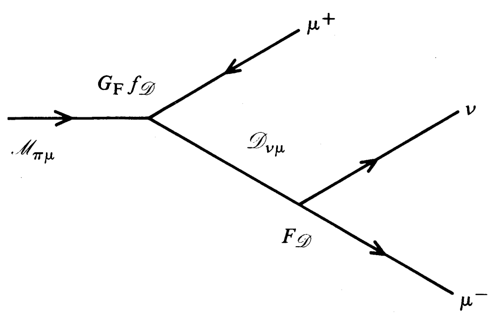

Fig. 3. The decay $\mathcal{M}_{\pi^+\mu^-} \rightarrow \mu^+\mu^-\nu$.

Ramm's analysis suggests that the overall rate for the decay (32b) is 2 to 5 times the rate of the decay (27). The decay (32b) proceeds via the diagram of Fig. 3, which gives an amplitude

$$M = G_\mathrm{F} f_\mathscr{D}\, \overline{C\mathrm{v}(\mu^+)}\, \gamma^\mu (1-\gamma_5)\, \mathrm{u}(\mathcal{M})\, \frac{q_\mu(\mathscr{D})\, q_\lambda(\mathscr{D})}{q^2(\mathscr{D}) - m^2(\mathscr{D})}$$

$$\times\, G\mathscr{D}_{\mu\nu}\, \bar{\mathrm{u}}(\mu^-)\, \gamma_5 \big(C\bar{\mathrm{u}}(\nu)^\mathrm{T}\big)\,, \tag{33}$$

where the $\mathscr{D} \to \mu\nu$ amplitude is written in the same form as (28), as is the $\mathscr{M} \to \mu^+\mathscr{D}$ amplitude. However, since we expect the $\mathscr{M} \to \mu^+\mathscr{D}$ amplitude to be a weak interaction as in Fig. 2*b*, we have explicitly included the G_F factor, and the V–A structure at this vertex. Then $f_{\mathscr{D}}$ would be expected to be of order f_π, perhaps modified by a different probability of finding the u and $\bar{\mathrm{d}}$ at the same point in the octet state compared with the singlet state.

In the narrow width approximation, we can integrate $\sum_{\text{spins}}|M|^2$ over the three-body phase space to obtain

$$\Gamma_{\mu\mu\nu} = \frac{1}{8\pi^2} G_F^2 f_{\mathscr{D}}^2 G_{\mathscr{D}\mu\nu}^2 \frac{m^2(\mathscr{M}) p_0 E_0}{m^{\frac{1}{2}}(\mathscr{D})\,\Gamma^{\frac{3}{2}}(\mathscr{D})} \frac{\{m^2(\mathscr{D}) - m_\mu^2\}^2}{m^3(\mathscr{D})}, \tag{34}$$

where p_0 and E_0 are the momentum and energy of the μ^- in the two-body decay $\mathscr{M} \to \mu^-\mathscr{D}^+$. Using the further approximation that

$$\Gamma(\mathscr{D}) \approx \Gamma(\mathscr{D}\to\mu\nu) = \frac{1}{16\pi} G_{\mathscr{D}\mu\nu}^2 \frac{\{m^2(\mathscr{D}) - m_\mu^2\}^2}{m^3(\mathscr{D})}, \tag{35}$$

we may rewrite (34) as

$$\Gamma_{\mu\mu\nu} = 4\left(\frac{2}{\pi}\right)^{\frac{1}{2}} \frac{G_F^2 f^2}{G_{\mathscr{D}\mu\nu}} \frac{m^2(\mathscr{M})\, p_0 E_0\, m(\mathscr{D})}{m^2(\mathscr{D}) - m_\mu^2} \tag{36a}$$

$$\approx 6\cdot 96 \frac{G_F^2 f^2}{G_{\mathscr{D}\mu\nu}} m^3(\mathscr{M}). \tag{36b}$$

For the $\mathscr{M} \to \mu\pi$ matrix element of equation (28), we may rewrite $\Gamma_{\mu\pi}$ as

$$\Gamma_{\mu\pi} = 7\cdot 59\times 10^{-3}\, G_{\mathscr{D}\mu\pi}^2\, m(\mathscr{M}), \tag{37}$$

so that the branching ratio is

$$B = \frac{\Gamma_{\mu\mu\nu}}{\Gamma_{\mu\pi}} = 917 \frac{G_F^2 f_{\mathscr{D}}^2\, m^2(\mathscr{M})}{G_{\mathscr{D}\mu\pi}^2\, G_{\mathscr{D}\mu\nu}}. \tag{38}$$

This can be of order from 2 to 5 only if one or both of the coupling parameters $G_{\mathscr{M}\mu\pi}$ or $G_{\mathscr{D}\mu\nu}$ are significantly smaller than the typical strength α_s we may expect for QCD interactions, or if $f_{\mathscr{D}}$ is significantly larger than the analogous parameter f_π. To make further progress we must estimate these parameters.

