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Low-lying Negative-parity Levels of ^{17}N and ^{18}N

F. C. Barker

Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, G.P.O. Box 4, Canberra, A.C.T. 2601.

Abstract

On the basis of a weak-coupling model, adjustments are made to the interactions used in the full shell model calculations of Millener in order to fit the experimental energies of the low-lying negative-parity levels of ^{16}N and of the low-lying positive-parity levels of ^{18}O and ^{19}O. The predicted energies of the low-lying negative-parity levels of ^{17}N then agree better with experiment, while those for ^{18}N lead to suggested spin assignments for the observed levels.

1. Introduction

In a recent study of ^{18}N with the reaction ^{18}O(^{7}Li, ^{7}Be)^{18}N, Putt *et al.* (1983) found strongly populated levels of ^{18}N at excitation energies of 121 and 747 keV, while the ground state and a level at 580 keV were weakly populated. Only the ground state has a definite spin–parity assignment, which is $J^\pi = 1^-$ (Olness *et al.* 1982).

Full $0\hbar\omega$ and $1\hbar\omega$ shell model calculations by Millener (Olness *et al.* 1982; D. J. Millener, personal communication) indicated that the low-lying negative-parity states of both ^{17}N and ^{18}N are well described by the weak coupling of a $p_{1/2}$ proton hole to the low-lying ^{18}O and ^{19}O states. This is illustrated in Fig. 1, which also shows the similar situation for the low-lying states of ^{15}N and ^{16}N (Millener and Kurath 1975). In the ^{18}N case, where the 2^- states can have mixed ^{19}O parentage, the lower 2^- state contains only a 12·4% admixture of the $\frac{1}{2}^-\otimes\frac{3}{2}^+$ configuration (Olness *et al.* 1982). The experimental energies of the low-lying $^{15-18}$N and $^{16-19}$O levels are given in Fig. 2.

Millener's calculations used the Millener–Kurath (MK) (1975) interaction between nucleons from the p and sd shells. This was derived to give a good account of the non-normal parity states of a number of nuclei from ^{11}Be to ^{16}O. It does not, however, give the experimental ordering of the low-lying ^{16}N levels or of the doublet members in ^{17}N. This suggests that the predicted energies of the low-lying ^{18}N levels might be significantly in error also.

We here adjust the MK interaction to fit the observed ^{16}N energies and use the adjusted interaction to calculate the ^{17}N and ^{18}N energies. This is done by calculating the adjustments on the basis of a weak-coupling structure of the $^{16-18}$N levels and simple descriptions of the $^{17-19}$O levels, and assuming that the same relationship between the adjustments would hold in the full shell model calculation. Since the

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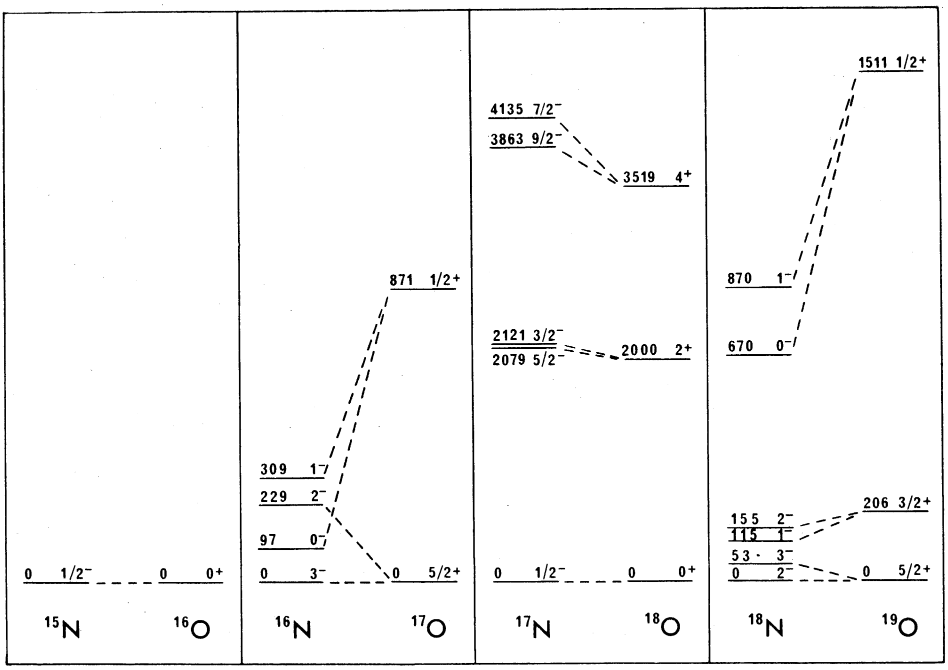

