

RUNGE SOLUTION OF THE CONTINUUM EQUATIONS FOR SPHERICAL ELECTROSTATIC PLASMA PROBES

By F. H. DORMAN*

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Abstract

An approximate but convergent method is used to determine the electric potential and charged particle density near non-small spherical probes in a weakly ionized continuum plasma. Quantities determined partly by experiment (ε , ρ_p , y_p) and partly from continuum theory (J_+ , J_-) are introduced into three continuum differential equations which are then solved using Runge's method. The initial conditions must be estimated iteratively until the sheath solution joins smoothly to the quasi-neutral solution. Two different types of solution curves are discussed.

The characterization of plasmas requires a knowledge of potentials and concentrations of charged particles. These quantities can be estimated from experimental measurements of currents from probes, but interpretation of the results is complicated by the variety of physical assumptions made about conditions surrounding the probes in the collisional case (Waymouth 1964). Thus, for example, we have the theories of Boyd (1951), Schulz and Brown (1955), Su and Lam (1963), Cohen (1963), Kiel (1969), and Baum and Chapkis (1970). Radbill (1966), after using a sophisticated computer approach to the continuum problem, drew attention to the desirability of simple hand calculation methods. The present note shows how this may be done by solving the continuum differential equations after obtaining estimates of the particle currents from Cohen's (1963) asymptotic theory. The nomenclature agrees with that of Cohen, with the additions that $y' = dy/d\zeta$ and y'_1 is the value at $\zeta = 1$.

Estimates of the required parameters may be derived as follows.

- (1) The ratio $\varepsilon = T_+/T_-$ may be obtained by assuming T_+ equivalent to the gas temperature and determining T_- from a double spherical probe current-voltage curve by the method of Johnson and Malter (1950). This has been discussed by Cozens and von Engel (1965) and Bradley and Matthews (1967), although the latter authors' corrections are hardly needed in view of the realizable accuracy.
- (2) The ratio $\rho_p = r_p/\lambda_D$ requires for the determination of the Debye length a value of N_0 , the density of charged particles at $r = \infty$. This may be obtained by calculating the saturation current and comparing it with the experimental value.
- (3) The dimensionless probe potential y_p in Cohen's (1963) theory may be derived from the "saturation" voltage that corresponds to the saturation current, and in practice an estimate is made of the sort that Sajben and Lee (1970) have

* Division of Mineral Chemistry, CSIRO, P.O. Box 124, Port Melbourne, Vic. 3207.

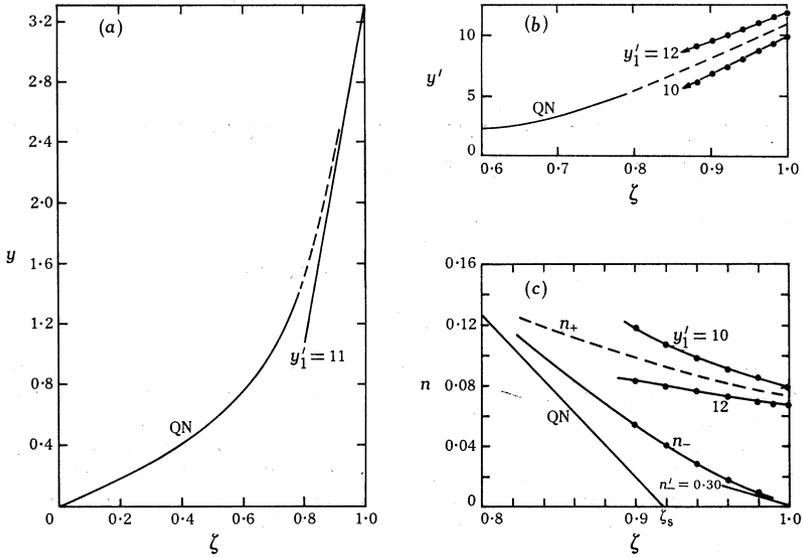


Fig. 1.—Plots of y , y' , and n against ζ for the first example (a nitrogen plasma at 10 Torr). The points are the calculated results for the indicated values of y_1' while the dashed curves show the computed solutions which smoothly join the quasi-neutral solutions (QN) given by the relations (1) with $\zeta_s = 0.92$.

