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A simple method for determination of depth of investigation characteristics in resistivity prospecting

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Abstract

For determination of depth of investigation characteristics in direct-current resistivity methods, individual contribution from an infinite horizontal sheet of infinitesimal thickness within a homogeneous half-space has been obtained through the solution of boundary value problems. In this study, no assumption is made of electrostatic equivalence or dipole polarization, which has been utilized by earlier workers.

Key words: depth of investigation characteristics, direct-current resistivity methods, homogeneous half-space.

Introduction

In the resistivity sounding method, the concept of depth of investigation was first introduced by Evjen (1938), where it has been defined as the depth at which a thin horizontal layer of ground contributes a maximum to the total measured signal. Following a 'quasimathematical' approach, Evjen found that for the Wenner electrode configuration, the depth of investigation is one-ninth of the distance between the current electrodes. The interest in the topic was revived by Roy and Apparao (1971) from a 'physically evident approach'. In their analysis the potential due to any infinitesimal horizontal sheet within a homogeneous half-space, the sheet extending from minus infinity to plus infinity in both x and y directions, has been determined (z axis points vertically downwards and x-y plane represents horizontal surface of observation). Then the individual contribution from all the infinitesimally thin layers are plotted against depth for a particular electrode array from which the depth investigation characteristic is obtained graphically for that electrode configuration.

To obtain the potential due to a thin infinitesimal horizontal sheet within a homogeneous half-space, Roy and Apparao used the concept of 'electrostatic equivalence' and obtained the contribution due to a volume element within the half-space. (A proportionality factor has been utilized by them for calculation of dipole moment of each elementary volume of the medium with *a posteriori* justification of the potential due to a homogeneous half-space. However, the justification for using the same proportionality factor was provided by Koefoed, in 1972.) The signal contribution due to the volume element is then integrated with respect to x and y, varying from minus infinity to plus infinity, which yields the individual signal from the infinitesimal horizontal sheet.

Following this basic approach, Roy (1972) extended the studies for different electrode arrays. Further, Roy (1974) determined the separate individual signal contributions for each layer in a two and three layer earth. Expressions for DIC (depth of investigation) for line electrodes, anisotropic half-space and gradient arrays have been derived by Apparao and Gangadhara Rao (1974), Bhattacharya and Sen (1981), and Bhattacharya and Dutta (1982).

In an interesting study of the generalized inversion of resistivity sounding data, Oldenburg (1978) derived the expression for the DIC characteristics of a homogeneous half-space from the concept of Frechet Kernel.

The theoretical basis of the study of DIC, generalized by Roy (1978), has recently been debated by Guptasarma (1981) and Guerreiro (1983), in favour of the 'quasiphysical' approach for determination of 'apparent contribution'. Although the concept of DIC is instructive for the 'teaching and understanding of geophysics' (Barker 1979), it has found practical application in recent times. Based on DIC studies, Edwards (1977) suggested a new technique of preparing pseudosections which have better correlations with actual sections. But possibly the most judicious use of the DIC concept has been put forward by Barker (1981). Based on his earlier study (1979) where the vertical signal contribution section was plotted for a homogeneous half-space for different electrode arrays, Barker (1981) developed an offset system of electrical resistivity sounding which minimized the near surface noise with the help of a patented multicore cable (Roy 1981).

In this study an attempt was made to derive the expression for DIC for homogeneous half-space. It has already been mentioned that the pivotal point for determination of DIC is to derive an expression for the signal from an infinitesimally thin horizontal sheet within a homogeneous half-space. This analysis has been made from a mathematical point of view based on the solution of boundary value problems for a horizontally stratified earth. It may be mentioned that in the present study, no assumption has been made about electrostatic equivalence or dipole polarization of the dielectric medium.

Mathematical analysis

A homogeneous half-space of resistivity ρ can be considered to consist of an infinite number of horizontal sheets of infinitesimal thickness dz, the sheets extending from $-\infty$ to $+\infty$ in both x and y directions. The potential (signal) due to a current source of strength $+I$ at a distance a from the source, which is equal to $\rho I/2\pi$a, may be supposed to be the sum of individual signals from the infinite number of horizontal sheets of infinitesimal thickness dz. In order to obtain an expression from any such individual sheet of thickness dz at a

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depth of z from the free surface, consider a three layer earth ($\rho_1 - \rho_2 - \rho_1$) where the intermediate layer is supposed to be sufficiently thin, the depth to the layers from the free surface being h_1 and h_2 ($h_2 = h_1 + h$), where h is a first order small quantity (Fig. 1).