Fig. 1. Spectra of low-lying negative-parity states of $^{15-18}$N and of low-lying positive-parity states of $^{16-19}$O calculated by Millener. Excitation energies are in keV. The weak-coupling parentage of the $^{15-18}$N states is indicated.

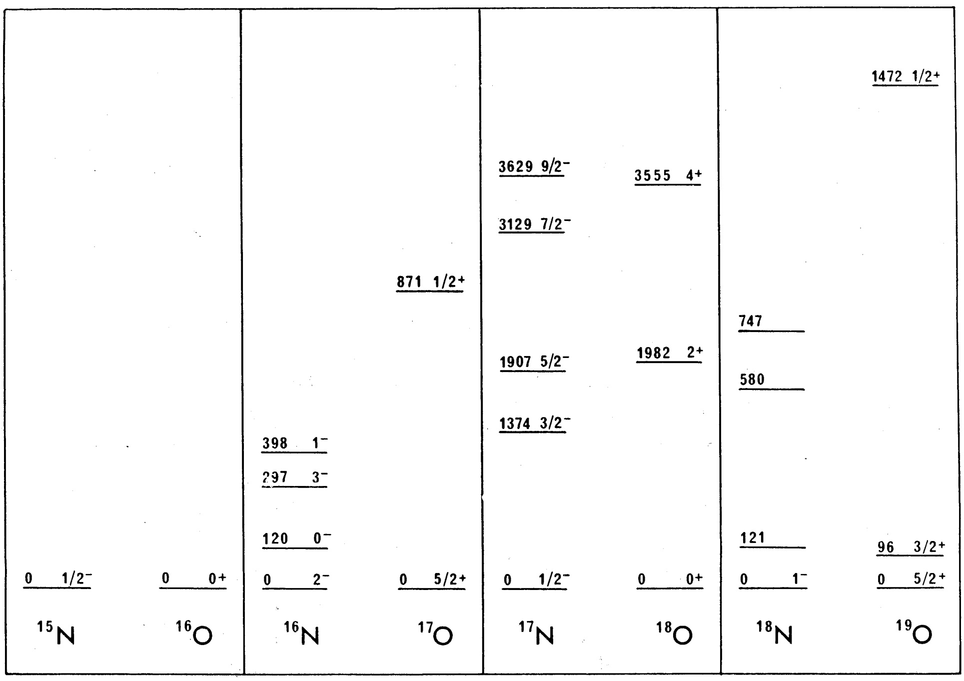

Fig. 2. Experimental spectra of low-lying negative-parity states of $^{15-18}$N and of low-lying positive-parity states of $^{16-19}$O (Ajzenberg-Selove 1982, 1983; Olness *et al.* 1982; Putt *et al.* 1983).

sd-shell interaction of Chung and Wildenthal (Chung 1976) used by Millener does not reproduce exactly the observed ^{18}O and ^{19}O level energies, we also adjust this interaction to fit the observed energies, as was previously suggested for ^{19}O by Olness *et al.* (1982).

