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E1 Decay of the First Excited State of ^{9}Be

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Abstract

The E1 decay strength of the $\frac{1}{2}^+$ first excited state of ^{9}Be is calculated using *R*-matrix formulae in which the excited state is correctly treated as unbound. Agreement with experiment is obtained for a reasonable choice of channel radius.

1. Introduction

In a study of some strong E1 transitions between low-lying states of ^{9}Be, ^{11}Be and ^{13}C, Millener *et al.* (1983) made full $0\hbar\omega$ and $1\hbar\omega$ shell model calculations for the negative- and positive-parity states respectively. With harmonic oscillator single-particle wavefunctions, they found that all the calculated strengths were less than the experimental values, by factors from about 2 to 30. With more realistic Woods–Saxon single-particle wavefunctions, they obtained increased strengths, which exceeded the experimental values by small factors (1·5–2) for the ^{11}Be and ^{13}C transitions. For these nuclei, all the levels involved are bound. In the ^{9}Be transition, the $\frac{1}{2}^+$ first excited state is slightly unbound for breakup into the ^{8}Be ground state plus an s-wave neutron, but Millener *et al.* treated the level as bound and studied the transition strength as a function of the binding energy. Their calculated values suggest that extrapolation to the experimental energy of the state would give agreement with the experimental strength, $B(\mathrm{E1}) = 0\cdot22 \pm 0\cdot09$ W.u. (1 W.u. = 2·79 mb) (Ajzenberg-Selove 1979), particularly in view of the large experimental error.

There are two difficulties with this result of Millener *et al.* for the ^{9}Be transition. The value of the experimental strength that they used was obtained from the inelastic electron scattering results of Clerc *et al.* (1968). By fitting recent $^9\mathrm{Be}(\gamma, \mathrm{n})^8\mathrm{Be}$ data, Barker (1983) obtained $B(\mathrm{E1}) = 1\cdot06^{+0.19}_{-0.16}$ mb $= 0\cdot38^{+0.07}_{-0.06}$ W.u., and attributed the small value obtained from electron scattering to inadequate allowance for the high-energy tail of the $\frac{1}{2}^+$ level. The calculated strengths of Millener *et al.* are clearly inconsistent with this new experimental value.

The other difficulty concerns the extrapolation of the calculated $B(\mathrm{E1})$ values across the energy region of the s-wave neutron threshold. This problem is considered in detail in Section 2. A calculation of $B(\mathrm{E1})$ for ^{9}Be in which the excited state is treated as unbound is given in Section 3.

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2. First Excited State of ^{9}Be Treated as Bound

In the calculation by Millener *et al.* (1983) of B(E1) for the $\frac{1}{2}^+ \to \frac{3}{2}^-$ transition in ^{9}Be, the main contribution involves the single-particle matrix element (SPME) for the $1s_{1/2} \to p_{3/2}$ neutron transition with a ^{8}Be ground-state core. This SPME was calculated as a function of the binding energy BE($1s_{1/2}$) of the $1s_{1/2}$ neutron, all other SPMEs being assumed constant. Now we note that the $1s_{1/2}\,p_{3/2}$ SPME may be written in terms of the initial and final neutron radial wavefunctions, $u_s(r)/r$ and $u_p(r)/r$, as

$$\mathrm{SPME} = \tfrac{16}{27}\int_0^\infty u_s(r)\,r\,u_p(r)\,\mathrm{d}r\Big/\left(\int_0^\infty u_s^2(r)\,\mathrm{d}r\int_0^\infty u_p^2(r)\,\mathrm{d}r\right)^{\frac{1}{2}}. \tag{1}$$

