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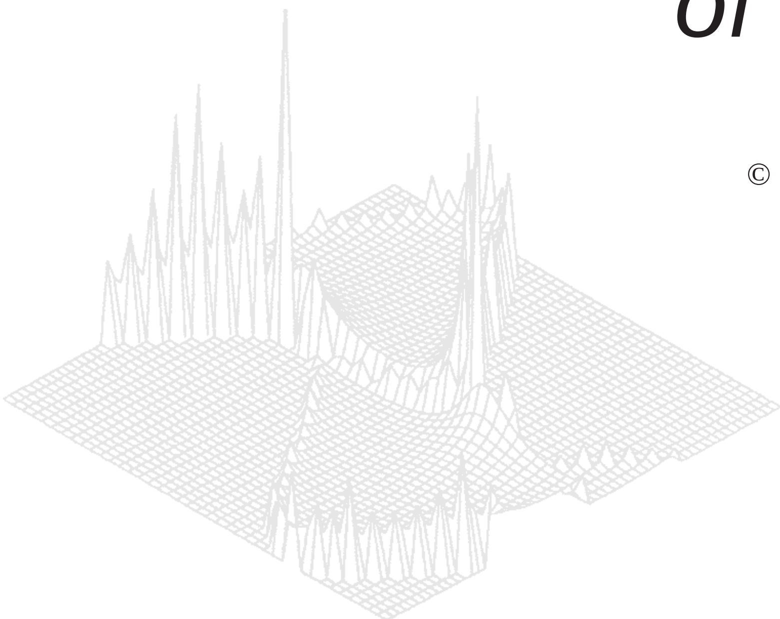

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Some Properties of the World Crystal in Fractal Spacetime Theory

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Abstract

We prove that the wave–particle duality, inertia and the Heisenberg uncertainty relation are properties of a fractal spacetime, self-structured by a gravitomagnetic background field, in the world crystal.

1. Introduction

It is well known that the geometrical tool that implements Einstein's general motion-relativity is the concept of a Riemannian curved spacetime. In a similar way, the concept of fractal spacetime (Nottale 1993) is a geometric tool adapted to construct the theory of scale-relativity (the fractal spacetime theory) (Nottale 1993, 1996). Fractal spacetime theory is based on a generalisation of Einstein's principle of relativity to the scale transformations. Namely, one redefines spacetime resolutions as characterising the state of scale reference systems, in the same way that velocity characterises their state of motion. Then one requires that the laws of physics apply, whatever the reference state of motion (principle of motion-relativity) or scale (principle of scale-relativity) of the system is.

The principle of scale-relativity is mathematically achieved by the principle of scale-covariance, requiring that the equations of physics keep their simplest form under transformations of resolutions (Nottale 1993).

In the fractal spacetime theory, the quantum behaviour becomes a manifestation of the fractal geometry of spacetime, in the same way that gravitation, in Einstein's theory of general relativity, is a manifestation of the curvature of spacetime. Therefore, the typical trajectories of quantum-mechanical particles are continuous but nondifferentiable, and can be characterised by a fractal dimension which jumps from $D = 1$ at large length scales to $D = 2$ at small length scales, the transition occurring at about the de Broglie scale (Nottale 1996). For instance, $D = 2$ is the fractal dimension of the Brownian motion (Nelson 1985). The effects of nondifferentiability (complex nature of the wavefunction) are accounted for by

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a scale-covariant derivative that transforms the equations of classical mechanics into the generalised Schrödinger equation (Nottale 1996).

Using this theory it was proved that motions in the solar system, in double galaxies and in the universe are quantised (Agop *et al.* 1998*a*; Nottale 1998). In the same context, by identifying space with a superconducting cosmic dust and by supposing that in the presence of a background gravitomagnetic field this 'matter' orders itself as a crystal, called the 'world crystal', we have shown that the physical spacetime is fractal (Agop *et al.* 1998*b*, 1998*c*). As a result, a specific mechanism of generating gravity by means of gravitational anions is obtained.