2. Calculation

For the purpose of the weak-coupling calculation, the assumed $^{17-19}O$ state descriptions, relative to a closed-shell $1s^4 1p^{12}$ ^{16}O ground state core, are

$$^{17}\mathrm{O},\ T=\tfrac{1}{2}:\quad \psi(\tfrac{5}{2}^+) = |\,\mathrm{d}_{5/2}, \tfrac{5}{2}\rangle, \quad \psi(\tfrac{1}{2}^+) = |\,\mathrm{s}_{1/2}, \tfrac{1}{2}\rangle,$$

$$^{18}\mathrm{O},\ T=1:\quad \psi(0^+) = |\,\mathrm{d}^2_{5/2}, 0\rangle, \quad \psi(2^+) = |\,\mathrm{d}^2_{5/2}, 2\rangle, \quad \psi(4^+) = |\,\mathrm{d}^2_{5/2}, 4\rangle,$$

$$^{19}\mathrm{O},\ T=\tfrac{3}{2}:\quad \psi(\tfrac{5}{2}^+) = |\,\mathrm{d}^3_{5/2}, \tfrac{5}{2}\rangle, \quad \psi(\tfrac{3}{2}^+) = |\,\mathrm{d}^3_{5/2}, \tfrac{3}{2}\rangle, \quad \psi(\tfrac{1}{2}^+) = |(\mathrm{d}^2_{5/2}, 0)\mathrm{s}_{1/2}, \tfrac{1}{2}\rangle.$$

The energy of the state $\psi(J^+)$ is denoted by $E(J)$. The energy $E_I(J)$ of the $^{16-18}N$ state of spin I^- obtained by weak coupling of a $p_{1/2}$ proton hole to the state $\psi(J^+)$ is then given by

$$^{16}\mathrm{N}:\quad E_I(J) = E(J) + M\{(l_j\, \mathrm{p}^{-1}_{1/2})I\},$$

$$^{17}\mathrm{N}:\quad E_I(J) = E(J) + 2\sum_{\bar{J}} U^2(\tfrac{5}{2}\tfrac{5}{2}I\tfrac{1}{2}; J\bar{J})\, M\{(\mathrm{d}_{5/2}\, \mathrm{p}^{-1}_{1/2})\bar{J}\},$$

$$^{18}\mathrm{N}:\quad \begin{cases} E_I(J) = E(J) + 3\sum_{\underline{J}\bar{J}} \langle \mathrm{d}^3_{5/2} J\{|\, \mathrm{d}^2_{5/2} \underline{J}, \mathrm{d}_{5/2}\rangle^2 U^2(\underline{J}\tfrac{5}{2}I\tfrac{1}{2}; J\bar{J})\, M\{(\mathrm{d}_{5/2}\, \mathrm{p}^{-1}_{1/2})\bar{J}\} \\ \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (J = \tfrac{3}{2}, \tfrac{5}{2}), \\ E_I(\tfrac{1}{2}) = E(\tfrac{1}{2}) + \tfrac{1}{6}\sum_{\bar{J}} (2\bar{J}+1)\, M\{(\mathrm{d}_{5/2}\, \mathrm{p}^{-1}_{1/2})\bar{J}\} + M\{(\mathrm{s}_{1/2}\, \mathrm{p}^{-1}_{1/2})I\}. \end{cases}$$

Here $M\{(l_j\, \mathrm{p}^{-1}_{1/2})I\}$ is the isospin 1 diagonal particle–hole matrix element ($l_j = \mathrm{d}_{5/2}$ or $\mathrm{s}_{1/2}$), and fractional parentage coefficients and Jahn U coefficients are involved. It is convenient to consider the mean energy of a doublet

$$\bar{E}(J) = \frac{1}{2(2J+1)} \sum_I (2I+1)\, E_I(J),$$

and the doublet splitting

$$D(J) = E_{J+\frac{1}{2}}(J) - E_{J-\frac{1}{2}}(J).$$

If one writes

$$S(J, J') = \bar{E}(J) - \bar{E}(J') - \{E(J) - E(J')\},$$

for the separation of the mean doublet energies in $^{16-18}N$ relative to the separation of the corresponding parent states in $^{17-19}O$, then one obtains the weak-coupling formulae