Writing

$$\int_0^\infty u_s^2(r)\,\mathrm{d}r = \int_0^R u_s^2(r)\,\mathrm{d}r + u_s^2(R)\int_R^\infty \{u_s(r)/u_s(R)\}^2\,\mathrm{d}r, \tag{2}$$

we choose $R > a$, where a is the channel radius, so that $u_s(r)$ has its asymptotic form for $r \geqslant R$, and put $u_s(r)/u_s(R) = \exp\{-\varepsilon(r-R)\}$ in the final integral in (2). Here $\varepsilon = \{2M_n\,\mathrm{BE}(1s_{1/2})/\hbar^2\}^{\frac{1}{2}}$, where M_n is the reduced mass in the ^{8}Be+n channel. Then we have

$$\int_0^\infty u_s^2(r)\,\mathrm{d}r = \int_0^R u_s^2(r)\,\mathrm{d}r + u_s^2(R)/2\varepsilon. \tag{3}$$

As BE($1s_{1/2}$) $\to 0$, then $\varepsilon \to 0$ and the second term on the RHS of (3) dominates, so that* SPME $\propto \varepsilon^{\frac{1}{2}} \propto \{\mathrm{BE}(1s_{1/2})\}^{\frac{1}{4}} \to 0$. Expressed in words, as the $1s_{1/2}$ neutron becomes less bound, its radial wavefunction extends out to larger distances, and the overlap with $ru_p(r)$ becomes smaller and vanishes in the limit of zero binding energy. This vanishing of SPME as BE($1s_{1/2}$) $\to 0$ obviously implies that extrapolation of the calculated B(E1) values to the experimental energy of the $\frac{1}{2}^+$ level is meaningless.

The reason for this type of behaviour is that a bound level possesses a ghost (Barker and Treacy 1962), so that not all the strength of the level is in the sharp peak below the threshold but some is in the ghost at energies above the threshold. As the binding energy of the level decreases, a smaller proportion of the strength remains in the sharp peak. For all except s-wave neutron channels, i.e. for all channels with a barrier, the proportion in the sharp peak remains nonzero as the binding energy becomes zero, and a slightly unbound level still has a sharp peak with a ghost at higher energies. For an s-wave neutron channel, the strength in the sharp peak approaches zero as the binding energy approaches zero and all the strength goes into the ghost; an unbound level then has only a broad peak at positive energies. The calculation of Millener *et al.* gave the strength in the sharp peak, which would be an appropriate basis for extrapolation to positive energies for any channel with a barrier, but in the present case of an s-wave neutron channel it is the strength in the ghost that continues smoothly into the strength of the unbound level.

* The numerical values of the SPMEs given by Millener *et al.* (1983) do not show the $\{\mathrm{BE}(1s_{1/2})\}^{\frac{1}{4}}$ dependence. We have recalculated these values and obtain $-2\cdot211$, $-2\cdot110$, $-1\cdot969$, $-1\cdot794$ and $-1\cdot543$ fm respectively for the five values of BE($1s_{1/2}$) considered by Millener *et al.* For BE($1s_{1/2}$) = 0·02 MeV, this value of the SPME is 15% less than that of Millener *et al.*; this discrepancy is presumably related to the long tail of the s-state wavefunction, with 6·5% of the state in the region $r > 50$ fm.

3. First Excited State of ^{9}Be Treated as Unbound

The E1 decay of the $\frac{1}{2}^+$ first excited state of ^{9}Be is similar to the E1 decay of the $\frac{1}{2}^+$ first excited state of ^{13}N, in that the excited state in each case is unbound for nucleon emission. This ^{13}N transition, along with other E1 transitions in ^{13}C and ^{13}N, was treated by Barker and Ferdous (1980) using R-matrix formulae that included external contributions to the E1 transition matrix elements, calculated using wavefunctions with the correct asymptotic forms. Certain approximations were required in order to make the calculation feasible, with the consequence that calculated quantities became dependent on the choice of the channel radius a. Similar formulae and approximations are used here for the ^{9}Be transition.

The approximations are that only $T = 0$ core states of ^{8}Be are important [as was found to be the case by Millener *et al.* (1983)], that external contributions are significant only in the ^{8}Be(g.s.)+n channel, and that the internal contribution may be calculated with harmonic oscillator single-particle wavefunctions with the upper limits of the radial integrals then extended from a to infinity.

