CHAPTER 1

NUMBERS

DISCOVER...

LEARN...

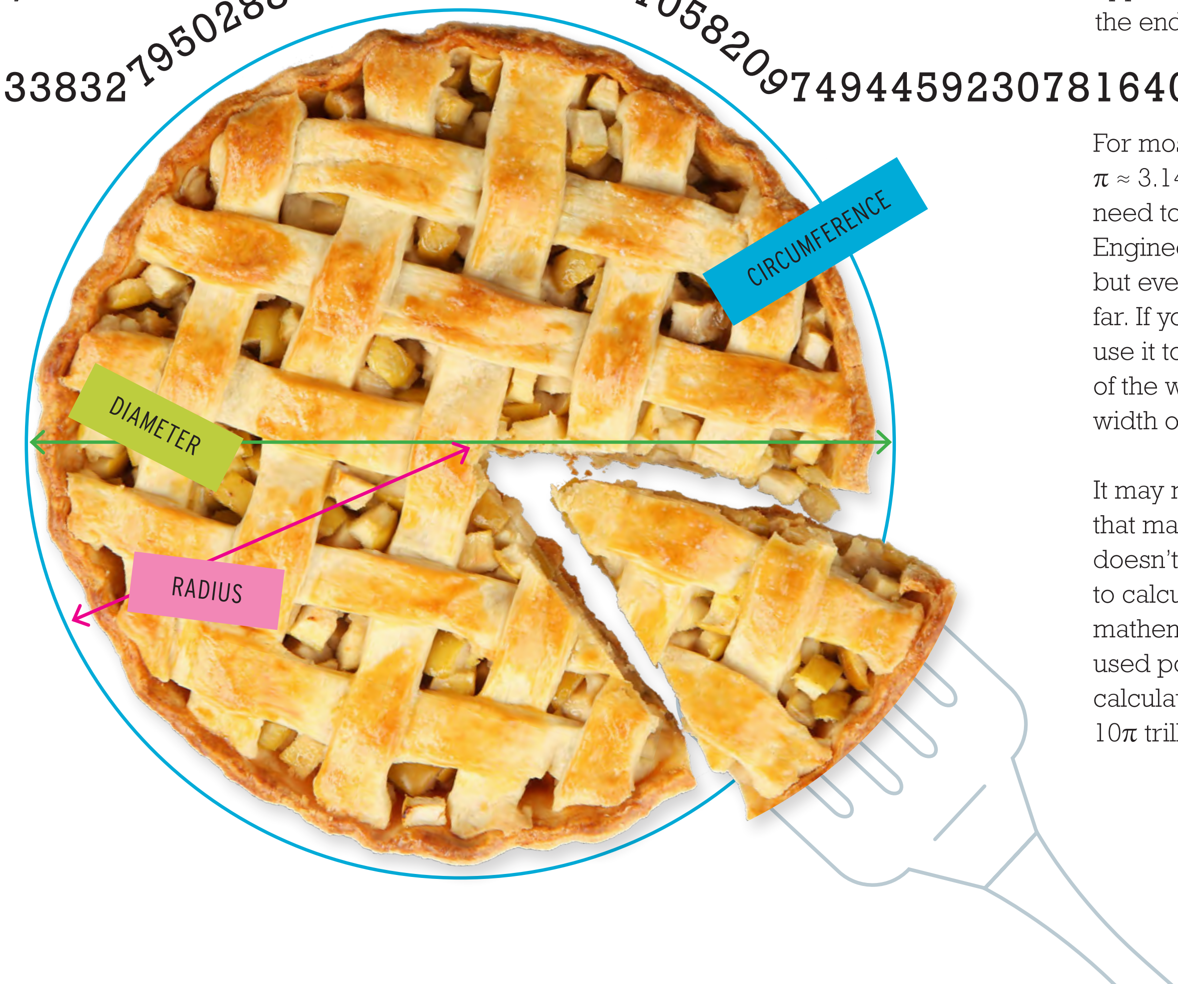
EXPERIMENT...

