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Excitation of the 9·50 MeV ($\frac{9}{2}+$) State of ^{13}C in Intermediate Energy Scattering Experiments

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Abstract

The differential cross section for the inelastic scattering of 135 MeV protons leading to the $\frac{9}{2}^+$ (9·5 MeV) state in ^{13}C has been measured. Microscopic DWBA analyses show that the angular distribution is well described by a pure (or nearly pure) $1p_{3/2}$ to $1d_{5/2}$ neutron transition with a total angular momentum transfer of 4, a description which is consistent with the previously observed large asymmetry in inelastic π^+ and π^- scattering to this state.

Introduction

Recent experiments on pion inelastic scattering from light nuclei, ^{12,13}C and ^{16}O in particular, have provided evidence on configuration and isospin mixing in nuclear states that otherwise are conjectured to be simple stretched configuration states. For example, the excitation, by π^+ and π^- scattering, of the 4^- states in ^{16}O at 17·8, 19·0 and 19·8 MeV can be understood only in terms of three state mixing (Barker *et al.* 1981). Such an explanation requires the consideration of $1p$–$1h$, 4^-, $T = 0$ and $T = 1$ configurations *and* the dominant $3p$–$3h$, 4^-, $T = 0$ basis state, and is able simultaneously to account for the relevant transfer reaction data as well as the pion and proton inelastic scattering data (Henderson *et al.* 1979; Barker *et al* 1981).

Data from transitions to such stretched configuration states in nuclei, particularly transitions induced by a variety of projectiles, may therefore provide useful information about the structure of those nuclear states. One such transition is that to the $\frac{9}{2}^+$ state in ^{13}C at 9·50 MeV excitation.

There has been uncertainty as to the spin and parity of this state as evidenced by an earlier compilation which assigns a $\frac{3}{2}^-$ spin–parity (Ajzenberg-Selove 1976; see also Holbrow *et al.* 1974; Anyas-Weiss *et al.* 1974). However, a recent assignment of $\frac{9}{2}^+$ (Ajzenberg-Selove 1981) is based primarily on the proton and pion scattering data discussed here. In particular, the inelastic proton scattering data reported here are clearly to be associated with a high (L=4) multipole transition rather than the low (L=2) multipole transition expected if the $\frac{3}{2}^-$ spin–parity assignment for the 9·5 MeV state is retained (see later discussion).

Assuming a $\frac{9}{2}^{+}$ assignment then, our interest in this particular transition was motivated by pion inelastic scattering data (Dehnhard *et al.* 1979) which revealed a marked asymmetry

$$\{\sigma(\pi^-)-\sigma(\pi^+)\}/\{\sigma(\pi^-)+\sigma(\pi^+)\} = 0\cdot8\pm0\cdot2,$$

consistent with a pure neutron excitation. A $\frac{9}{2}^{+}$ ($T=\frac{1}{2}$) state constructed for the 13-nucleon system will involve only a few configurations and its excitation from the ground state will occur in only one simple excitation process. Thus, if the pion asymmetry data are complemented by inelastic proton scattering data in an analysis, a stringent test of the structure of this $\frac{9}{2}^{+}$ state may be possible.

Experiment

Data on the differential cross section for the scattering of 135 MeV protons from ^{13}C have been obtained using the Indiana University Cyclotron Facility. The proton beam bombarded a $9\cdot5\ \mathrm{mg\,cm^{-2}}$ elemental carbon target enriched in ^{13}C to 91%. The inelastically scattered protons were detected with the QDDM magnetic spectrograph of the IUCF, to which the beam analysis system was dispersion matched. The excitation energy bite at the focal plane of the spectrograph was about $6\frac{1}{2}$ MeV, and the data reported here were obtained from measurements taken with the spectrograph setting centred on a proton energy corresponding to 9·0 MeV excitation. The focal plane detector was composed of a helical delay-line gas counter for position measurement (Officer *et al.* 1975), followed by two plastic scintillators. The differential energy loss of detected particles in the two scintillators was used in particle identification; a triple coincidence from the detectors was required. Overall resolution was about 80 keV FWHM.

The differential cross section for inelastic proton scattering to the 9·50 MeV state of ^{13}C was calculated from the number of counts in the peak using the known acceptance solid angle of the spectrograph, the quantity of charge collected in the particular run and the ^{13}C target thickness. The statistical accuracy of the differential cross sections calculated in this way is better than $\pm3\%$ except where explicit error bars are shown in Fig. 1. The absolute accuracy of the cross section points has been estimated by comparing differential cross sections measured at different times, using different ^{13}C targets. The absolute cross section values are expected to be accurate to $\pm6\%$.