The reduced transition probability B(E1) is related to the observed radiation width $\Gamma_\gamma^0(\frac{1}{2}^+ \to \frac{3}{2}^-)$ by

$$\Gamma_\gamma^0(\tfrac{1}{2}^+ \to \tfrac{3}{2}^-) = \tfrac{16}{9}\pi(E_\gamma/\hbar c)^3 B(\mathrm{E1}), \tag{4}$$

where (Barker and Ferdous 1980, equation 26)

$$\Gamma_\gamma^0(\mathrm{i}\to\mathrm{f}) = f_{\mathrm{if}}(a^2/N_\mathrm{i} N_\mathrm{f})\,|\mathcal{M}_{\mathrm{if}} + 2\Theta_\mathrm{i}\Theta_\mathrm{f} J_{\mathrm{if}}|^2 . \tag{5}$$

In the present case

$$\begin{aligned} f_{\mathrm{if}} &= \tfrac{4}{3}(\tfrac{4}{9})^2 e^2 (E_\gamma/\hbar c)^3 (0100\,|\,10)^2 U^2(11\tfrac{1}{2}\tfrac{1}{2};0\tfrac{3}{2}) \\ &= (128/729)e^2(E_\gamma/\hbar c)^3 , \end{aligned} \tag{6}$$

and other quantities are as defined by Barker and Ferdous. Thus, we get

$$B(\mathrm{E1}) = (8/81\pi)e^2(a^2/N_\mathrm{i} N_\mathrm{f})\,|\mathcal{M}_{\mathrm{if}} + 2\Theta_\mathrm{i}\Theta_\mathrm{f} J_{\mathrm{if}}|^2 . \tag{7}$$

We now consider the values to be used for the various quantities in equation (7), mostly as functions of the channel radius a. The internal contribution $\mathcal{M}_{\mathrm{if}}$ is obtained from a shell model calculation, using harmonic oscillator single-particle wavefunctions with length parameter b_0. Such a calculation by Woods and Barker (1984), using the Millener and Kurath (1975) interaction between nucleons from the p and sd shells and either the (8–16)2BME interaction of Cohen and Kurath (1965) or the Barker (1966) interaction within the p shell, gives $\mathcal{M}_{\mathrm{if}} = 0\cdot221 b_0/a$ and $0\cdot187 b_0/a$ respectively. The calculation of Millener *et al.* (1983), which used a different p-shell interaction and allowed excitations out of the 0s shell, gave a somewhat larger value,

$$\mathcal{M}_{\mathrm{if}} = \tfrac{8}{9}(0\cdot621/1\cdot617)b_0/a = 0\cdot341 b_0/a .$$

We use $\mathcal{M}_{\mathrm{if}} = 0\cdot2 b_0/a$, with $b_0 = 1\cdot75$ fm (Woods and Barker 1984). We also have $N_\mathrm{i} = 1$, since the shift factor $S(E)$ is identically zero for an s-wave neutron channel with $E > 0$ [all energies are measured from the ^{8}Be(g.s.)+n threshold].* The quantities N_f and J_{if} are given by

$$N_\mathrm{f} = 1 + \Theta_\mathrm{f}^2(\zeta a)^{-1}\{1-(1+\zeta a)^{-2}\}, \tag{8}$$

$$aJ_{\mathrm{if}} = (\zeta - \mathrm{i}\eta)^{-1} + \zeta(1+\zeta a)^{-1}(\zeta - \mathrm{i}\eta)^{-2}, \tag{9}$$

* For an s-wave neutron channel with $E < 0$, one has $S(E) \propto |E|^{\frac{1}{2}}$, so that $N \equiv 1 + \gamma^2 \mathrm{d}S/\mathrm{d}E \propto |E|^{-\frac{1}{2}}$ as $E \to 0$. Thus for the $\frac{1}{2}^+$ state treated as bound, equation (7) gives B(E1) vanishing as $\{\mathrm{BE}(1\mathrm{s}_{1/2})\}^{\frac{1}{2}}$ as $\mathrm{BE}(1\mathrm{s}_{1/2}) \to 0$.