DISCOVER: PI(E)

In a book about kitchen maths, you might expect one of the obvious things to include would be pie – but not the kind you eat! Pi, written using the symbol π (the sixteenth letter in the Greek alphabet), represents a very specific number – and a particularly useful and important one.

The ancient Egyptians, Babylonians and Greeks all had their own approximations of π, which they used in practical calculations for farming and taxes; 3.125, 3.1605 and 3.143 were all used as early values for π. Nowadays, the value of π is known incredibly precisely. It's:

3.141592653589793238462643383279502884197169 399375105820974944592307816406286 208998628034825



WHAT IS IT GOOD FOR?

You might have encountered π in the formulas for the dimensions of a circle. For example, the circumference, C (the distance around the outside of a circle) is π times the diameter, D (the distance across the middle):

$$C = \pi \times D$$

You can also use π to work out the area, A, of a circle. If you know its radius, r (half of the diameter, or the distance from the middle to any point on the edge), you can find the area of the circle using this formula:

$$A = \pi \times r^2$$

π also crops up in the equations for the surface area and volume of a sphere, which is like a 3-D circle, so you'd expect π to be involved somehow.

π is useful in lots of other ways, too – it appears in the formula used to calculate the period of a pendulum (the amount of time it takes to swing), and in the formula for the buckling force of a beam (how much force you can apply to it before it crumples). π is used in building and construction, communication, medicine, air travel and even spaceflight.

HOW MANY DIGITS?

You might have noticed that the numerical expression of π goes off the edge of the page and you can't see the end of it. That's because it doesn't have an end! The digits of π continue on forever, and it's impossible to write the whole thing. If you ever stopped writing it, the number you'd have wouldn't exactly equal π, so you couldn't use the equal sign. You can instead use ≈, which means 'is approximately', or you can put '. . .' at the end of the number!

For most practical purposes, using $\pi \approx 3.14$ is fine, especially if you don't need to be particularly accurate. Engineers use more precise values, but even then, you don't need to go too far. If you know 39 digits of π, you can use it to calculate the circumference of the whole universe, accurate to the width of a single atom!

It may not be necessary to know that many digits of π – but that doesn't stop people from trying to calculate more. In March 2019, mathematician Emma Haruka Iwao used powerful supercomputers to calculate 31,400,000,000,000 (around 10π trillion) digits of π: a world record.

EXPERIMENT: FINDING π USING STICKS

One great thing about the number π is that it crops up in interesting places. For example, it's related to the probability of a randomly dropped stick crossing a line drawn on a piece of paper. By dropping lots of sticks and counting how many cross a line, you can get an approximation for π.

YOU WILL NEED:

- A dozen or more sticks. These could be matchsticks, pretzel sticks, toothpicks – it could be anything, as long as you can find at least a dozen of them and they're all the same length
- Paper, pen and ruler

WHAT TO DO:

1. Measure the length of a stick. This distance will be the 'stick length'.

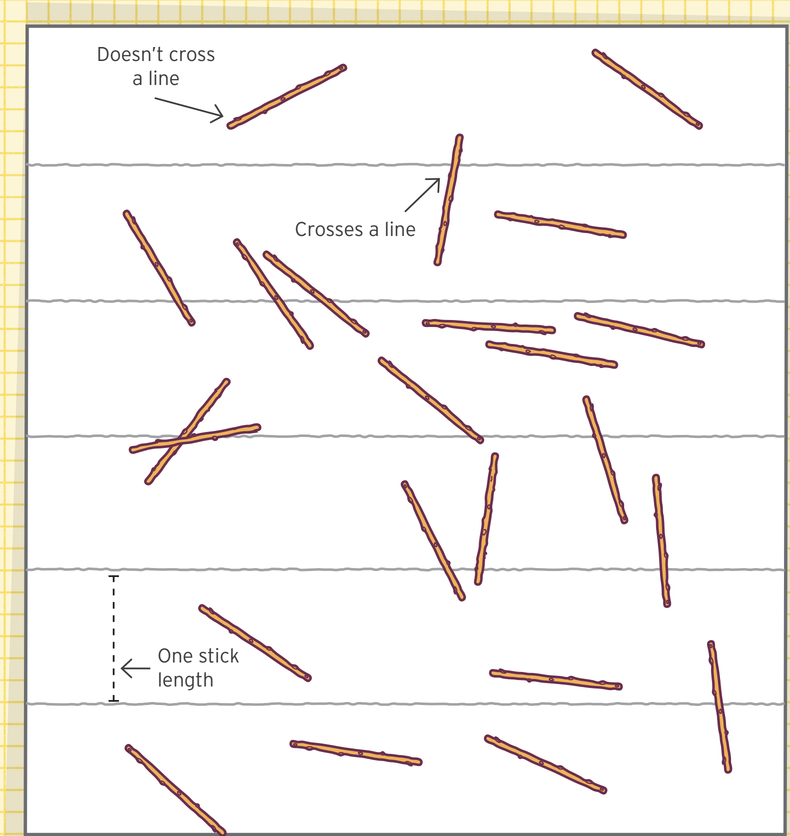
2. Draw a line across the width of the page, parallel to the top edge of the page, that is one stick length away from the top edge.