In the present work we suppose that the physical spacetime is fractal and that it orders itself as a crystal in the presence of a background gravitomagnetic field (Ciubotariu and Agop 1996; Agop *et al.* 1998*b*, 1998*c*, 1998*d*). Since geodesics correspond to a generalised Schrödinger equation with a periodic field imposed by the spatial lattice, we prove that the wave–particle duality, inertia and the Heisenberg uncertainty relation are properties induced by the fractal spacetime.

2. Dynamics of Particles in the World Crystal: Wave–particle Duality as a Property of Space

Considering space to be structured as a periodic one-dimensional lattice of period Λ, where $\Lambda = (c^2/16\pi G\rho)^{1/2}$ is the gravitational fundamental length (Agop *et al.* 1998*c*), the wave function associated with a particle moving in this lattice is of Bloch type (Licea 1986):

$$\Psi_x(x) = u_k(x)\mathrm{e}^{\mathrm{i}kx}\,, \tag{1}$$

$$u_k(x + N\Lambda) = u_k(x), \quad N = \pm 1, \pm 2, ... \tag{2}$$

i.e. a plane wave modulated by a periodic function with the direct lattice period. The 'states' of the particle in the periodic field of the spatial lattice are specified by the wavevector $\mathbf{k}$.

We will identify any particle by its mean speed

$$\langle \nu \rangle = \frac{\langle p \rangle}{m_0} = \frac{2D}{\mathrm{i}} \int_\Omega \Psi_k^*(x)\partial_x \Psi_k(x)\mathrm{d}x\,, \tag{3}$$

where the function $\Psi_k(x)$ is assumed to be normalised in the 'volume' $\Omega = N\Lambda$ of the space crystal and D is a diffusion coefficient, depending on the fractal dimension (Nottale 1996). Since $\partial_x\Psi_k(x) = \mathrm{i}k\Psi_k(x)+\mathrm{e}^{\mathrm{i}kx}\partial_x u_k(x)$, relation (3) becomes

$$\langle \nu \rangle = 2Dk + \frac{2D}{\mathrm{i}} \int_\Omega u_k^*(x)\partial_x u_k(x)\mathrm{d}x\,. \tag{4}$$

This relation is calculated taking into account the generalised Schrödinger equation (Nottale 1996)

$$[-2m_0 D^2(\partial_x + \mathrm{i}k)^2 + V(x)]u_k(x) = \epsilon_k u_k(x)\,, \tag{5}$$

where $V(x)$ is the potential induced by the spatial lattice and ϵ_k the energy eigenvalue. Differentiating with respect to k gives

$$[4m_0D^2\mathrm{i}(\partial_x+\mathrm{i}k)+\partial_k\epsilon_k]u_k=0\,. \tag{6}$$

Multiplying equation (6) by $u_k^*(x)$ and calculating the volume integral over the volume Ω with the normalisation condition

$$\int_\Omega u_k^*(x)u_k(x)\mathrm{d}x=1\,, \tag{7}$$

one gets

$$2Dk+\frac{2D}{\mathrm{i}}\int_\Omega u_k^*(x)u_k(x)\mathrm{d}x=\frac{1}{2m_0D}\partial_k\epsilon_k\,, \tag{8}$$

from which, identifying equation (4) with (8), it results that

$$\langle\nu\rangle=\frac{1}{2m_0D}\partial_k\epsilon_k=\partial_p\epsilon_k\,. \tag{9}$$

Since the group speed of the Bloch wave packet is given by

$$\nu=\partial_k\omega=\frac{1}{2m_0D}\partial_k(2m_0D\omega)=\frac{1}{2m_0D}\partial_k\epsilon_k\,, \tag{10}$$

where we considered that $\epsilon_k=2m_0D\omega$ (Nottale 1996; Agop *et al*. 1998*a*), it results that the mean speed of a particle in the spatial crystal (equation 9) may be understood as the group speed of the Bloch waves (equation 10). Thus the wave–particle duality is not an intrinsic feature of the particle but a characteristic of the spatial crystal.

3. Particle Energy Spectrum in the World Crystal: Inertia as a Property of Space

Let us suppose that the potential energy of the particle in the spatial crystal is small compared to the kinetic energy. One can then use perturbation theory to obtain the corrections of the energy $\epsilon_k^0(\mathbf{r})$ and wavefunction $\Psi_k^0(\mathbf{r})$ of the unperturbed state, considering $V(\mathbf{r})$ as a perturbation of the free-particle Hamiltonian.