We would expect the $\mathscr{M} \to \mu\pi$ decay to proceed through colour rearrangement, for example by the process illustrated in Fig. 4*a*, which we may represent phenomenologically by Fig. 4*b*. Evaluating this diagram with a cut-off Λ, we estimate that

$$G_{\mathscr{M}\mu\pi} \approx \frac{\surd 2}{3} \frac{\alpha_s}{8\pi} \left\{ \ln\left(1 + \frac{\Lambda^2}{M_8^2}\right) - \frac{\Lambda^2}{\Lambda^2 + M_8^2} \right\} S_\mu^{\frac{1}{2}} S_\pi^{\frac{1}{2}}, \tag{39}$$

where $\frac{1}{3}\surd 2 = \frac{1}{3}\surd\frac{1}{8}\,\mathrm{Tr}(T_a T_a)$ is a colour factor which arises from the colour structure $\mathrm{Tr}(T_a T_a)$ and normalisation factors ($\surd\frac{1}{3}$ for each 3–$\bar{3}$ colour singlet state and $\surd\frac{1}{8}$ for

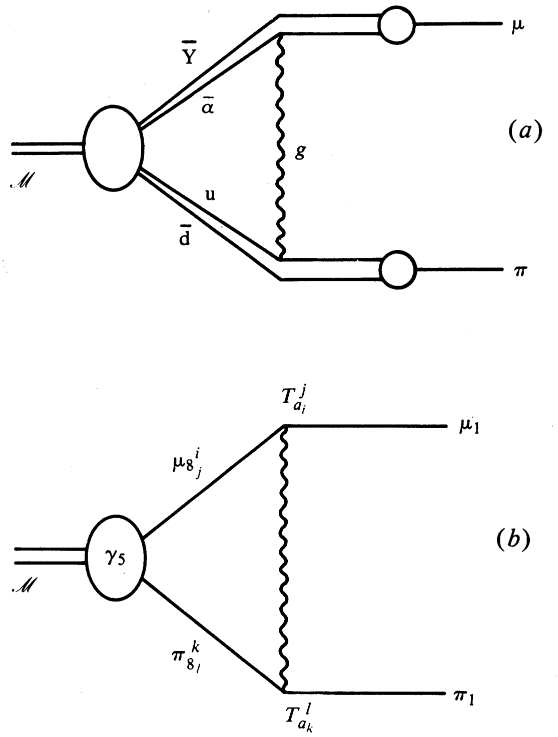

Fig. 4. (*a*) One of the composite-level gluon exchange processes which underlie $\mathscr{M} \to \mu\pi$ decay. (*b*) A phenomenological representation of $\mathscr{M} \to \mu\pi$ decay through gluon exchange.

the 8–$\bar{8}$ colour singlet state), and the other factor derives from the loop integral which is cut off at a scale Λ, so that Λ^{-1} is the size parameter of the $\mathscr{M}$. Further, M_8 is a typical mass parameter of the octet states, and $S_\mu^{1/2}$ and $S_\pi^{1/2}$ are spatial overlap factors for the octet to singlet states, e.g.

$$S_\pi^{\frac{1}{2}} = \langle \pi_8 | \pi_1 \rangle = \int d^3 r\, \psi_8^*(r)\, \psi_\pi(r).$$

In the nuclear physicist's language, the S are 'spectroscopic factors' (see e.g. McCarthy 1968). If we assume that the π bound states are characterised by size parameters $R_{\pi 1}$ and $R_{\pi 8}$, then

$$S_\pi^{\frac{1}{2}} \approx \left| \frac{2 R_{\pi 1} R_{\pi 8}}{R_{\pi 1}^2 + R_{\pi 8}^2} \right|^{\frac{3}{2}}, \tag{40}$$

where the expression is exact for gaussian wavefunctions. A similar expression holds for $S_\mu^{1/2}$.

We remark that the octet and singlet muon are both bound by the hypercolour forces, and we would therefore expect that $R_{\mu 1} \sim R_{\mu 8}$ and $S_\mu \sim 1$. However, the colour forces are responsible for the binding of the quarks into the octet pion and the singlet pion, so we may expect S_π to be significantly different from 1. In this case we make the approximations $R_{\pi 8} \ll R_{\pi 1}$ and $S_\pi \approx (2R_{\pi 8}/R_{\pi 1})^3$.