3. Draw an identical line below the first one. It should be parallel to it, and one stick length away. Repeat until the entire page is filled with lines.

4. Place the paper on the floor. While standing, drop the sticks randomly onto the paper.

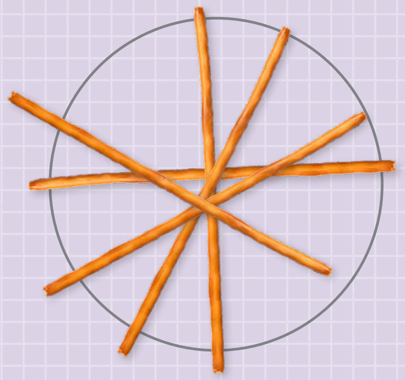
5. Count how many sticks cross a line and how many do not. (To make it easier to keep track, remove each stick after it has been counted.)

CROSSING A LINE



BUFFON'S NEEDLE

This experiment is called Buffon's Needle. It originates in the eighteenth century, when a mathematician named Buffon wondered what the probability of a dropped stick crossing a line would be. Because a stick lands flat and can point in any direction, the probability is related to the circle of area it might cover, and where you see circles, π is always nearby!



FINDING π

When the results are in, the following numbers will be needed:
A = the total number of sticks dropped
B = the number of sticks that crossed a line

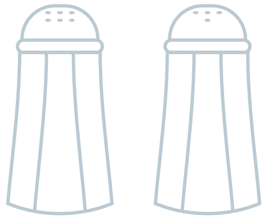
To calculate an approximation of π, the following formula will be needed:

$$\pi \approx \frac{2A}{B}$$

This means that π is approximately equal to 2 multiplied by A, divided by B. So, if 20 sticks were dropped, and 13 of them crossed a line, A would be 20, and B would be 13. The formula would then give: $(2 \times 20) \div 13 = 40 \div 13$, which gives about 3.077.

Since this method depends on a real-world experiment, and things that are random are hard to predict, it won't give an answer of $\pi = 3.14159$... But if the answer is between 3 and 3.5, it's a fairly close approximation! Also, the more sticks that are dropped, the closer the answer will be to π.

LEARN ABOUT: HOW MANY POSSIBLE BURGERS?



Some restaurants sell themselves on how many delicious items you can choose from their menu. With maths, you can work out exactly how many possible combinations there are, and it turns out you don't need a lot of items on a menu to have an impressive number of meal variations.

COUNTING BURGERS

Imagine you're making burgers. You serve each burger as a single- or a double-beef patty, and a customer can have cheese if they want. With these options, the possible combinations are:

- Single burger (no cheese)
- Double burger (no cheese)
- Single cheeseburger
- Double cheeseburger

There are two choices to make, and each can be one of two options: single or double, and cheese or no cheese. So, the number of possible burgers is $2 \times 2 = 4$.

But if a third option is added – for example, having lettuce – this doubles the number of possible combinations, since each of the existing four burgers can be with or without lettuce.

