



ation are small, by simply carrying out a power expansion on the apparent amplitude spectra and then conducting another non-linear inversion on the resultant spectra to find the true electrical parameters of the source.

A seven-channel truck-mounted spectral IP system has been constructed in which an on-board computer is used in the acquisition, reduction and interpretation of IP data. The frequency range which is normally used is 0.037 Hz to 2187 Hz, or eleven frequencies spaced a factor of 3 apart. Since the current and 6 n-spacing voltages are recorded simultaneously, survey work can proceed rapidly. Only 20 to 30 minutes usually elapse between current dipoles. A total of twelve 1000 ft. dipoles, $n = 1$ to 6, have been measured in one day for an end-to-end line coverage of 19,000 feet.

The introduction of portable, low-cost computers is significantly changing electrical methods of geophysics. With microprocessors and minicomputers to monitor multichannel data acquisition systems, we will continue to see more and more measurements being made at a faster rate. This should lead to productivity increases which more than compensate for the increased cost of equipment, while the increased number of frequencies would result in a factor of 10 decrease in the cost per data point.

Since the highest frequency used is well into the EM range, spectral IP becomes more than just IP, and the high frequency data can be used for more than just coupling removal. The boundaries between EM and IP are only artificial: better interpretation can be made of both low frequency (IP) and high frequency (EM) data by using the combined spectra. Data acquisition hardware, mass storage capability, and portable computing power are already technologically adequate. The challenge of the present and future is combining the three into workable, efficient systems and creating the software to interpret the results.

3D NUMERICAL MODELLING OF ELECTRICAL AND ELECTROMAGNETIC METHODS

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Almost a decade has passed since solutions to the general two-dimensional electrical and electromagnetic problem appeared in the literature (e.g. Jepsen (1969), Coggon (1971), Hohmann (1971), Swift (1971) and Vozoff (1971)). In that time three-dimensional solutions have been obtained on the computer but they often require large computational resources and have not been widely distributed. As a result exploration geoscientists have had little benefit from three-dimensional modelling algorithms. It is timely to review the computational difficulties associated with these problems, illustrate the usefulness of three-dimensional computer modelling, and offer an opinion on the direction such modelling will take in the 'eighties'.

In many of the modelling techniques, the unknowns (electric fields, magnetic fields or potentials) are approximated by known functions of position with unknown coefficients. By enforcing some criteria, a linear system of equations in the unknown coefficients is obtained. In the integral equation methods the unknowns need to be approximated only where the conductivity is anomalous, whereas in the differential equation methods (finite element, finite difference) the unknowns have to be approximated in the entire region. The matrix produced by the integral equation methods contains almost all zeros. Differential equation methods have the advantage of being able to model more complex distributions of conductivity than integral equation methods, but require more computational resources than the latter methods.

The table illustrates the number of unknowns and computer storage required to solve the typical two-dimensional resistivity and frequency-domain electromagnetic problems that could be solved in the late 'sixties' and early 'seventies'. These problems have in common the fact that they may be formulated with just a scalar unknown, either electric potential in the case of resistivity, or one complex component of electric or magnetic field, in the case of electromagnetic problems. In general the solution time for these problems is the order of 2 minutes or less of c.p.u. time on modest computers such as the Univac 1108 or the CDC 6400.

Three-dimensional resistivity modelling is more complicated than two-dimensional modelling only because the scalar unknowns need to be approximated over a further dimension. Nevertheless, for differential equation methods the number of unknowns, and computer storage, as illustrated in the table, are significantly larger. Typically 40 minutes of c.p.u. time are required for a problem with no planes of symmetry, and the algorithms do not fit easily on modest computers. This is to be contrasted with a few minutes, or less, for a single body problem solved with integral equation techniques.