Let the generalised Schrödinger equation of a free particle be (Nottale 1996)

$$-2m_0D\Delta\Psi_k^0(\mathbf{r})=\epsilon_{\mathbf{k}}^0(\mathbf{r})\Psi_{\mathbf{k}}^0(\mathbf{r})\,, \tag{11}$$

with the normalised eigenfunctions

$$\Psi_{\mathbf{k}}^0(\mathbf{r})=\frac{1}{\sqrt{\Omega}}\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \tag{12}$$

and eigenvalues

$$\epsilon_{\mathbf{k}}^0(\mathbf{r})=2m_0D^2\mathbf{k}^2\,. \tag{13}$$

Now, using perturbation theory (Licea 1986), one obtains the eigenfunctions

$$\Psi_{\mathbf{k}}(\mathbf{r}) = \frac{1}{\sqrt{\Omega}} e^{i\mathbf{k}\cdot\mathbf{r}} \left(1 + \sum_{\mathbf{K}_N} \frac{V_{\mathbf{k}_N}}{\epsilon^0_{\mathbf{k}} - \epsilon^0_{\mathbf{k}+\mathbf{K}_N}} e^{i\mathbf{K}_N\cdot\mathbf{r}} \right) \tag{14}$$

and the eigenvalues

$$\epsilon_{\mathbf{k}} = \epsilon^0_{\mathbf{k}} + \sum_{\mathbf{K}_N} \frac{|V_{\mathbf{K}_N}|^2}{\epsilon^0_{\mathbf{k}} - \epsilon_{\mathbf{k}+\mathbf{K}^0_N}}\,, \tag{15}$$

where $V_{\mathbf{K}_N}$ is the potential of the particle in comparison with the reciprocal spatial lattice $\mathbf{K}_N$. Expressions (14) and (15) are valid only for $\epsilon^0_{\mathbf{k}} \neq \epsilon^0_{\mathbf{k}+\mathbf{K}_N}$.

If $\epsilon^0_{\mathbf{k}} = \epsilon^0_{\mathbf{k}+\mathbf{K}_N}$, then from the eigenvalues (Licea 1986)

$$\epsilon^{(\pm)}_{\mathbf{k}} = \frac{\epsilon^0_{\mathbf{k}} + \epsilon_{\mathbf{k}+\mathbf{K}^0_N}}{2} \pm \left[\frac{(\epsilon^0_{\mathbf{k}} + \epsilon_{\mathbf{k}+\mathbf{K}_N})^2}{4} + |V_{\mathbf{K}_N}|^2 \right]^{\frac{1}{2}} \tag{16}$$

we get the jump $\Delta\epsilon_{\mathbf{k}} = \epsilon^{(+)}_{\mathbf{k}} - \epsilon^{(-)}_{\mathbf{k}} = 2|V_{\mathbf{K}_N}|$, and for a quadratic dependence of the energy on the wavevector, i.e.

$$2m_0 D^2 \mathbf{k}^2 = 2m_0 D^2 (\mathbf{k} + \mathbf{K}_N)^2\,, \tag{17}$$

the restriction

$$\mathbf{k}\cdot\frac{\mathbf{K}_N}{2} = \left(\frac{\mathbf{K}_N}{2}\right)^2. \tag{18}$$

Equation (18) defines the gravitational Brillouin zones of the world crystal. Hence:

(i) the energy of particles from the spatial crystal experiences a jump of $2|V_{\mathbf{K}_N}|$ at the edges of the gravitational Brillouin type zone;

(ii) since the energy of the particle cannot take values from the energetic interval $(2m_0 D^2\mathbf{k}^2 - |V_{\mathbf{K}_N}|,\ 2m_0 D^2\mathbf{k}^2 + |V_{\mathbf{K}_N}|)$, its energy spectrum will contain forbidden domains. These are a consequence of perturbation of the states of a free particle by the periodic field of the world crystal;

(iii) the existence of forbidden zones in the energy spectrum of the particle may be understood as a 'Bragg reflection' of their associated Bloch waves at the edges of the gravitational Brillouin zones of the space crystal.