In fact, for each extra item that is added to a burger, the number of combinations doubles. This means the number of possible burgers can grow very quickly. You can see in the table below that 10 options yields more than one thousand possible burgers.

Then you can add in possibilities with more choices. For example, somewhere that offers three kinds of cheese. In a restaurant where you're allowed to choose one of these three options, that will multiply the total by three (or four, if you include 'no cheese').

NUMBER OF OPTIONS	POSSIBLE COMBINATIONS
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1,024

OVER TO YOU

For each of the following menus, work out how many possible burgers you could have (if you're not sure, write out all the combinations you can think of and then check if you've missed any):

MCBURGER'S

BURGERS: single- or double-beef patty
OPTIONAL CHEESE (CHOOSE ONE): cheddar or mozzarella
OPTIONAL SAUCES (NONE, ONE, TWO OR ALL THREE): ketchup, mustard, BBQ sauce
OPTIONAL EXTRAS: lettuce, tomato, pickle

BURGER WORLD

BURGERS: beef, chicken or veggie patty
BUNS (CHOOSE ONE): white, rye, brioche or wholemeal bun
OPTIONAL EXTRAS: burger sauce, lettuce, pickle, cheese

THE MEAT FACTORY

BURGERS: Angus beef burger, steak burger, sausage patty
SAUCES (CHOOSE ONE): burger sauce, BBQ sauce, hot sauce
OPTIONAL EXTRAS (ADD UP TO TWO OF EACH): bacon, onion rings, fried egg

LINDA'S BURGER PALACE

BURGERS: tofu steak, portobello mushroom, bean burger, veggie patty
BUNS (CHOOSE ONE): with or without sesame seeds
SAUCES (CHOOSE UP TO TWO): ketchup, mayo, mustard, BBQ sauce, sweet chilli sauce
OPTIONAL EXTRAS (CHOOSE UP TO ONE): onion rings, fried egg, Camembert cheese

Next time you have guests round for dinner, figure out how many possible meals they can choose from – they might be impressed!

IN 2002, a leading fast-food chain introduced a menu with eight items you could choose from. They claimed there were '40,312 possible combinations'. Was this correct? If not, how many combinations are there?

A HOTEL breakfast buffet claims you can have 'more than a billion different breakfasts'. Assuming you can have one of each item, how many different items are in the buffet?

DISCOVER: FACTORIALS!

When you have a selection of five delicious treats to eat, it can be difficult to decide which to eat first, which to eat second and so on. The possibilities seem endless! Well, they're not. There's a neat mathematical way to work out how many options you have – it's called a factorial.

When picking which of five snacks to eat first, there are five choices, since none of these five options have been eaten yet. But once one has been eaten, there remain four options for what to eat next, then three options for what to eat third, then two options for what to eat fourth, then only one thing left to eat last.

We can work out the number of possible ways like this:

$$5 \times 4 \times 3 \times 2 \times 1 = 120$$

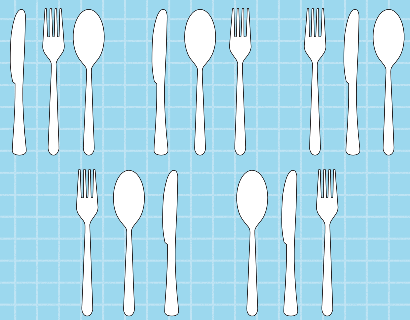
This is called five factorial, and written as '5!' So, if you see an exclamation mark after a number, you should first check whether it's a really exciting number. But if it isn't, it means someone is asking you to multiply that number by each of the numbers smaller than it, all the way down to 1.