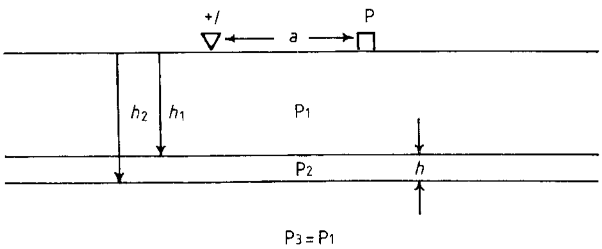

Fig 1 A three-layer earth with an intermediate thin layer.

The electrical potential $V(\mathrm{a})$ for such a model is given by Stefanesco's (Stefanesco *et al* 1930) equation:

$$V(\mathrm{a}) = \frac{\rho_1 I}{2\pi}\Big[\frac{1}{\mathrm{a}} + 2\int_0^\infty \theta(\lambda) J_0(\lambda \mathrm{a})\,\mathrm{d}\lambda\Big]$$

where the Kernel function, $\theta(\lambda)$, is

$$\theta(\lambda) = \frac{\mathrm{k}_1 \mathrm{e}^{-2\lambda h_1} + \mathrm{k}_2 \mathrm{e}^{-2\lambda h_2}}{1 - \mathrm{k}_1 \mathrm{e}^{-2\lambda h_1} - \mathrm{k}_2 \mathrm{e}^{-2\lambda h_2} + \mathrm{k}_1 \mathrm{k}_2 \mathrm{e}^{-2\lambda(h_2 - h_1)}} \quad (1)$$

Since $\mathrm{k}_1 = -\mathrm{k}_2$ for a $\rho_1 - \rho_2 - \rho_1$ sequence, equation (1) can be simplified by retaining only the first power of h, thus:

$$\theta(\lambda) = \frac{2\mathrm{k}_1 h}{1 - \mathrm{k}_1^2} \lambda \mathrm{e}^{-2\lambda h_1} \quad (2)$$

The well known Weber's integral is:

$$\int_0^\infty \mathrm{e}^{-\lambda x} J_0(\lambda y)\,\mathrm{d}\lambda = 1/(x^2 + y^2)^{1/2}$$

Differentiating both sides with respect to x:

$$\int_0^\infty \lambda \mathrm{e}^{-\lambda x} J_0(\lambda y)\,\mathrm{d}\lambda = x/(x^2 + y^2)^{3/2}$$

Thus, Stefanesco's equation for this particular model reduces to:

$$V(\mathrm{a}) = \frac{\rho_1 I}{2\pi}\Big[\frac{1}{\mathrm{a}} + \frac{8\mathrm{k}_1}{1 - \mathrm{k}_1^2} \cdot \frac{h_1 h}{(\mathrm{a}^2 + 4h_1^2)^{3/2}}\Big] \quad (3)$$

Now designating h_1 by z, h by $\mathrm{d}z$ and k_1 by simply k, equation (3) becomes:

$$V(\mathrm{a}, z) = V(z) = \frac{\rho_1 I}{2\pi}\Big[\frac{1}{\mathrm{a}} + \frac{8\mathrm{k}}{1 - \mathrm{k}^2} \cdot \frac{z\mathrm{d}z}{(\mathrm{a}^2 + 4z^2)^{3/2}}\Big] \quad (4)$$

Assuming that: $\rho_2 = \rho_1 + \mathrm{d}\rho_1$, then the right hand side of equation (4) becomes (after substituting resistivity values for k):

$$\frac{I\rho_1}{2\pi \mathrm{a}} + \frac{I}{2\pi} \cdot \mathrm{d}\rho_1 \cdot \frac{4z\mathrm{d}z}{(\mathrm{a}^2 + 4z^2)^{3/2}}$$