Analysis of Results

Inelastic proton scattering at an incident energy of 135 MeV may be analysed by direct reaction theory using the two nucleon *t* matrix, since at these energies any competing reaction mechanisms may be ignored (Geramb *et al.* 1975; Love 1980). Further, distortion effects are not so critical as at lower energies, though they cannot be ignored. Thus, if the relevant *t* matrix can be specified, medium energy (p, p′) studies may be good tests of nuclear structure. At 135 MeV, the *t* matrix should be well defined by the model form ascertained from previous analyses of inelastic proton scattering data on the excitation of unnatural parity 4^- states in ^{16}O (Amos *et al.* 1978; Henderson *et al.* 1979; Lindgren *et al.* 1979; Barker *et al.* 1981) and 6^-

states in ^{24}Mg and ^{28}Si (Amos *et al.* 1978; Lindgren *et al.* 1979). This is expected since the simplest mode of formation of the $\frac{9}{2}^+$, $T = \frac{1}{2}$ state in ^{13}C is to form a 4^- *p–h* excitation in the ^{12}C core, the $p_{1/2}$ neutron remaining as a spectator.

The model *t* matrix used in some of the earlier calculations consisted of a central even-state interaction and a tensor force, of the form given by Eikemeier and Hackenbroich (1971), but with the strength of the tensor force reduced to 70% of that specified by them. In the distorted wave approximation, the transition amplitude has the form (Geramb *et al.* 1975; Amos *et al.* 1978; Lindgren *et al.* 1979; Love 1980)

$$T_{\mathrm{if}} = \sum_{j_1 j_2 I \alpha} (2J_{\mathrm{f}}+1)^{-\frac{1}{2}} \langle J_{\mathrm{i}} I M_{\mathrm{i}} N \,|\, J_{\mathrm{f}} M_{\mathrm{f}} \rangle\, S^{(\alpha)}(j_1 j_2; J_{\mathrm{i}} J_{\mathrm{f}}; I)\, \mathscr{M}^{(\alpha)}_{j_1 j_2}. \tag{1}$$

In this expression, $S^{(\alpha)}(j_1\, j_2;\, J_{\mathrm{i}}\, J_{\mathrm{f}};\, I)$ is the spectroscopic amplitude for excitation of a state involving an angular momentum change I, occurring by means of a single particle excitation $j_1 \to j_2$. The nuclear angular momentum change is $J_{\mathrm{i}} \to J_{\mathrm{f}}$, and α specifies a neutron or proton transition, being the third component of isospin. These spectroscopic amplitudes weight the two-nucleon matrix elements

$$\mathscr{M}^{(\alpha)}_{j_1 j_2} = \sum_{m_1 m_2} (-)^{j_1 - m_1} \langle j_1 j_2\, m_1\, -m_2 \,|\, I - N \rangle$$
$$\times \langle \chi_{\mathrm{f}}^{(-)}(0)\, \phi_{j_2 m_2 \alpha}(1) \,|\, t(01) \,|\, \chi_{\mathrm{i}}^{(+)}(0)\, \phi_{j_1 m_1 \alpha}(1) \rangle, \tag{2}$$

in which $\chi^{(\pm)}$ are the distorted wavefunctions describing the incident and emitted protons and the $\phi_{jm\alpha}$ are appropriate bound state wavefunctions. For the latter, Woods–Saxon wavefunctions have been used. All relevant parameters of the potentials are given in Table 1.

Table 1. Potential parameter values

	Optical model	Bound state Woods–Saxon $p_{3/2}$	Bound state Woods–Saxon $d_{5/2}$
V_0 (MeV)	14·23	50·56	68·75
r_0 (fm)	1·315	1·25	1·25
a_0 (fm)	0·629	0·65	0·65
W_0 (MeV)	4·476		
$4W_{\mathrm{d}}$ (MeV)	0·0		
r_{d} (fm)	1·572		
a_{d} (fm)	0·82		
V_{so} (MeV)	3·4	5·3	4·5
W_{so} (MeV)	−2·3		
r_{so} (fm)	0·93	1·25	1·25
a_{so} (fm)	0·5	0·65	0·65
r_{c} (fm)	1·326	1·25	1·25