In the one-dimensional case, and taking $K_N = 2\pi N/\Lambda$, from equation (18) one finds $k = \pi N/\Lambda$, where $N = \pm 1, \pm 2, \ldots$. Its values force the limits of the gravitational Brillouin zones of the spatial crystal. Thus for $N = \pm 1$ one gets the first gravitational Brillouin zone, namely $-\pi/\Lambda \leq k \leq \pi/\Lambda$.

Let us now assume the energy to be of the form

$$\epsilon(x) = 2m_0 D^2 k^2 \pm |V_{\mathbf{K}_N}(x)|\,. \tag{19}$$

When the term $|V(x)|$ is absent, the dependence $\epsilon(x) = 2m_0 D^2\mathbf{k}^2$ is a parabola (see the dashed line in Fig. 1). The presence of this term causes the energy to show discontinuities for $K = \pm\,\pi/\Lambda,\ \pm\,2\pi/\Lambda,\ \ldots$.

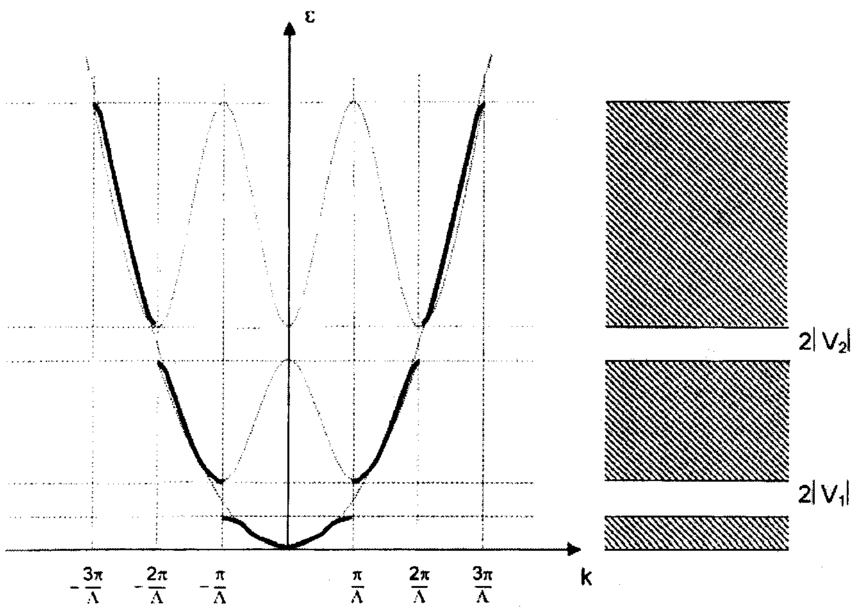

Fig. 1. Energy spectrum of particles in the world crystal.

Therefore, the energy spectrum of the particle in the spatial crystal has a band structure, with allowed bands being separated by forbidden bands. The width of a forbidden energy band is $2|V_{\mathbf{K}_N}|$ and it increases with increasing potential energy $V(\mathbf{r})$. Once the energy ϵ_k increases, the width of the forbidden energy band decreases, and the width of the allowed energy band increases.

For small values of k, the term $|V_{\mathbf{K}_N}|^2$ under the square root in equation (16) may be neglected compared with the first term, and thus we have

$$\epsilon_{\mathbf{k}}^{(+)} = \begin{cases} \epsilon_{\mathbf{k}}^0 = 2m_0 D^2 \mathbf{k}^2 \\ \epsilon_{\mathbf{k}+\mathbf{K}_N}^0 = 2m_0 D^2 (\mathbf{k}+\mathbf{K}_N)^2 \,. \end{cases} \tag{20}$$