KEEPING ORDER

Using this rule, you can make predictions. For example, if you have a knife, fork and spoon, how many ways can you arrange them in order on the table? There are:

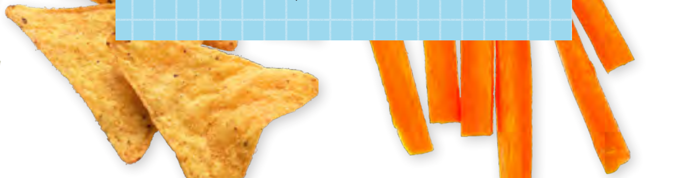
$$3! = 3 \times 2 \times 1 = 6$$

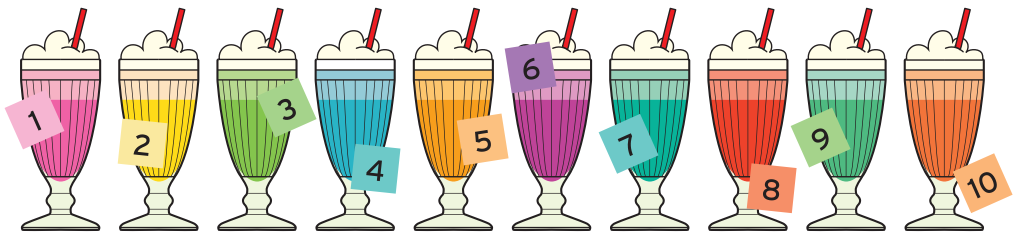
possible ways. So, let's check:



Here are five different ways to order them, so one is missing. What should it be? (See page 144 for the answer.)

If you had four things, you'd have **4! = 4 × 3 × 2 × 1 = 24** combinations. Pick four things you can see in the room now, and write out all the possible arrangements (see page 144 for the answers).





A TALL ORDER

Since the factorial of a number gets bigger pretty quickly as the number itself gets larger, the various ways to combine something can be huge. For example, if you have 10 things: **10! = 10 × 9 × 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 = 3,628,800**. That's more than 3 million ways!

If you take a deck of cards and shuffle them, how many possibilities are there? With 52 cards in a deck, there must be 52! different possible orderings of the cards. And how many is that?

52! = 80,658,175,170,943,878,571, 660,636,856,403,766,975,289,505, 440,883,277,824,000,000,000,000

This is an unbelievably large number! If someone had shuffled a deck of cards into a different order once a second since the Big Bang (which created the universe 13.8 billion years ago), they still wouldn't have seen them all. In fact, they'd only be...

This means every time you pick up a pack of cards and shuffle them to deal out a game of Go Fish, you will put them in an order that is almost certainly unique and has never been seen before by anyone (and will likely never be seen again)... unless you play a *lot* of games of Go Fish.

$$\frac{1}{5844795302242310041424683830174186012702138075426324480000}$$

...of the way through.

EXPERIMENT: SECRET NUMBERS IN YOUR FOOD

Sequences of numbers often have patterns in them, and sometimes those patterns give you a glimpse of how things are connected to one another. Fibonacci numbers have an interesting pattern, and they pop up in some interesting places – like on a pineapple!

FIBONACCI NUMBERS

The Fibonacci sequence starts with a 1. It then continues ... with another 1. Because of the way it's defined, it's necessary to give the first two numbers, but after that, we use this rule:

Each number in the sequence is found by adding the number before it to the number before that.

Another way to write this might be to say that if the Nth number in the sequence is called A_N, then $A_N = A_{N-1} + A_{N-2}$. So, you start with two 1s, which are added together to get the third number, 2. Then the fourth number will be 1 + 2 = 3, and then 2 + 3 = 5 and so on.

Add these two...

1 1 2 3 5 _ _ _ _ _

... to get the next one!

Can you work out the next five numbers?

FIBONACCI IN A PINEAPPLE

YOU WILL NEED:

- **A whole pineapple**
- **Marker pens, or brightly coloured tape, in different colours**