Only the spectroscopic amplitudes remain to be specified and to do so necessitates definition of a model of nuclear structure. Two models of the structure have been considered. They are the p-s-d shell model for ^{13}C which will be described later and a convenient simple model in which the ^{13}C ground state is treated as a neutron outside the ^{12}C ground state, and the $\frac{9}{2}^{+}$ state as a $d_{5/2}\,p_{3/2}^{-1}$ excitation based upon that ground state. Specifically, the simple model states can be written:

(i) for the ground state $(\frac{1}{2}^{-};T=\frac{1}{2})$

$$|^{13}\mathrm{C};\tfrac{1}{2}^{-}\rangle = N_{\mathrm{i}}\,a^{\dagger}_{\frac{1}{2}m_{\mathrm{i}}\frac{1}{2}}|0\rangle\,; \tag{3a}$$

(ii) for the 9·50 MeV state $(\frac{9}{2}^{+};\frac{1}{2})$

$$|^{13}\mathrm{C};\tfrac{9}{2}^{+}\rangle = N_{\mathrm{f}}\sum_{T} C_T(2T+1)^{\frac{1}{2}}[[a^{\dagger}_{5/2}\times a_{3/2}]^{4;T}\times a^{\dagger}_{1/2}]^{9/2;1/2}|0\rangle; \tag{3b}$$

where the isospin coefficients are constrained to satisfy

$$|C_0|^2+|C_1|^2 = 1\,.$$

In these expressions, the $a^{\dagger}_{jm\alpha}$ are proton $(\alpha=-\frac{1}{2})$ and neutron $(\alpha=\frac{1}{2})$ creation operators for an angular momentum state (j,m) and the square brackets denote standard angular momentum coupling. With the C_0 and C_1 coefficients constrained as noted above, the normalization factors N may be defined in terms of the fractional occupancies σ_j of the jj coupling single-particle states of the 12-particle core ground state, represented by $|0\rangle$ in equations (3). This gives the relations

$$N_{\mathrm{i}} = \{(1-\sigma_{1/2})\}^{-\frac{1}{2}}, \tag{4a}$$

$$N_{\mathrm{f}} = \{(1-\sigma_{5/2})\sigma_{3/2}(1-\sigma_{1/2})\}^{-\frac{1}{2}}. \tag{4b}$$

The spectroscopic amplitude $S^{(\alpha)}(j_1 j_2;J_{\mathrm{i}}J_{\mathrm{f}};I)$ is then written for excitation of the 9·50 MeV $(\frac{9}{2}^{+})$ state of ^{13}C

$$\begin{aligned} S^{(\alpha)}(\tfrac{3}{2}\tfrac{5}{2};\tfrac{1}{2}\tfrac{9}{2};4) = S_{3/2,5/2,4} &= \langle ^{13}\mathrm{C},\tfrac{9}{2}\,\|\,[a^{\dagger}_{5/2}\times a_{3/2}]^{4}\,\|\,^{13}\mathrm{C},\tfrac{1}{2}\rangle \\ &= \{5\sigma_{3/2}(1-\sigma_{5/2})\}^{\frac{1}{2}}\{C_0+(-)^{\frac{1}{2}+\alpha}C_1\}. \end{aligned} \tag{5}$$

The values of C_0, C_1 and $\sigma_{3/2}$ essentially specify the spectroscopic amplitude, since both theoretically (Bassichis *et al.* 1967; Boeker 1968; Smith *et al.* 1979) and experimentally (Spicer 1973) the d-shell occupancy $(\sigma_{5/2})$ in the ^{12}C ground state is near zero.

The recent pion inelastic scattering data (Dehnhard *et al.* 1979) are of particular value in that the same form of scattering amplitude as given in equation (1) is required in the data analysis and, as asymmetry has been measured, we then have a test of the nuclear structure independent of overall normalization. Specifically for the inelastic scattering of 164 MeV pions, the relevant amplitudes can be evaluated assuming 3–3 resonance dominance. In this case, the two-body matrix elements scale as follows:

$$\langle\chi^{(-)}\phi_{j_2}|\,t(\pi\alpha)\,|\chi^{(+)}\phi_{j_1}\rangle \equiv g_{\pi\alpha}f_{j_1j_2}(Q),$$

with

$$g_{\pi^-,+\frac{1}{2}} = g_{\pi^+,-\frac{1}{2}} = 3g_{\pi^+,+\frac{1}{2}} = 3g_{\pi^-,-\frac{1}{2}} = g\,. \tag{6}$$