The two states are equivalent, since $\epsilon(\mathbf{k}) = \epsilon(\mathbf{k}+\mathbf{K}_N)$, and express the parabolic dependence typical of a free particle. In the close vicinity of the edges of the first gravitational Brillouin zone, i.e. for $k = \pm\pi/\Lambda$, the term $|V_{\mathbf{K}_N}|^2$ mentioned above is large, and hence

$$\epsilon_{\mathbf{k}}^{(\pm)} = \tfrac{1}{2}[2m_0 D^2 \mathbf{k}^2 + 2m_0 D^2 (\mathbf{k}+\mathbf{K}_N)^2] \pm |V_{\mathbf{K}_N}| \left(1 \pm 2m_0 D^2 \frac{\mathbf{K}_N(2\mathbf{k}+\mathbf{K}_N)}{8|V_{\mathbf{K}_N}|^2}\right) . \tag{21}$$

Then, in compliance with the equation $(1/m^*)_{\mu\sigma} = (4m_0^2 D^2)^{-1} \partial^2\epsilon/\partial k_\mu \partial k_\sigma$ (Licea 1986), one gets

$$\left(\frac{1}{m^*}\right)_{\mu\sigma} = \frac{1}{m_0}\left(1 \pm m_0 D^2 \frac{K_N^2}{|V_{\mathbf{K}_N}|}\right) , \tag{22}$$

from which it results that, for the first gravitational Brillouin zone of the spatial crystal, where $k_N = 2\pi/\Lambda$, the mass is

$$m^* = m_0 \left(1 \pm \frac{4m_0 D^2}{|V_{\mathbf{K}_N}|}\left(\frac{\pi}{\Lambda}\right)^2\right)^{-1} . \tag{23}$$

By analogy with a crystalline solid (Licea 1986), we will name m^* the 'effective mass', i.e. the mass that results from the particle's interaction with other 'material systems' of the world crystal.

The signs '+' and '−' correspond to the energies $2m_0D^2\mathbf{k}^2 - |V_{\mathbf{K}_1}|$ and $2m_0D^2\mathbf{k}^2 + |V_{\mathbf{K}_N}|$, respectively, i.e. to some minimal and maximal values of the energy in an allowed energy band. Let us evaluate m^* with $\hbar = 10^{-34}$ J s, $\pi/\Lambda = 5\times10^{-11}$ m^{-1}, $m_0 = 10^{-30}$ kg and $|V_{\mathbf{K}_N}| = 1$ eV. One gets $m^* = m_0/(1\pm6)$, which means that for microscopic systems the effective mass is positive at the lower zone of the energy band and negative at the higher one. Extrapolating the result (23) to a cosmological scale, one obtains, for a system characterised by (Agop *et al.* 1998*c*; Agnese and Festa 1997) $\hbar \sim 10^{67}$ J s, $\pi/\Lambda \sim 10^{-23}$ m^{-1}, $m_0 \sim 10^{40}$ kg and $|V_{\mathbf{K}_N}| \sim 10^{49}$ J, a value of $m^* \sim m_0/(1\pm2)$, that is, the effective mass at this level may be positive as well as negative. In such a context, the negative effective mass (i.e. negative energy) implies the existence of some distinct spacetime structures as wormholes (Visser and Hochberg 1997) and cosmic strings (Gott 1991).

The effective mass concept constitutes an adequate description of the particles in the spatial crystal, but also generates some characteristics different from Newtonian mechanics. For explicitness, we show in Fig. 2, in the one-dimensional case, the dependence of the energy, speed and effective mass on the wavenumber k, for the first gravitational Brillouin zone. For $k = 0$, we have $\epsilon(0) = 0$ and for $k = \pm\pi/\Lambda$, $\epsilon(\pm\pi/\Lambda) =$ const. (see Fig. 2*a*). It results that $\nu(0) = 0$ and $\nu(\pm\pi/\Lambda) = 0$, thus the speed curve has maxima and minima at the inflexion points of the energy curve (Fig. 2*b*). Consequently $m^*(0) > 0$ and $m^*(\pm\pi/\Lambda) < 0$, whilst at the energy inflexion points $m^* \to \infty$ (Fig. 2*c*). In other words, the effective mass is positive at the central gravitational Brillouin zone of the spatial crystal, i.e. at the lowest edge of the corresponding energy band, and negative at the extremities of the gravitational Brillouin zone, i.e. at the higher edges of the energy band.

