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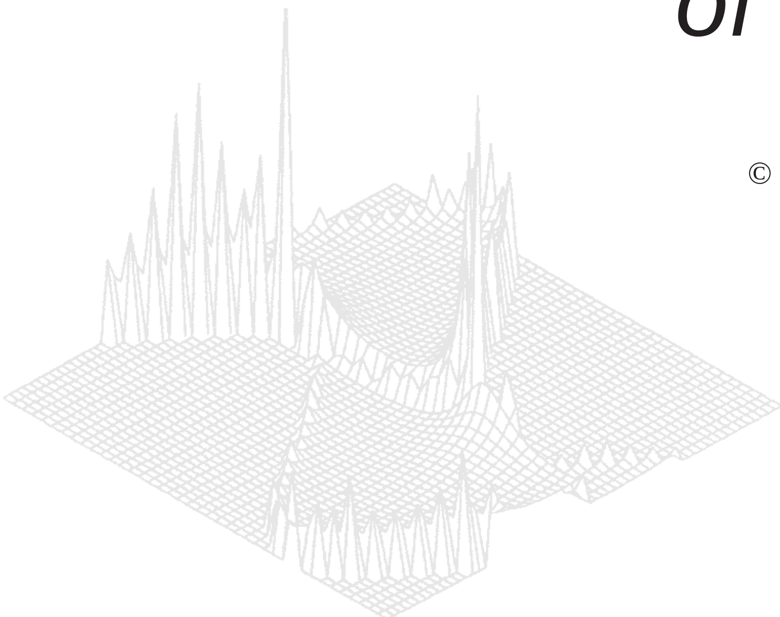

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Bulk Viscosity and Particle Creation in Brans–Dicke Theory

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Abstract

The effect of bulk viscosity on the evolution of the spatially flat Friedmann–Lemaitre–Robertson–Walker (FLRW) models in the context of open thermodynamical systems, which allow for particle creation, is analysed within the framework of Brans–Dicke (BD) theory. The BD field equations are modified with the incorporation of a creation pressure and bulk viscous stress. A class of physically plausible models has been taken into consideration. The behaviour of the particle number density and bulk viscosity is discussed with the evolution of the Brans–Dicke scalar field.

1. Introduction

Recently there has been growing interest in alternative theories of gravitation, especially scalar–tensor theories of gravity, which are very useful tools in understanding early universe models. In a pioneering work Mathiazhagan and Johri (1984) and La and Steinhardt (1989) showed that the old inflationary idea with a first order phase transition can be made to work if one considers Brans–Dicke (BD) theory instead of Einstein's theory. Hyperextended inflation (Steinhardt and Accetta 1990) generalises the results of extended inflation in BD theory and solves the graceful exit problem in a natural way, without recourse to any fine tuning as required in relativistic models. The renewed interest in BD theory is also due to the inadequacy of general relativity to contribute to the super unification of the basic interactions and to explain satisfactorally the evolution of galactic structure.

It has been shown by Padmanabhan and Chitre (1987) that the presence of bulk viscosity leads to inflationary-like solutions in general relativity. Another peculiar characteristic of bulk viscosity is that it acts like a negative energy field in an expanding universe (Johri and Sudharsan 1988). There are many circumstances during the evolution of the universe in which bulk viscosity could arise (Maartens 1995, and references therein): (i) when neutrinos decouple from the cosmic fluid; (ii) when photons decouple from matter; (iii) at the time of formation of galaxies; and (iv) during particle creation in the early universe. These various

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processes giving rise to bulk viscosity could lead to an effective mechanism for entropy production. Some authors have already obtained cosmological solutions with bulk viscosity in BD theory (Johri and Sudharsan 1989; Pimentel 1994; Beesham 1996). Cosmological models with non-causal and causal thermodynamics have been reviewed by Grøn (1990) and Maartens (1995) respectively. Recently Banerjee and Beesham (1996) have considered Brans–Dicke cosmology with a causal viscous fluid.