The first term of this expression is the potential due to a half-space. The effect of the infinitesimal sheet of thickness $\mathrm{d}z$ and resistivity contrast $\mathrm{d}\rho_1$ is contained in the second term. To obtain the individual potential (signal) due to an infinitesimal sheet of thickness $\mathrm{d}z$ and resistivity ρ_1, the second term should be integrated with respect to ρ_1. ΔV, which is the DIC, denotes this second term:

$$\Delta V = \frac{\rho_1 \mathrm{I}}{\pi} \cdot \frac{2z\mathrm{d}z}{(\mathrm{a}^2 + 4z^2)^{3/2}} \quad (5)$$

The normalized DIC for various electrode configurations, both collinear and dipolar, can be obtained by considering different current electrodes and potential electrodes simultaneously. For example, the expressions for the normalized DIC (NDIC) for two-electrode and Wenner configurations (Fig. 2) are given by:

$$\mathrm{NDIC_{TE}} = \frac{4\mathrm{a}z}{(\mathrm{a}^2 + 4\mathrm{z}^2)^{3/2}} \cdot \mathrm{d}z$$

$$\mathrm{NDIC_W} = 8\mathrm{a}z\Big[\frac{1}{(\mathrm{a}^2 + 4z^2)^{3/2}} - \frac{1}{(4\mathrm{a}^2 + 4z^2)^{3/2}}\Big]\mathrm{d}z$$

where a is the distance between two consecutive electrodes in both two-electrode and Wenner configurations. These equations are exactly the same as obtained by Roy and Apparao (1971). For other electrode arrays, the expressions for NDIC are similar to what was obtained by Roy and Apparao (1971). It is rather interesting that the expression for DIC (equation 5) is the same as that obtained by Roy and Apparao (1971) and Oldenburg (1978).

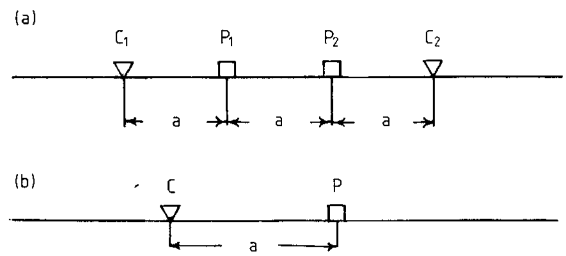

Fig 2 Schematic diagram showing (a) Wenner and (b) two-electrode configurations.

Results and discussion

For maintaining uniformity for plotting the NDIC against depth for Wenner and two-electrode configurations, it is instructive to express NDIC in terms of L, where L is the distance between the outermost electrodes. For Wenner configurations, L = 3a, while for the two-electrode configurations L = a.

Figure 3 shows the plots of NDIC for Wenner and two-electrode configurations against z/L, where z denotes the depth from the free surface of the infinitesimal sheet of thickness dz (assumed to be equal to 1) from which the response is calculated. From the figure, the DIC of Wenner and two-electrode configurations are 0.11 L and 0.35 L respectively; obviously the same as those obtained by Roy and Apparao.

The 'effective depth' (Edwards 1977), which has been defined as that depth up to which the signal equals half of the total signal due to the semi-infinite earth, has been calculated for Wenner and two-electrode arrays and are shown in the same figure. They are 0.17 L and 0.87 L for Wenner and two-electrode arrays, respectively.

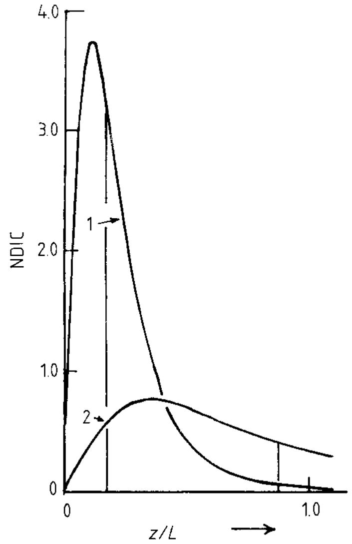

Fig 3 NDIC for (1) Wenner and (2) two-electrode configurations. The 'effective depths' for these two arrays are 0.17 L and 0.87 L respectively.

Conclusion

Thus, while Roy and Apparao's (1971) analysis for the determination of DIC, besides being complicated and lengthy, is also controversial, the present analysis is fairly simple and follows from the potential in a stratified earth. This approach can be extended to a layered earth by suitably changing the Kernel function of Stefanesco's equation, which is subsequently being studied by the authors.

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