Finally, we note that, since $\mathscr{M}$ is conjectured to be light, whereas we would perhaps expect large octet masses M_8, the compositeness scale Λ of the $\mathscr{M}$ should be of the same order as M_8. We therefore set the term $\ln(1+\Lambda^2/M_8^2)-\Lambda^2/(\Lambda^2+M_8^2)$ in equation (39) to 1, and estimate

$$G_{\mathscr{M}\mu\pi} \approx \frac{\sqrt{2}}{3}\frac{\alpha_s}{8\pi}\left(\frac{2R_{\pi 8}}{R_{\pi 1}}\right)^{\frac{3}{2}}. \tag{41}$$

A similar argument leads to the estimate

$$G_{\mathscr{D}\mu\nu} = \frac{\sqrt{2}}{3}\frac{\alpha_s}{8\pi}. \tag{42}$$

The final parameter we require is $f_{\mathscr{D}}$, which we obtain following the reasoning used by Van Royen and Weisskopf (1967) to estimate f_π, giving

$$f_{\mathscr{D}} \approx f_\pi\left(\frac{\psi_{\pi 8}(0)}{\psi_{\pi 1}(0)}\right)^{\frac{1}{2}} \approx f_\pi\left(\frac{R_{\pi 1}}{R_{\pi 8}}\right)^{\frac{3}{2}}. \tag{43}$$

We see that if the octet pion is smaller than the singlet pion, $G_{\mathscr{M}\mu\pi}$ is reduced and $f_{\mathscr{D}}$ is increased, so that the branching ratio B can be increased to the experimental level. However, the enhancement required from the size mismatch is not very large. Setting $B=3$ gives

$$R_{\pi 8}/R_{\pi 1} \sim (10^{-5}\,\alpha_s^{-3})^{\frac{1}{6}}. \tag{44}$$

Our analysis does not define the Q^2 at which we should evaluate α_s. For $\alpha_s = 0{\cdot}5$ we find $R_{\pi 8}/R_{\pi 1} \approx 0{\cdot}2$, which is a reasonable range of values for the size ratio.

We have another consistency check on our estimate of $G_{\mathscr{M}\mu\pi}$, in that $G^2_{\mathscr{M}\mu\pi}/4\pi$ should be less than the limit derived above from the anomalous magnetic moment of the muons for the case $|a|=0$, $|b|=1$. Our estimate is

$$\frac{G^2_{\mathscr{M}\mu\pi}}{4\pi} = \frac{\alpha_s^2}{144\pi^3}\left(\frac{R_{\pi 8}}{R_{\pi 1}}\right)^3 \approx 7{\cdot}08\times 10^{-7}\,\alpha_s^{\frac{1}{2}}, \tag{45a, b}$$

which is within the limits given in Table 1.

The smaller size of the $(u\bar{d})_8$ state corresponds in a bag type of picture to an enhanced energy of the colour-octet state relative to the colour-singlet state, a result which should not be surprising. Taking this result seriously suggests that the π_8 state has a mass of order 1 GeV, which would be regarded as very light for an octet state. This breakdown of our intuition may be regarded as a defect of the model, or it may simply be a consequence of the naivety of the models we have used to estimate the parameters. For example, it could be argued that G_F in (33) should be replaced by $G_F^{(8)} \sim G_F\, m_W^2/m_{W8}^2$, where m_{W8} is the effective mass of a colour octet W. To fit the datum on the branching ratio, $R_{\pi 8}/R_{\pi 1}$ must then be multiplied by $(m_W^2/m_{W8}^2)^{2/3}$, and the colour-octet $(u\bar{d})_8$ state is presumably even smaller and higher in energy. The ratio $R_{\pi 8}/R_{\pi 1} \sim 0{\cdot}3$ should be interpreted as an upper limit on the relative dimensions of the colour octet and singlet states.

We can also use the lower limit on the coupling constant from Table 1, namely

$$G^2_{\mathscr{M}\mu\pi}/4\pi \gtrsim 2\cdot 4\times 10^{-11}\,, \tag{46}$$

to obtain a lower bound on the size ratio

$$R_{\pi 8}/R_{\pi 1} \gtrsim 6\cdot 0\times 10^{-3}\,\alpha_s^{-\frac{2}{3}} = 0\cdot 017\,, \quad \alpha_s = 0\cdot 5 \tag{47a}$$

$$= 0\cdot 19\,, \quad \alpha_s = 0\cdot 1\,. \tag{47b}$$

In our model this can be combined with equation (44), reinterpreted as a value for $(R_{\pi 8}/R_{\pi 1})(m_{W8}/m_W)^{2/3}$, to give a bound on m_{W8}/m_W which is

$$m_{W8}/m_W \lesssim 24\cdot 5\alpha_s = 12\cdot 3\,, \quad \alpha_s = 0\cdot 5 \tag{48a}$$

$$= 2\cdot 5\,, \quad \alpha_s = 0\cdot 1\,. \tag{48b}$$

Thus, we see that consistency of our model requires rather light effective masses for the colour-octet states.