where $\zeta = (2M_n E_g/\hbar^2)^{\frac{1}{2}}$ and $\eta = (2M_n E_e/\hbar^2)^{\frac{1}{2}}$, with E_g the binding energy of the $\frac{3}{2}^-$ ground state and E_e the energy of the $\frac{1}{2}^+$ excited state. We take $E_g = 1\cdot666$ MeV (Ajzenberg-Selove 1984) and initially take E_e as the peak energy $E_m = 0\cdot031$ MeV (Barker 1983).

Table 1. Values of the spectroscopic factor $\mathscr{S}_f$ from one-neutron pickup reactions on ^{9}Be

$\mathscr{S}_f$	Reference	$\mathscr{S}_f$	Reference
	Reaction (p, d)		Reaction (d, t)
0·58	Towner (1969)	0·51	Fitz *et al.* (1967)
0·81	Sundberg and Källne (1969)	0·37	Darden *et al.* (1976)
0·67	Schoonover *et al.* (1971)	0·29	Tanaka (1978)
~0·6	Hudson *et al.* (1972)		Reactions (^{12}C, ^{13}C) and (^{16}O, ^{17}O)
0·80	Darden *et al.* (1976)		
1·23	Becchetti *et al.* (1981)	0·84	Barker *et al.* (1970)
1·53	Becchetti *et al.* (1981)		

We still require values of the dimensionless reduced width amplitudes Θ_i and Θ_f. In Barker and Ferdous (1980), values of Θ_i and Θ_f for the ^{13}C and ^{13}N states were obtained as functions of a by fitting experimental data and were compared with shell model values. We use a similar approach for the ^{9}Be case. The fit (Barker 1983) to ^{9}Be$(\gamma, n)^8$Be data gave values of $\varepsilon_R \equiv 2\hbar^2\Theta_i^4/M_n a^2$, which can be used to give Θ_i as a function of a; the best fit value of ε_R was 0·192 MeV and the acceptable range 0·11–0·34 MeV. From shell model calculations, one gets a value of the spectroscopic amplitude $\mathscr{S}_i^{\frac{1}{2}}$, which is related to Θ_i by

$$\Theta_i = \mathscr{S}_i^{\frac{1}{2}}\, u_i(a)\left(\tfrac{1}{2}a \Big/ \int_0^a u_i^2(r)\,dr\right)^{\frac{1}{2}}. \tag{10}$$

We calculate $u_i(r)$ for a Woods–Saxon potential with radius parameter $r_0 = 1\cdot75$ fm and diffuseness $a_0 = 0\cdot65$ fm (Millener *et al.* 1983) and depth adjusted to fit E_m. The shell model calculation (Woods and Barker 1984) gives $\mathscr{S}_i^{\frac{1}{2}} = 0\cdot781$ for the Cohen and Kurath interaction and 0·778 for the Barker interaction. Values of Θ_i that fit $\varepsilon_R = 0\cdot192$ MeV give $\mathscr{S}_i^{\frac{1}{2}} = 0\cdot37$–$0\cdot57$ for a = 4–7 fm. We use the latter values of Θ_i, and note that they are smaller than the values suggested by shell model calculations. Similarly we may use the measured value of the width of the ^{9}B ground state $\Gamma^o = 0\cdot54 \pm 0\cdot21$ keV (Teranishi and Furubayashi 1964), together with the relations

$$\Gamma^o = 2\gamma_f^2 P(1 + \gamma_f^2\, dS/dE)^{-1}, \qquad \gamma_f^2 = \Theta_f^2(\hbar^2/M_n a^2), \tag{11a, b}$$

to obtain Θ_f as a function of a. Values of Θ_f^2 that fit $\Gamma^o = 0\cdot54$ keV give $\mathscr{S}_f = 0\cdot36$–$0\cdot61$ for a = 4–7 fm. Values of $\mathscr{S}_f$ obtained from one-neutron pickup reactions on ^{9}Be targets are given in Table 1. Values of $\mathscr{S}_f^{\frac{1}{2}}$ from shell model calculations are 0·730 (Barker 1966), 0·762 (Cohen and Kurath 1967) and 0·751 (Kumar 1974). We use the mean calculated value $\mathscr{S}_f^{\frac{1}{2}} = 0\cdot75$ ($\mathscr{S}_f = 0\cdot56$), which is reasonably consistent with the experimental values, considering their spread.