Thus, in terms of the isoscalar and isovector mixing coefficients, we have

$$T^{(\pm)}_{\mathrm{if}}(\tfrac{9}{2}^{+}) \approx (2C_0\pm C_1)F(Q), \tag{7}$$

where $F(Q)$ is a function dependent on momentum transfer, and the $\pm$ symbols refer to scattering by π^+ and π^- mesons respectively. Then, since the $\sigma(\pi^{\pm})$ are proportional to $|T_{\mathrm{if}}^{(\pm)}|^2$, the asymmetry between π^- and π^+ inelastic scattering becomes (choosing C_1 to be positive)

$$A = \frac{\sigma(\pi^-)-\sigma(\pi^+)}{\sigma(\pi^-)+\sigma(\pi^+)} = -\frac{4C_0\,|(1-C_0^2)^{\frac{1}{2}}|}{3C_0^2+1}. \tag{8}$$

The measured value, $A = 0\cdot 8 \pm 0\cdot 2$, thus determines C_0 to be

$$C_0 = -0\cdot 243 \qquad (-0\cdot 164 \text{ to } -0\cdot 447)$$

$$\text{or} \quad C_0 = -0\cdot 707 \qquad (-0\cdot 447 \text{ to } -0\cdot 832). \tag{9}$$

Thus we have a constraint upon the isospin coefficients and the (p, p′) data are now analysed to determine whether there is a preference for one of these values over the other. The magnitude of the calculated (p, p′) cross sections is, however, also dependent upon the shell occupancies σ_j in the ^{12}C ground state, as given in equation (5). The occupancy $\sigma_{5/2}$ is taken as zero, so that the magnitude of the cross section depends on the value taken for $\sigma_{3/2}$. Its value varies between 0·81, given by a 0p shell model calculation, and 0·59, given by a multi-shell Hartree–Fock calculation, so that calculations of the cross section magnitude will vary by up to a factor of 2 according to the choice of nuclear model.

Discussion

The results of the calculation displayed in Fig. 1*a* were obtained using values of C_0 and $\sigma_{3/2}$ of $-0\cdot 243$ and 0·49 respectively in a fully antisymmetrized direct reaction theory (Geramb *et al.* 1975; Amos *et al.* 1978; Lindgren *et al.* 1979; Love 1980), and using Woods–Saxon potential wavefunctions for the single nucleon. For simplicity, equation (6) does not include the antisymmetrization. The calculation reproduces the general feature of the data, namely a broad peak in the differential cross section; it does give a peak that is somewhat too narrow.

The value of $\sigma_{3/2}$ (0·49) required to determine the magnitude is lower than that given by the Hartree–Fock description of the ^{12}C ground state (0·59) but is acceptable considering the simplicity of the model of ^{13}C structure chosen. We have therefore used the value of 0·49 for $\sigma_{3/2}$ in all analyses which use this model of the ^{13}C states. This parameter being fixed, the variation in differential cross section magnitudes and shapes from those given in Fig. 1*a* is due solely to the relative weights (C_0, C_1) of the isoscalar and isovector transition strengths.

With the previous results (Amos *et al.* 1978; Lindgren *et al.* 1979; Barker *et al.* 1981) as a guide, a broader distribution than that given in Fig. 1*a* is expected if the isovector transition strength is weakened. This occurs if the alternative value of $C_0 = -0\cdot 707$ which fits the pion asymmetry is used. The results of this calculation are shown in Fig. 1*b*; an improved fit results, again with the use of Woods–Saxon wavefunctions.

In both of these results, the calculated differential cross sections are dominated by the contribution of the tensor force, in agreement with earlier studies of the excitation of 4^- states. The tensor force contribution to the cross section is an order of magnitude larger than that due to the central even-state force alone, and furthermore is essential in predicting the peak angle correctly. The fit to the proton inelastic scattering data does prefer one of the two possible solutions, resulting from consideration of the pion asymmetry data, over the other.