General three-dimensional electromagnetic modelling is far more difficult than the corresponding resistivity problem because the unknowns are complex vectors rather than real scalars. A solution which gives accurate results for a wide class of three-dimensional models has yet to be reported in the literature. Good agreement has been obtained between

a finite element solution and Hohmann's integral equation solution for a horizontal loop source and a body that is 30 times more conductive than the host rock. However the agreement is very poor when the conductivity of the body is 1000 times that of the host — a more realistic massive sulphide model. The disagreement is probably due to the large size of the cells in the integral equation solution. The integral equation solution took 5 minutes of c.p.u. time on the Univac 1008 while the finite element solution took 3 hours of c.p.u. time. The computational resources required for each solution are given in the table. Clearly the differential equation solutions require enormous computational resources and fit on modest computers only by using a large amount of I-O time.

For a large number of useful three-dimensional earth models, in both resistivity and electromagnetic problems, the integral equation method gives accurate results for a fraction of the effort needed to solve the corresponding differential equation problem. However on increasing the conductivity contrast of the body, or on increasing the volume of the body, the computational resources required by the integral equation solution increase dramatically, whereas the resources required by the differential equation solution do not increase at the same rate.

The finite element method, or indeed any of the differential equation methods, is well suited to examine the effects of irregular conductive overburden on the apparent resistivity and p.f.e. patterns of a massive sulphide body on a faulted contact. A moderate conductivity contrast between the faulted blocks overshadows, to a significant extent, the apparent resistivity signature of the massive sulphides. The presence of irregular, polarizable material in the overburden swamps the polarization response of the sulphides. In mise-a-la-masse surveys, an irregular overburden does not change, significantly, the shape of the contours of electric potential, but it does distort the contours of the apparent resistivity.

A class of methods, known as hybrid methods, appear to offer some relief for the excessive computational resources required by three-dimensional electrical and electromagnetic

modelling. Hybrid methods are characterised by the use of a differential equation method in and around the inhomogeneities. The fields or potentials external to the region are represented either by eigen-functions or Green's functions. The method of summary representation (Polozhii, Nabighian) uses a finite difference approximation in the core and eigen-functions in the external region; the Unimoment method (Mei and students) uses a finite element approximation in the core and eigen-functions in the external region; and the EMMMA program of W.L. Scheen uses a finite element approximation in the core and terminates the mesh through the use of Green's functions. Hybrid methods have the advantage that it is relatively easy, compared with the integral equation methods, to obtain higher order approximations in the inhomogeneity, and the sparsity of the matrix can be maintained. In addition the large number of unknowns necessary for the differential equation methods can be reduced significantly.

Future work in numerical modelling should include investigation of:

- (i) Special three-dimensional models where because of symmetry or geometry the unknowns are no longer vectors in three-dimensional space and accurate solutions may be obtained relatively cheaply. The work in thin plate models by Lajoie and in sphere models by Lee are good examples. Such models cannot describe all common field geometries but are valuable for the insights they offer into the physics of these problems.
- (ii) Hybrid methods.
- (iii) Higher order approximations for both integral and differential equation methods.

It is unlikely that the solution to a model with an arbitrarily complex distribution of conductivity will ever be a trivial computational problem. Routine, cost-effective solutions to this class of problem await the next generation of computers and the time when computational resources are offered as an overhead, in much the same way as electric power.

Table — Modelling Methods

		Approximate Number of Unknowns (Real)	Approximate Storage Required (Real)
2D	Resistivity		
	Integral Equation	< 50	< 5,000
	Differential Equation	1,000	20,000
	E.M.		
3D	Resistivity		
	Integral Equation	< 200	< 20,000
	Differential Equation	3,000	100,000
	E.M.		
	Integral Equation	< 100	< 10,000
	Differential Equation	14,000	200,000 or 3,500,000*
	E.M.		
	Integral Equation	540	150,000
	Differential Equation	45,000 ⁺	1,500,000 or 36,000,000*

Number of unknowns and computer storage required to solve electrical and electromagnetic problems.

* Storage required for over-relaxation or direct solution to system of equations.

+ Assumes problem has 1 plane of symmetry.