$$^{16}\mathrm{N}:\quad S(\tfrac{1}{2}, \tfrac{5}{2}) = X, \tag{1a}$$

$$D(\tfrac{5}{2}) = \Delta_{32}, \qquad D(\tfrac{1}{2}) = \Delta_{10},$$

$$^{17}\mathrm{N}: \quad S(2,0) = 0, \qquad S(4,0) = 0, \tag{1b}$$

$$D(2) = \tfrac{5}{6}\Delta_{32}, \qquad D(4) = \tfrac{3}{2}\Delta_{32},$$

$$^{18}\mathrm{N}: \quad S(\tfrac{3}{2},\tfrac{5}{2}) = 0, \qquad S(\tfrac{1}{2},\tfrac{5}{2}) = X, \tag{1c}$$

$$D(\tfrac{5}{2}) = \Delta_{32}, \qquad D(\tfrac{3}{2}) = \tfrac{2}{3}\Delta_{32}, \qquad D(\tfrac{1}{2}) = \Delta_{10},$$

where

$$\Delta_{JJ'} = M\{(l_j \mathrm{p}_{1/2}^{-1})J\} - M\{(l'_{j'} \mathrm{p}_{1/2}^{-1})J'\},$$

$$X = \tfrac{3}{4}\Delta_{10} - \tfrac{7}{12}\Delta_{32} - \Delta_{20}.$$

Table 1. Values of energy separations and splittings in $^{16-18}$N

Nucleus	Quantity	Value (keV) Experiment[A]	Shell model[B]	Adjusted
^{16}N	$S(\frac{1}{2},\frac{5}{2})$	−716	−710	−716
	$D(\frac{5}{2})$	297	−229	297
	$D(\frac{1}{2})$	278	212	278
^{17}N	$S(2,0)$	−288	96	96
	$S(4,0)$	−148	465	465
	$D(2)$	533	−42	396
	$D(4)$	500	−272	517
^{18}N	$S(\frac{3}{2},\frac{5}{2})$		−117	−117
	$S(\frac{1}{2},\frac{5}{2})$		−730	−736
	$D(\frac{5}{2})$		34	560
	$D(\frac{3}{2})$		21	372
	$D(\frac{1}{2})$		200	266

[A] Ajzenberg-Selove (1982, 1983).
[B] Olness *et al.* (1982); D. J. Millener, personal communication.

Values of $S(J,J')$ and $D(J)$ obtained from experimental energies (Ajzenberg-Selove 1982, 1983) and from the full shell model calculations of Millener are given in the third and fourth columns of Table 1. The values for ^{18}N make use of energies corresponding to unmixed 2^- states of pure parentage, $E(2^-;\frac{1}{2}^-\otimes\frac{5}{2}^+) = 19$ keV and $E(2^-;\frac{1}{2}^-\otimes\frac{3}{2}^+) = 136$ keV, which with a mixing matrix element of ± 51 keV give eigenstates with the energies and 12·4% mixing calculated by Millener.

We assume that adjustments to the MK interaction will give adjustments to the values of $S(J,J')$ and $D(J)$ for $^{16-18}$N that are related in the same way as in the weak-coupling formulae (1). Thus, adjustment of the MK interaction to make the calculated values of $S(J,J')$ and $D(J)$ for ^{16}N agree with the experimental values implies changes in the values of X, Δ_{32} and Δ_{10} of −6, 526 and 66 keV respectively, and these imply definite changes in the calculated values for ^{17}N and ^{18}N. These adjusted values of $S(J,J')$ and $D(J)$ are given in the last column of Table 1. Adjusted values of the $^{16-18}$N level energies are then obtained by using these adjusted values from Table 1, together with experimental values of $^{17-19}$O energies. Since the experimental ^{17}O energies were fitted in the calculation of Millener and Kurath (1975), the adjusted ^{16}N energies agree with the experimental values. By using experimental values for the ^{18,19}O energies, we are effectively adjusting the sd-shell interaction used in the shell model calculations. The resultant adjusted ^{17,18}N level energies are shown in Fig. 3. The 2^- levels shown are the result of mixing the states of pure parentage with a mixing matrix element of ± 51 keV, giving a lower 2^- level containing only $1\cdot3\%$ of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ configuration.