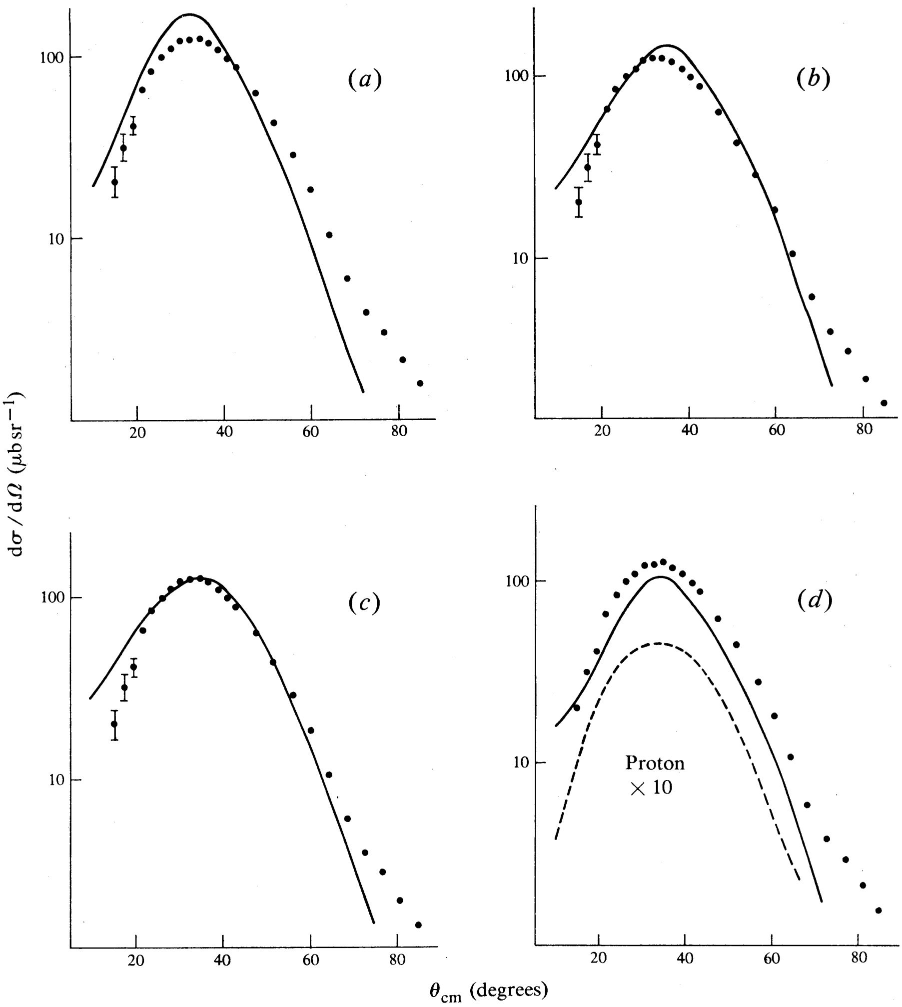

Fig. 1. Calculated (curves) and experimental (dots) differential cross sections for the inelastic scattering of 135 MeV protons leading to the $\frac{9}{2}^+$ state of 9·5 MeV excitation in ^{13}C. Isoscalar 4^- *p–h* amplitudes C_0 of (*a*) $-0{\cdot}243$, (*b*) $-0{\cdot}707$ and (*c*) $-0{\cdot}97$ were used in the specification of the final ($\frac{9}{2}^+$) nuclear state. Part (*d*) is the same as (*b*) except that wavefunctions were used for the ^{13}C ground state and 9·5 MeV state as given by a full p-s-d shell model calculation (see text). The dashed curve in (*d*) is the proton excitation contribution to the proton scattering cross section, enhanced by a factor of 10.

A calculation assuming a dominantly isoscalar core excitation (say, $C_0 = -0{\cdot}97$) was used in a further calculation of the (p, p′) cross section. A good fit to the data results, as is shown in Fig. 1*c*, although, like the two previous results, the calculation is too large at forward angles. However, if this description of the excitation of the $\frac{9}{2}^+$ state were correct, the asymmetry A in the pion scattering results would be 0·25, which clearly rules out this value of C_0. Thus both the proton inelastic scattering and the pion asymmetry measurements are necessary to indicate the isospin admixture in the excitation of the 9·50 MeV state.