In recent years a phenomenological macroscopic approach for particle production in terms of bulk viscous stresses has been described in the literature (Hu 1982; Barrow 1988; Sudharsan and Johri 1994; Triginer and Pavon 1994). Prigogine *et al.* (1989) have investigated the role of irreversible processes in the creation of matter out of gravitational energy which may play an important role in the evolution of the early universe. Detailed studies of the thermodynamics of matter creation have been under taken by Calvao *et al.* (1992) and Johri and Kalyani (1994). In this context, it is relevant to consider other effects which are important in the evolution of the early universe within the framework of BD cosmological theory.

In this paper we have investigated the role of particle creation and bulk viscosity as separable irreversible processes in the framework of BD theory. As argued in Triginer and Pavon (1994), we have considered the adiabatic particle production processes in which the entropy per particle σ_c associated with matter creation processes is constant and only dissipative processes can change the entropy per particle in the cosmic fluid.

2. Basic Equations

The effective energy–momentum tensor of the cosmic fluid in the presence of the creation of matter and bulk viscosity includes the creation pressure term p_c and the bulk viscous stress Π, and may be written as

$$T_{ab} = (\rho + P_{\text{eff}})u_a u_b - P_{\text{eff}}\, g_{ab}\,. \tag{1}$$

Here ρ is the energy density and P_{eff} stands for the effective pressure which may be defined as

$$P_{\text{eff}} = p + p_c + \Pi\,, \tag{2}$$

where p, p_c and Π represent the equilibrium pressure, creation pressure and bulk viscous stress respectively. The creation pressure p_c is associated with the creation of matter out of the gravitational field (Prigogine *et al.* 1989).

The gravitational field equations with usual notation in BD theory have the form

$$G_{ab} = -\frac{8\pi}{\phi}T_{ab} - \frac{\omega}{\phi^2}[\phi_{;a}\phi_{;b} - \tfrac{1}{2}g_{ab}\phi_{;c}\phi^{;c}] - \frac{1}{\phi}[\phi_{;a;b} - g_{ab}\Box^2\phi]\,, \tag{3}$$

$$\Box^2\phi = \frac{8\phi}{3+2\omega}T\,. \tag{4}$$

For a homogeneous and isotropic model of the universe, represented by the FLRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{5}$$

with the barotropic equation of state

$$p = \gamma\rho, \qquad 0 \leq \gamma \leq 1, \tag{6}$$

the BD field equations (3) and (4) now become

$$3\left(\frac{\dot{R}}{R}\right)^2 + 3\frac{k}{R^2} + 3\frac{\dot{R}\dot{\phi}}{R\phi} - \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 = \frac{8\pi\rho}{\phi}, \tag{7}$$

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{\ddot{\phi}}{\phi} + \frac{\omega}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + 2\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} + \frac{k}{R^2} = -\frac{8\pi}{\phi}(\gamma\rho + p_c + \Pi), \tag{8}$$

$$\frac{\ddot{\phi}}{\phi} + 3\frac{\dot{R}}{R}\frac{\dot{\phi}}{\phi} = \frac{8\pi}{(3+2\omega)\phi}[\rho - 3(\gamma\rho + p_c + \Pi)]. \tag{9}$$

Equations (7)–(9) lead to the continuity equation

$$\dot{\rho} + (1+\gamma)\rho\Theta = -(p_c + \Pi)\Theta, \tag{10}$$

where $\Theta = u^a_{;a} = 3\dot{R}/R$ stands for the expansion scalar and u_a is the four velocity vector.

The simplest and most commonly used linear relation between the bulk viscous stress Π and the divergence of the four velocity vector u_a as given in Eckart theory is

$$\Pi = -\xi u^a_{;a} = -\xi\Theta. \tag{11}$$

Here the bulk viscosity coefficient ξ is in general a function of time. Although Eckart theory has some shortcomings, in contrast to extended irreversible thermodynamics (EIT), we are considering it as a first step to study the role of bulk viscosity in adiabatic particle production processes under the framework of scalar–tensor theory. A more complete analysis of the EIT of dissipative process with adiabatic particle creation within the framework of BD theory is under investigation.