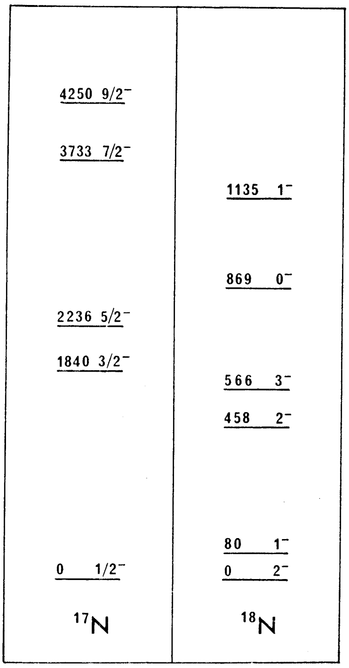

Fig. 3. Spectra of low-lying negative-parity states of ^{17}N and ^{18}N calculated using adjusted shell model interactions.

3. Discussion

It is seen from Table 1 and Fig. 3 that the adjusted interactions give better agreement with the experimental level energies of ^{17}N than did the original Millener calculation, at least as far as the ordering and separations of the doublet members are concerned. This suggests that the predicted ^{18}N energies for the adjusted interactions should be more accurate than those given by Olness *et al.* (1982).

There is still the difficulty, however, that the predicted ground state of ^{18}N is 2^-, whereas the observed ground state is 1^- (Olness *et al.* 1982). An argument for expecting a 1^- ground state has been given by Sheline (1983) on the basis of the collective model. Putt *et al.* (1983) found the ground state and 580 keV level of ^{18}N to be weakly populated in the reaction ^{18}O(^{7}Li, ^{7}Be)^{18}N, while the 121 and 747 keV levels were strongly populated. Since the $\frac{3}{2}^+$ level of ^{19}O is weakly populated relative to the $\frac{5}{2}^+$ ground state in ^{18}O(d, p)^{19}O (Wiza and Middleton 1966), it is reasonable to suppose that ^{18}N states of $\frac{1}{2}^- \otimes \frac{3}{2}^+$ structure would be populated weakly compared with those of $\frac{1}{2}^- \otimes \frac{5}{2}^+$ structure in ^{18}O(^{7}Li, ^{7}Be)^{18}N. The requirements of minimal changes to the ^{18}N spectrum of Fig. 3, of a 1^- ground state, of weak population of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ states, and of small mixing of the $\frac{1}{2}^- \otimes \frac{3}{2}^+$ and $\frac{1}{2}^- \otimes \frac{5}{2}^+$ 2^- states lead to suggested spin assignments of 2^-, 2^- and 3^- for the observed 121, 580 and 747 keV levels of ^{18}N respectively. The main change to the spectrum of Fig. 3 is a reduction of the energy of the lower 1^- state by about 200 keV. The non-observation of the $\frac{1}{2}^- \otimes \frac{1}{2}^+$ 0^-

and 1^- levels in $^{18}O(^7Li, ^7Be)^{18}N$ is not surprising, since the similar reaction $^{16}O(^7Li, ^7Be)^{16}N$ populates the low-lying 0^- and 1^- levels, of $\frac{1}{2}^- \otimes \frac{1}{2}^+$ structure, very weakly relative to the low-lying 2^- and 3^- levels, of $\frac{1}{2}^- \otimes \frac{5}{2}^+$ structure (L. K. Fifield, personal communication).

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