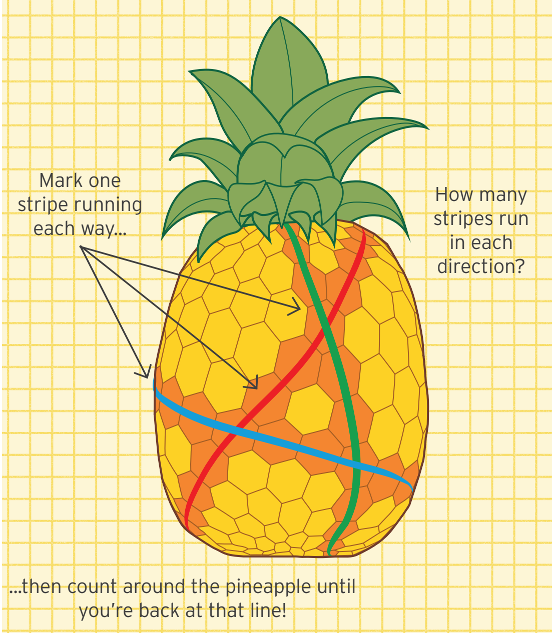
The surface of a pineapple is split into segments, which run in diagonal stripes around the fruit. You're going to count the stripes, but make sure you keep track of where you started!

WHAT TO DO:

1. Pick a segment somewhere near the top of the body of the pineapple, and find the diagonal stripe it's a part of. Drawing a line with a marker, or using a strip of tape, follow this diagonal line all the way around the pineapple.

2. Pick out another segment somewhere on the pineapple. Find the diagonal line that this piece belongs to, making sure it goes in a different direction to the first line, and mark it all the way around using a different colour. It might cross your existing line!

FIBONACCI PINEAPPLE



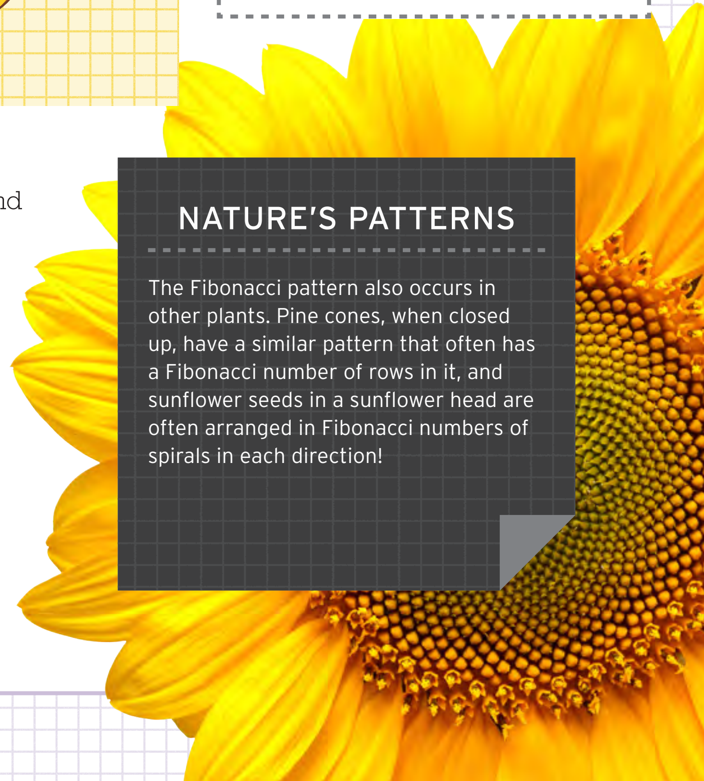
3. Look for a diagonal stripe running in a third direction around the pineapple – on a different angle – and mark this using a third colour.

When you've finished, you should have a pretty jazzy-looking pineapple, but also, you'll be able to easily count how many stripes there are in each direction (starting from a marked one and continuing until you get back there). What do you find?

WHAT HAPPENS?

The number of stripes running in each direction on a pineapple will often (but not always!) be a Fibonacci number. This is because these numbers give the best angle to arrange the segments so the pineapple can fit them in most efficiently.

Since fruit is natural and grows differently depending on the conditions, it can sometimes have non-Fibonacci numbers of stripes. If you find your pineapple doesn't give a Fibonacci number, try going back to the supermarket and counting the stripes on other pineapples there (but maybe don't use a marker).



NATURE'S PATTERNS

The Fibonacci pattern also occurs in other plants. Pine cones, when closed up, have a similar pattern that often has a Fibonacci number of rows in it, and sunflower seeds in a sunflower head are often arranged in Fibonacci numbers of spirals in each direction!