Thus, on the basis of this simple model, the combination of pion and proton inelastic scattering to the 9·50 MeV ($\frac{9}{2}^+$) state does indicate a preference for one isospin mixture in its excitation. The *p–h* description for the equally weighted, but opposite in sign, isoscalar and isovector core excitation is the preferred solution of the three in accounting for both pion and proton scattering data. This description is equivalent to a pure $1p_{3/2} \rightarrow 1d_{5/2}$ neutron excitation with a total angular momentum change of 4, since the proton and neutron spectroscopic amplitudes are 0 and $-2 \cdot 06$ respectively (with $\sigma_{3/2}$ taken as 0·49).

The second model of ^{13}C structure was given by a full p-s-d shell model calculation with a composite set of matrix elements used earlier (Morrison *et al.* 1978) in a calculation of the low excited states of ^{18}O. This calculation gave proton and neutron spectroscopic amplitudes for the excitation of the $\frac{9}{2}^+$ state of 0·113 and $-1 \cdot 954$ respectively. The neutron transition clearly dominates, consistent with the pion scattering asymmetry measurement. Fig. 1*d* shows the calculation of the proton inelastic scattering differential cross section compared with the experimental data. In this case there is no free normalizing factor, as $\sigma_{3/2}$ is fixed by the shell model calculation and we have taken $C_0 = -0 \cdot 707$. Specifically, this model accounts for effects due to the polarization of the ^{12}C core from the presence of the thirteenth nucleon, effects which were explicitly neglected in the simple model. The effect of including core polarization will be to vary the occupation probabilities $\sigma_{1/2}$ and $\sigma_{3/2}$ from those appropriate to the ground state of ^{12}C. These variations will seriously affect the ^{13}C ground state description.

The fit to the data, shown in Fig. 1*d*, is regarded as acceptable given that there are no free parameters in the calculation. A normalization upwards of 15% would make this a very good fit. Also shown in Fig. 1*d* (dashed curve) is the proton excitation contribution to the proton scattering cross section, enhanced by a factor of 10, for comparison with the total cross section.

This p-s-d shell model calculation specifies the isospin mix in the excitation of the $\frac{9}{2}^+$ state as $C_0 = -0 \cdot 666$ and $C_1 = 0 \cdot 746$, in fair agreement with the equal mixture given earlier by the simple excitation model. The pion asymmetry specified by these values is 0·85. Thus the pion scattering asymmetry A, and the magnitude and shape of the proton inelastic scattering cross section are all accounted for in a distorted wave calculation combined with wavefunctions from a full p-s-d shell model calculation for the ground and $\frac{9}{2}^+$ states of ^{13}C.

Finally, we return to the question of the spin–parity assignment of the 9·5 MeV state. The pion and inelastic proton scattering data analysed here clearly are consistent with the stretched *p–h* values of $\frac{9}{2}^+$ preferred by particle transfer reaction studies (Anyas-Weiss *et al.* 1974) to the older but tentative $\frac{3}{2}^-$ assignment. Given that these are the only possible assignments, the (p, p′) data clearly dismiss the $\frac{3}{2}^-$ possibility as is illustrated in Fig. 2, where arbitrarily normalized pure neutron excitation calculations for an $L = 2$ natural parity transition to a $\frac{3}{2}^-$ final state are compared with the tensor force dominated $\frac{9}{2}^+$ state excitation. The two distributions are different irrespective of the relative contributions of central and tensor forces required by an appropriate $\frac{3}{2}^-$ model spectroscopy, and whether the excitation is restricted to a pure neutron excitation or not.

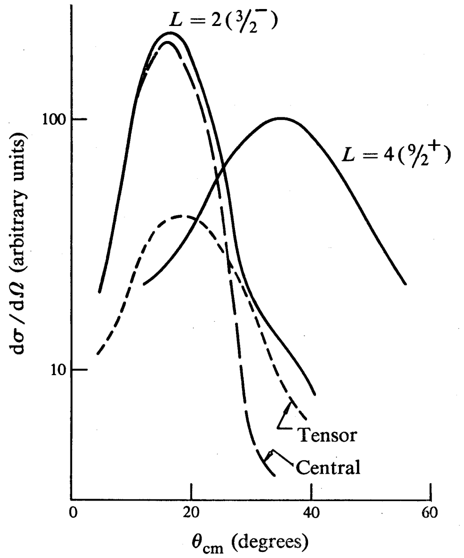

Fig. 2. Calculated differential cross sections for pure neutron transitions in the inelastic scattering of 135 MeV protons leading to final states of spin–parity assignment $\frac{3}{2}^-$ and $\frac{9}{2}^+$.

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