The particle number density flow and entropy flow vectors take the form

$$N^a = nu^a; \qquad S^a = \sigma n u^a, \tag{12}$$

where n is the particle number density and σ is the entropy per particle. The particle number density flow vector N^a is supposed to satisfy the balance equation

$$N^a_{;a} = \dot{n} + n\Theta = \Gamma\,. \tag{13}$$

The source term Γ will be positive or negative depending on whether there is production or annihilation of particles. This term plays an important role in models with particle non-conservation. In the case of particle conservation, the source term Γ vanishes.

The second law of thermodynamics suggests

$$S^a_{;a} = n\dot{\sigma} + \sigma\Gamma \geq 0\,. \tag{14}$$

The Gibbs equation for an open thermodynamical system can be written as

$$nT\dot{\sigma} = \dot{\rho} - (\rho + p)\frac{\dot{n}}{n}\,, \tag{15}$$

where T is the fluid temperature. Using equations (10), (11) and (13), from equation (15) we get the entropy rate per particle

$$\dot{\sigma} = -\frac{p_c\Theta}{nT} + \frac{\xi\Theta^2}{nT} - \frac{(1+\gamma)\rho}{n^2T}\Gamma\,. \tag{16}$$

As stated above, in the adiabatic particle production processes the entropy per particle associated with matter creation is constant (refer to Triginer and Pavon 1994), so that only viscous phenomena may change the entropy per particle, and hence

$$p_c = -\frac{(1+\gamma)\rho}{n\Theta}\Gamma = -\frac{(1+\gamma)\rho}{\Theta}\left[\Theta + \frac{\dot{n}}{n}\right]. \tag{17}$$

This is the mathematical expression of the adiabatic criterion, relating the pressure arising from matter creation to the rate of particle production.

By use of (17), equation (16) reduces to

$$\dot{\sigma} = \frac{\xi\Theta^2}{nT}\,. \tag{18}$$

Further, using this relation, we have

$$S^a_{;a} = \sigma\Gamma + \frac{\xi\Theta^2}{T}\,, \tag{19}$$

and the second law of themodynamics (14) implies

$$\Gamma \geq -\frac{\xi\Theta^2}{T\sigma}\,. \tag{20}$$

By a combination of equations (10), (11) and (17), we obtain

$$\frac{\dot{n}}{n} = \frac{1}{(1+\gamma)\rho}[\dot{\rho} - \xi\Theta^2]\,, \tag{21}$$

which on integration gives

$$n^{1+\gamma} = L\rho\left[\exp\left(-\int \xi\rho^{-1}\Theta^2 dt\right)\right]\,. \tag{22}$$

Here L is an integration constant.

Maartens (1996) has suggested that the Gibbs integrability condition shows explicitly that one cannot independently specify an equation of state for the pressure and temperature. If the equation of state for pressure is barotropic [i.e. $p = p(\rho)$] then the equation of state for temperature should be barotropic [i.e. $T = T(\rho)$] and may be written as

$$T \propto \exp\int \frac{dp}{\rho(p)+p}\,, \tag{23}$$

which with the help of equation (6) gives

$$T = T_0\rho^{\gamma/(\gamma+1)}\,, \tag{24}$$

where T_0 is a proportionality constant.

From equations (18) and (24) we get

$$\dot{\sigma} = \frac{\xi\Theta^2}{nT_0\rho^{\gamma/(\gamma+1)}}\,. \tag{25}$$

This equation gives the entropy rate per particle for corresponding values of ξ, Θ, n and ρ. Further, equation (25) shows that if the bulk viscosity is zero then the entropy per particle is constant which is the case in Prigogine *et al*. (1989) and has been pointed out by Calvao *et al*. (1992).