In an Eötvös-like experiment,

$$|V_{K_N}| \sim \frac{GM_\rho m_0}{R_\mathrm{P}} = m_0 g_\mathrm{P} R_\mathrm{P} \,, \qquad \frac{\hbar}{m_0} \to 2D = R_\mathrm{P}^2 \frac{2\pi}{T_\mathrm{P}} \,, \tag{24}$$

and the effective mass (23) becomes

$$m^* = m_0 \left(1 \pm \frac{4\pi^2 R_\mathrm{P}^3}{g_\mathrm{P} T_\mathrm{P}^2}\left(\frac{\pi}{\Lambda}\right)^2\right)^{-1} , \tag{25}$$

where g_P is the gravitational acceleration at the surface of the Earth, R_P the radius of the Earth and T_P the period of revolution about its axis. Taking $g_\mathrm{P} \sim 10$ m s^{-2}, $R_\mathrm{P} \sim 6{\cdot}3\times10^6$ m, $T_\mathrm{P} \sim 8{\cdot}64\times10^4$ s, and considering that the matter in the universe has an influence on this mass (Mach's principle), i.e. $\Lambda \sim 10^{27}$ m, one gets $m^* \sim m_0/(1 \pm 10^{-42})\ m_0$. Consequently, the positive effective mass is equal to the inertial mass (local equivalence principle). This means that inertia is in fact a space property.

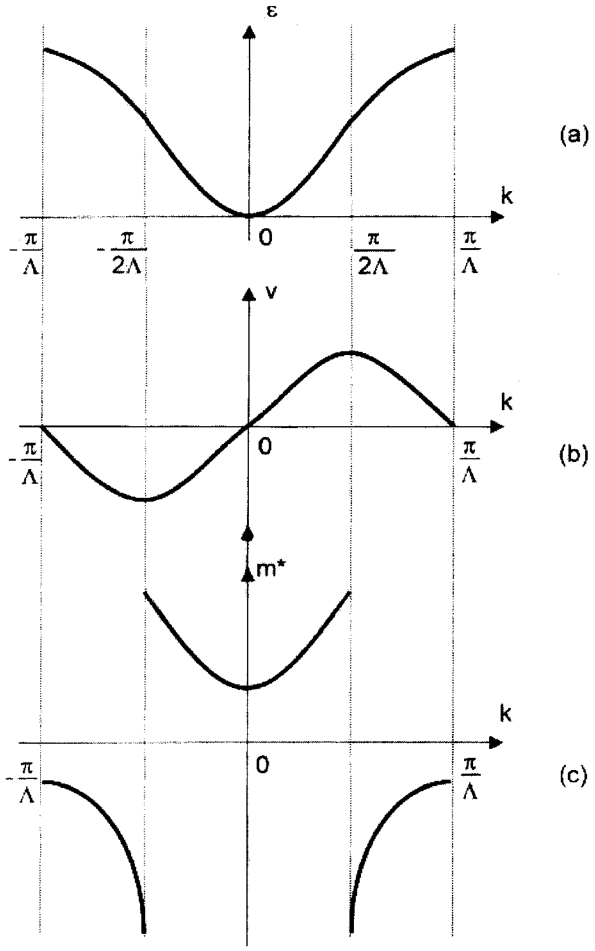

Fig. 2. Variation of the energy, speed and effective mass of particles in the world crystal.

4. Heisenberg Uncertainty Relation as a Property of Space

The ordering of space as a crystal confers wave properties on any moving particle, its mean speed being identified with the group velocity of the Bloch wave packet. If the potential energy of the particle in the periodic field of the spatial crystal is small compared to its kinetic energy, the energy and momentum of the particle may be characterised by relations similar to those for the free particle, replacing the mass m_0 by the positive effective mass m^*, i.e.

$$E = \frac{p^2}{2m^*}, \qquad p = m^* v \,. \tag{26}$$

Then the Bloch wave packet reduces to the de Broglie wave packet (Titeica 1984)

$$\Psi(x,t) = a\exp[-\mathrm{i}(E_0 t - p_0 x)/2m^* D]\frac{\sin\xi}{\xi}\,, \tag{27}$$

with

$$\xi = \frac{\Delta p}{4m^* D}(x - t\partial_{\mathrm{P}} E) \tag{28}$$

obtained by overlapping an ensemble of plane harmonic waves, for which the momentum p of the particle lies in the interval

$$p_0 - \Delta p/2 \leq p \leq p_0 + \Delta p/2\,. \tag{29}$$

Taking into account equation (26), it results that the group velocity of the de Broglie wave packet, namely

$$\nu_{\mathrm{g}} = \partial_{\mathrm{P}} E = \nu\,, \tag{30}$$

coincides with the speed of the particle.