5. Summary

We have first shown that the light neutral $\mu\pi$ resonance suggested by the recent analyses by Ramm (1972, 1982, 1985) and Ballagh *et al.* (1984) is not in conflict with QED, and that it could, if confirmed, be regarded as a possible neutral excited lepton, which would be expected from composite models of leptons and quarks.

As a particular example we point out that a whole octet of meson–lepton resonances are to be expected on the basis of a composite model proposed by Fritzsch and Mandelbaum, of which the Ramm resonance could be the 'pion' member. This model also predicts dilepton resonances, which Ramm (1985) has suggested as the explanation of the lobe structure seen in the data. We have made models of the necessary three point functions, and find that, if the weak decay of the $\mathscr{M}_{\mu\pi}$ resonance to the $\mathscr{M}_{\mu\nu}$ and μ does not introduce new weak scale, we can account for the data on the assumption that the $(\mathrm{ud})_8$ state is about three times smaller than the $(\mathrm{u\bar{d}})_1$ state. If, however, the $\mathscr{M} \to \mathscr{D}_\mu$ strength is reduced by a factor of m^2_W/m^2_{W8}, consistency with the branching ratio implied by the lobe structure requires the $(\mathrm{u\bar{d}})_8$ to be even smaller, with

$$R_{\pi 8}/R_{\pi 1} \sim 0\cdot 3(m_W/m_{W8})^{\frac{2}{3}}\,. \tag{49}$$

In this case the absolute value of the branching ratio is reduced by the ratio $(m_W/m_{W8})^4$, and the mass of the $(\mathrm{u\bar{d}})_8$ state is increased by $(m_{W8}/m_W)^{2/3}$. Nevertheless, consistency of the model permits us to limit $m_{W8}/m_W \lesssim 12$, so that neither the octet W nor the octet pion can become very massive in our model.

While this may be regarded with surprise, on the scale of hypercolour, both colour and electromagnetic interactions may be viewed as small perturbations, as pointed out by Fritzsch (1983, 1984). In a chirally invariant hypercolour theory, the 't Hooft mechanism will give rise to massless states when colour and electromagnetic interactions are switched off. Masses are then generated by a chiral symmetry breaking mechanism. However, if this mechanism is independent of the colour interactions the colour forces do not contribute to the mass and one expects singlet and octet leptons to have masses of the same order. An example of this process is the Baur–Fritzsch (1984) mechanism in which chiral symmetry breaking occurs through electromagnetic self-energy interactions. In these circumstances the $[(\bar{q}q)_8 \times l_8]_1$ state could occur with a relatively low mass, and the masses of its constituent octet states could also be relatively low.

Finally, we make some brief remarks about other processes in which one could usefully look for manifestations of this and similar exotic states. Ramm (1972) has found evidence of the $\mu\pi$ enhancement in $K^0_{\mu 3}$ decays, but further studies in this system would be helpful. On the basis of the model discussed in Section 4, it would be surprising if there were not also a πe resonance. This would be difficult to see in neutrino induced reactions because of the absence of good ν_e beams with sufficient energy, but it could show up in an analysis of K^0_{e3} decays. We encourage a search of these decays for a πe enhancement. Because we have no model which accounts for the mass of the $\pi\mu$ resonance, we are unable to predict the mass of the conjectured πe state. If the model of Section 4 is appropriate one could surmise that the $(\pi\mu)$–(πe) mass separation may not be as large as the μ–e mass separation, because one would expect the mass to be largely determined by the colour interaction between the $(u\bar{d})_8$ and l_8 states, and to not be sensitive to the flavour of the octet lepton.

We conclude by emphasising that our purpose in this paper is to stimulate further experimental work pertaining to possible lepton–pion and dilepton resonances, even in the low mass region, and to advocate that an open mind be kept about their interpretation. The present evidence by Ramm is not at all universally accepted, but it should not be discarded on the grounds that such states have no place in our present model of the natural world. We have sought to indicate how these states may be described in the framework of composite models, and to link the 'hints' provided by Ramm to the parameters of the model and hence to other possible reactions.

While some aspects of our interpretation, particularly about the size and mass of the octet states, may be regarded as naive, we emphasise that it is precisely such properties of the constituents that we would hope to study through the characteristics of any confirmed resonance.

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