3. The Models

We consider a power law relation between the scale factor R and the scalar field ϕ of the form

$$\phi = KR^{\alpha}\,, \tag{26}$$

where K is a proportionality constant and α is the power index. This commonly used assumption leads to constant deceleration parameter models which are the most well known models in both general relativity and BD theory (Berman and Gomide 1988; Berman and Som 1990; Beesham 1993; Johri and Kalyani 1994). This gives us the motivation to study such models.

Using equations (11) and (26), a combination of field equations (7)–(9) for the FLRW flat ($k = 0$) model yields

$$\frac{\ddot{R}}{R} + \beta \left(\frac{\dot{R}}{R}\right)^2 = 0\,, \tag{27}$$

where

$$\beta = \frac{\omega\alpha^2 + 4\omega\alpha - 6}{2(\omega\alpha - 3)} = \text{constant}\,. \tag{28}$$

Equation (27) may be rewritten as

$$\beta = -\frac{R\ddot{R}}{\dot{R}^2}\,. \tag{29}$$

Here β has a definition similar to the constant deceleration parameter q.

The solution to equation (27) is given by

$$R = (A + MDt)^{1/M}\,, \tag{30}$$

where $M = 1 + \beta$; $\beta \neq -1$ and A and D are integration constants. This will give us a number of models. For $\beta = -1$, equation (27) gives an exponential inflationary model of the unverse. Polynomial inflation in models is possible if $-1 < \beta < 0$. By virtue of equation (28), this constrains α to lie either in the range

$$-2 + 2\left(1 + \frac{4}{3\omega}\right)^{\frac{1}{2}} < \alpha < -3 + 3\left(1 + \frac{4}{3\omega}\right)^{\frac{1}{2}} \tag{31}$$

or in the range

$$-3 - 3\left(1 + \frac{4}{3\omega}\right)^{\frac{1}{2}} < \alpha < -2 - 2\left(1 + \frac{4}{3\omega}\right)^{\frac{1}{2}}\,. \tag{32}$$

When $\alpha = -2 \pm 2[1 + (3/2\omega)]^{\frac{1}{2}}$, we have $\beta = 0$ and from (30) we can see that R grows linearly with time. The behaviour of R is independent of ω.

By use of (30), equation (26) gives

$$\phi = K(A + MDt)^{\alpha/M}\,. \tag{33}$$

Using equations (30) and (33), from (7) we get

$$\rho = \frac{KD^2}{8\pi}\left(3 + 3\alpha - \frac{\omega}{2}\alpha^2\right)(A + MDt)^{(\alpha - 2M)/M}\,. \tag{34}$$

With the help of (34), from equation (24) we get an expression for the temperature in all models

$$T = T_0 \left[\frac{KD^2}{8\pi} \left(3 + 3\alpha - \frac{\omega}{2}\alpha^2 \right) (A + MDt)^{(\alpha - 2M)/M} \right]^{\gamma/(\gamma+1)} . \tag{35}$$

As $\alpha/M \leq 1$, equations (34) and (35) indicate the energy density and temperature are decreasing with the evolution of the universe.

With the scale factor $R(t)$ given by equation (30), the energy density and temperature in general relativity take the form

$$\rho = \frac{3}{8\pi G} \frac{D^2}{(A + MDt)^2} ,$$

$$T = T_0 \left(\frac{3D^2}{8\pi G} \right)^{\gamma/(\gamma+1)} (A + MDt)^{-2\gamma/(\gamma+1)} .$$

Assuming that α lies in the range given by the relation (31), equation (33) shows that the scalar field is increasing, while (34) and (35) indicate that in BD theory the energy density as well as the temperature are decreasing slower than in general relativity. When α is considered to lie in the range mentioned by the relation (32), the scalar field decreases, and the energy density as well as the temperature are decreasing more rapidly than in general relativity.