At a moment t, the wave packet (27) extends over a distance Δx, obtained from the relation

$$\Delta\xi = \frac{\Delta p \Delta x}{4m^* D} \geq \pi \tag{31}$$

or

$$\Delta p \Delta x \geq 4\pi m^* D\,. \tag{32}$$

Equation (32) defines the fractal uncertainty relations. If $D = \hbar/2m^*$, then (32) reduces to Heisenberg's uncertainty relation.

Condition (32) corresponds to a 'Bragg diffraction' on the planes of the spatial crystal or, more precisely, at the edges of the gravitational Brillouin zone. Indeed, let

$$A_\varphi = a\frac{\sin\Delta\xi}{\Delta\xi} \tag{33}$$

be the amplitude of the de Broglie wave packet at a moment t. Now, if one allows a diffraction of the de Broglie wave packet on the spatial crystal, the diffraction minima are obtained by cancelling the intensity

$$I_\varphi = a^2\frac{\sin^2\Delta\xi}{\Delta\xi^2}\,, \tag{34}$$

which implies

$$\Delta\xi = \pm N\pi\,. \tag{35}$$

Since diffraction is an elastic interaction process between the de Broglie wave and the spatial crystal lattice, $k = k'$, equation (31) written as an identity

$$\Delta p \Delta x = 4\pi N m^* D\,, \tag{36}$$

with

$$\Delta p = |2m^* D\mathbf{k}' - 2m^* D\mathbf{k}|, \qquad |\mathbf{K}_N| = \frac{2\pi N}{\Lambda}, \qquad \Delta x = \Lambda, \tag{37}$$

reduces to the momentum conservation law

$$2m^* D\mathbf{k}' - 2m^* D\mathbf{k} = 2m^* D\mathbf{K}_{\mathrm{N}}\,. \tag{38}$$

Squaring this and bearing in mind that $(-\mathbf{K}_N)$ is also a vector of the reciprocal lattice, one gets the diffraction condition (18).

Thus one defines the planes of the spatial crystal on which 'Bragg reflection' takes place as planes corresponding to the edges of the gravitational Brillouin zone. Therefore:

(i) the identity relation (36) corresponds to a 'Bragg diffraction' of the de Broglie waves on the planes of the spatial crystal;
(ii) the uncertainty relations (32) correspond to a 'Bragg diffraction' on the first planes, $N = \pm 1$, of the spatial crystal;
(iii) the fractal uncertainty relations are a feature of space.

5. Conclusions

Considering a fractal spacetime self-structured as a crystal, called the world crystal, we can conclude that:

(i) the space geodesics correspond to a generalised Schrödinger equation in a periodic field imposed by the spatial mesh;
(ii) in this space the waves associated with the particles are of Bloch type, and thus the mean speed of the particles is the group speed of the Bloch wave packet. Hence the wave–particle duality is not an intrinsic property of the particle, but a spatial property;
(iii) considering that the potential energy of the particle in the world crystal is much smaller than its moving energy, we can say that the particle's energy spectrum in the world crystal will have both allowed and forbidden energy zones. The existence of the forbidden zones is interpreted as a 'Bragg reflection' of the Bloch wave packet on the edges of the Brillouin gravitational zones of the world crystal. In an Eötvös experiment it results that the positive effective mass is the same as the inertial mass; accordingly, the inertia is a property of space and not a property of the particle;
(iv) by substituting in the world crystal the inertial mass with the positive effective mass, the Bloch wave packet is replaced by a de Broglie wave

packet. From the spatial extension of the de Broglie wave packet at a certain time, we can deduce the fractal uncertainty relations. These relations are interpreted as a 'Bragg diffraction' of the de Broglie wave packet on the world crystal planes. Therefore, spatial uncertainty is a property of space.

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