Table 2 gives as functions of channel radius a the calculated values of various relevant quantities, and the resultant values of B(E1) are given in column A. The values in column B are discussed below. The values of B(E1) are in better agreement

Table 2. Values of B(E1) for the $\frac{1}{2}^+ \rightarrow \frac{3}{2}^-$ transition in ^{9}Be

a (fm)	Θ_i	Θ_f	N_f	aJ_{if} (fm)	B(E1) (W.u.) A	B
4·0	0·426	0·675	1·327	5·39+0·98i	1·037	0·710
5·0	0·476	0·500	1·153	5·19+0·92i	0·797	0·551
6·0	0·522	0·374	1·075	5·03+0·88i	0·575	0·400
7·0	0·563	0·285	1·038	4·91+0·84i	0·412	0·289

with the experimental value of $0{\cdot}38^{+0.07}_{-0.06}$ W.u. for the larger values of a ($\gtrsim 7$ fm). Previously, Barker and Ferdous (1980) found acceptable fits to properties of low-lying levels of ^{13}C and ^{13}N for $a \approx$ 4–6 fm. Agreement with the results of Millener *et al.* (1983) for the ^{13}C transitions is obtained by use of the present formulae with their values of spectroscopic amplitudes (i.e. of their OBDMEs) for $a \approx$ 5–6 fm, and for the ^{11}Be transition with $a \approx 4{\cdot}3$ fm. For such values of a ($\approx$4–6 fm), the present calculated values of B(E1) for ^{9}Be are too large. This discrepancy is not removed by reasonable changes in the value taken for $\mathscr{M}_{if}$, nor in the value of ε_R used to obtain Θ_i, since the calculated B(E1) values scale roughly as $\varepsilon_R^{0.43}$ while the experimental values of B(E1) (Barker 1983) scale as $\varepsilon_R^{0.29}$. A smaller value of $\mathscr{S}_f^{\frac{1}{2}}$ would improve the agreement. Also, a marked and reasonable reduction in the calculated values of B(E1) is obtained when allowance is made for the appreciable width of the $\frac{1}{2}^+$ excited state. We do this by weighting the B(E1) values with a density-of-states function (Barker and Treacy 1962), so that

$$B(\mathrm{E1}) = \int_0^\infty \mathrm{d}E_e\, B(\mathrm{E1}, E_e)\pi^{-1}\frac{(\varepsilon_R E_e)^{\frac{1}{2}}}{(E_R - E_e)^2 + \varepsilon_R E_e}, \tag{12}$$

where $B(\mathrm{E1}, E_e)$ is now calculated at the energy E_e of the $\frac{1}{2}^+$ state. We take $\varepsilon_R = 0{\cdot}192$ MeV as before and $E_R = 0{\cdot}067$ MeV (Barker 1983). The resultant values of B(E1) are given in column B of Table 2. Agreement with experiment is obtained for $a \approx 6$ fm, which is a reasonable value since ^{9}Be is a larger-than-average nucleus.

4. Summary

The first excited state of ^{9}Be is slightly unbound for breakup into the ^{8}Be ground state plus an s-wave neutron. A previous calculation of the E1 decay strength of this state by treating it as bound and extrapolating the calculated strength as a function of binding energy to the energy of the actual state fails, because of the singular properties possessed by an s-wave neutron threshold. An R-matrix calculation in which the state is treated as unbound involves approximations that make the calculated strength dependent on the choice of the channel radius, but a reasonable choice leads to agreement with the experimental strength.

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