As we have only six basic equations, viz. (6), (7), (22), (24), (26) and (27), and seven unknowns, viz. R, ϕ, ρ, p, n, T and ξ, in order to solve for ξ and n, we require one more physically reasonable relation (condition) amongst the variables. In the following subsections, we consider, in turn, a viscosity energy density, a uniform particle density, an ideal gas and a second order correction term separately.

(3a) Models with Bulk Viscosity Energy Density Law

In most of the investigations involving bulk viscosity, the coefficient of bulk viscosity is assumed to be a simple power function of the energy density (see e.g. Pavon *et al*. 1991; Zimdahl 1996; Maartens 1996):

$$\xi = \xi_0 \rho^m ,$$

where ξ_0 and m are positive constants. If $m = 1$, then this may correspond to a radiative fluid, whereas $m = 1{\cdot}5$ may represent a string dominated universe (Murphy 1973; Santos *et al*. 1985). In this subsection we assume the bulk viscosity energy density relation above, which from equation (34), leads to

$$\xi = \xi_0 \left[\frac{KD^2}{8\pi} \left(3 + 3\alpha - \frac{\omega}{2}\alpha^2 \right) (A + MDt)^{(\alpha - 2M)/M} \right]^m . \tag{36}$$

With the help of (30), (34) and (36), equation (22) yields

$$n = L_1(A+MDt)^{(\alpha-2M)/(1+\gamma)M} \exp\left[L_2(A+MDt)^{[(m-1)(\alpha-2M)-M]/M}\right], \qquad (37)$$

where L_1 and L_2 are given by

$$L_1 = \left[\frac{LKD^2}{8\pi}\left(3+3\alpha-\frac{\omega}{2}\alpha^2\right)\right]^{1/(1+\gamma)},$$

$$L_2 = \frac{9MD\xi_0[KD^2(3+3\alpha-\frac{1}{2}\omega\alpha^2)]^{m-1}}{(8\pi)^{(m-1)}(1+\gamma)[(m-1)(\alpha-2M)-M]}.$$

Further, as $\alpha/M < 1$, equations (36) and (37) suggest the bulk viscosity and particle number density are decreasing with the evolution of the universe. If we take $\xi_0 = 0$, then our solution reduces to that of Johri and Kalyani (1994).

(3b) Models with Uniform Particle Number Density ($n = n_0$)

As suggested by Triginer and Pavon (1994) in this subsection we consider the particle number density to be uniform ($\dot{n} = 0$) during evolution of the universe, which leads to the result that the particle production source term (Γ) is determined by the expansion rate,

$$\Gamma = n_0\Theta. \qquad (38)$$

Using the condition $\dot{n} = 0$, equation (21) yields

$$\xi = \frac{\dot{\rho}}{\Theta^2}. \qquad (39)$$

With the help of (30) and (34), equation (39) reduces to

$$\xi = \frac{KD}{72\pi}(\alpha-2M)\left(3+3\alpha-\frac{\omega}{2}\alpha^2\right)(A+MDt)^{(\alpha-M)/M}. \qquad (40)$$

Equations (40) suggest that the bulk viscosity is decreasing while the universe is expanding.

(3c) Models for an Ideal Gas

In this subsection we consider

$$N^a_{;a} = \dot{n} + n\Theta = 0. \qquad (41)$$

This is the equation for conservation of total particle number in standard cosmology. Equation (41) with (30) leads to

$$n = CR^{-3} = C(A+MDt)^{-3/M}. \qquad (42)$$

As the total particle number is conserved, the source term Γ and hence the creation pressure p_c vanish in this case and therefore we are dealing with cosmological models with bulk viscosity only.

By use of (41), equation (21) becomes

$$\xi = \frac{\dot{\rho}}{\Theta^2} + (1+\gamma)\frac{\rho}{\Theta}\,, \tag{43}$$

which with the help of equations (30) and (34) reduces to

$$\xi = \frac{KD}{72\pi}[(3+3\gamma+\alpha-2M)\left(3+3\alpha-\frac{\omega}{2}\alpha^2\right)](A+MDt)^{(\alpha-M)/M}\,. \tag{44}$$

It can be very easily seen that in these models particle number density and bulk viscosity are decreasing functions of time. If we put $\alpha = M$, then our solution reduces to that of Beesham (1994), and hence our solution is a generalisation of the latter.