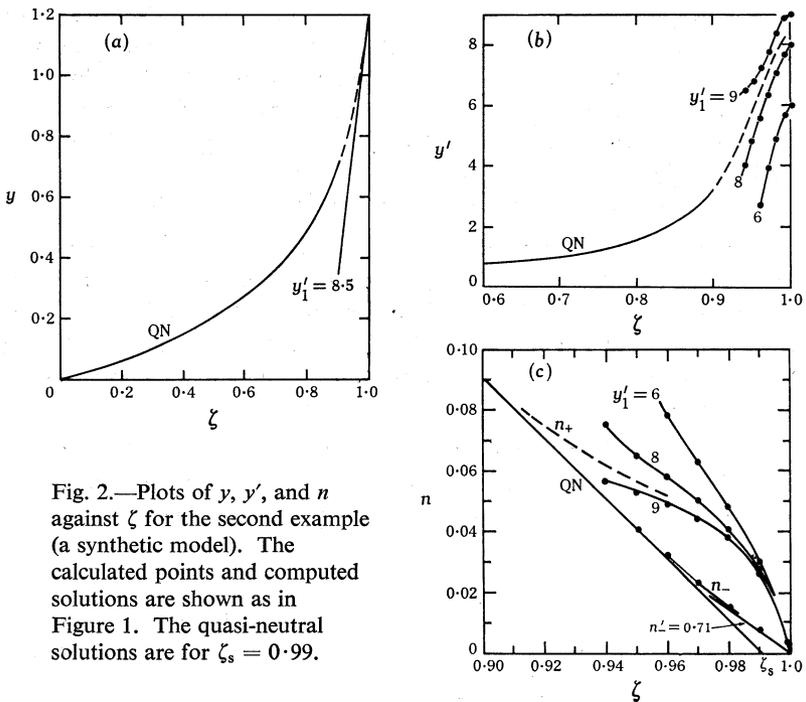


Fig. 2.—Plots of y , y' , and n against ζ for the second example (a synthetic model). The calculated points and computed solutions are shown as in Figure 1. The quasi-neutral solutions are for $\zeta_s = 0.99$.

criticized. However, provided the slope of the linear saturation ion current is not too large, a useful estimate is possible when the effect of the leads is ignored (Whitman and Chien 1971).

- (4) The currents J_+ and J_- can be read to two-figure accuracy from Cohen's (1963) Figures 3 and 4, which may be improved by considering a set of $\varepsilon J_+ + J_-$ for different values of y_p . In as much as $\varepsilon J_+ + J_-$ determines the point ζ_s where the quasi-neutral solution for n cuts the ζ axis, it should be known to 2%. The present work has shown that extrapolated values may be estimated to about $\rho_p = 10$ since, although Cohen's asymptotic theory uses inverse powers of $\rho_p^{2/3}$, the continuum equations are smooth for finite ρ_p .

Using the above procedure, Cohen's (1963) example (his Fig. 1(b)) was initially calculated with $J_- = 0$. Complete agreement was obtained and a value of $y'_1 = 50 \pm 0.5$ was estimated. Seven further examples were then investigated, two of which are considered here.

The results for the first example are illustrated in Figure 1. The parameters $\rho_p = 19$, $y_p = 3.3$, $\varepsilon^{-1} = 114$, $\varepsilon J_+ = 0.80$, and $J_- = 0.30$ were estimated from experimental double probe results with a nitrogen plasma at 10 Torr pressure excited by 900 MHz microwave power (200 W). The quasi-neutral solutions (QN) given by the relations

$$y = \{-\varepsilon(J_+ - J_-)\zeta_s/(1 + \varepsilon)\} \ln n, \quad n = 1 - \zeta \zeta_s^{-1}, \quad (1)$$

are also plotted in Figure 1. The difference between the n_+ curve of Figure 1(c) and that in Cohen's Figure 1(b) is due to the larger value of ε^{-1} in his equation (4), whilst the n_- curve is raised to be more nearly linear due to its initial gradient of 0.30. From Figure 1, an estimate of sufficient accuracy for our purpose of where the Runge solution joins the quasi-neutral solution is at $\zeta = 0.78$, so that, taking this as the sheath "edge", then for a probe radius of 0.225 cm and $\lambda_D = 0.012$ cm the sheath thickness is $(5 \pm 1)\lambda_D$.

The second example (Fig. 2) was a synthetic one, with parameters $\rho_p = 50$, $y_p = 1.2$, $\varepsilon^{-1} = 10$, $\varepsilon J_+ = 0.40$, and $J_- = 0.71$, and illustrates a model nearer to Figure 1 of Cohen (1963).

From a consideration of the seven examples so far calculated the following conclusions have been drawn.

- (1) y'_1 is inversely related to ρ_p , other parameters being substantially constant.
- (2) There are two types of n_+ curve depending on the value of ε^{-1} : (i) For small ε^{-1} the curve has a relatively slow approach to $\zeta = 1$ (see Fig. 2(c)). (ii) For large ε^{-1} there is a steep drop from $\zeta \approx 0.999$ (see Fig. 1(c)); comparison of this with Figure 2 of Su and Lam (1963) shows that there is only a very thin "diffusion layer" and no proper "ion sheath" for the lower value of y_p used here.

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