(3d) Creation with Second Order Correction in H

To consider particle non-conservation in BD theory, we assume in this subsection the simple relation

$$\frac{\dot{n}}{n} + 3H = bH^2\,, \tag{45}$$

where b is a constant and $H = \dot{R}/R$ is the Hubble parameter. This is a simple and physically reasonable expression which generalises the conservation of total particle number in standard cosmology to the non-conservation of total particle number by considering the Taylor expansion of $\dot{n}/n = f(H)$ up to second order in H (Triginer and Pavon 1994).

Using (45), equation (13) gives

$$\Gamma = bnH^2\,. \tag{46}$$

Equation (46) suggests that for $b > 0$, $b = 0$ and $b < 0$, we have, respectively, creation, no creation and annihilation of particles. In the context of open thermodynamic systems, we have

$$\frac{\dot{n}}{n} + 3H = \frac{\dot{N}}{N} = \frac{\dot{S}}{S} \geq 0\,. \tag{47}$$

Here N is the number of particles in a given volume V. From equations (45) and (47) it can be very easily seen that $b \geq 0$, i.e. there is either creation or no creation. Using (30), equation (45) after integration gives

$$n = C(A+MDt)^{-3/M}\exp\left[-\frac{bD}{M(A+MDt)}\right]\,, \tag{48}$$

where C is an integration constant.

From (21) and (45) we obtain

$$\xi = \frac{1}{\Theta^2}[\dot{\rho} + (\Theta - bH^2)(1+\gamma)\rho]\,. \tag{49}$$

Now, using (30) and (34), equation (49) gives an explicit form of ξ as a function of time:

$$\xi = \frac{KD(6+6\alpha-\omega\alpha^2)}{144\pi(A+MDt)^{1-(\alpha/M)}}\left[3+3\gamma+\alpha-2M-\frac{bD(1+\gamma)}{(A+MDt)}\right]. \tag{50}$$

In this case also the viscous stress and particle number density are decreasing with the evolution of the universe.

4. Conclusion

In the present paper, we have studied Brans–Dicke cosmological models with non-causal thermodynamics of a dissipative homogeneous and isotropic universe in the context of particle creation. In all models the energy density, creation pressure, bulk viscosity, temperature and particle number density are decreasing functions of time.

From equation (30), for $A = 0$, we get $R(t) \sim t^{1/M}$ and these models have a big-bang singularity. In this case particle horizons do not exist. On the other hand, for $A \neq 0$ we have non-singular models of the universe and in contrast to the singular models, particle horizons exist in this case.

Present observational data indicate that $\omega \geq 500$ (Will 1993). Taking $\omega = 500$, $\alpha = 0{\cdot}0035$ which is in the range of (31), and the age of the universe as $t \sim 10^{10}$ yr $\sim 3\times 10^{17}$ s, equation (35) yields $T \sim 10^{-9}$ MeV for the present value of the cosmic microwave background radiation temperature. This is in fair agreement with the measured value.

When $\beta = -1$, equation (27) yields the exponential solution $R \sim e^{Bt}$, where B is an integration constant. In this case the system of equations (7)–(9) suggests that the scalar field ϕ and energy density ρ are proportional to the power function of scale factor R^α, with $\alpha = (-6\omega \pm \sqrt{36\omega^2 - 48\omega})/2\omega$ and $\omega \geq \frac{4}{3}$. This value of β leads to $\dot{H} = 0$ which implies a greater value of the Hubble parameter and a correspondingly faster rate of expansion of the universe as compared to the relation (30). In the case of exponential inflationary models also one can see that the BD scalar field, energy density, creation pressure, bulk viscosity, temperature and particle number density are decreasing functions of time.

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