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Nonlinear Equation in (2+1) Dimensions for a Plasma with Negative Ions

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Abstract

A new form of coupled nonlinear evolution equation is derived for a plasma with negative ions in (2+1) dimensions. This system of equations can be considered to be an extension of the usual Davey–Stewartson equation. A modified version of reductive perturbation has been used. It is also shown that this set of equations can sustain both cnoidal type and the usual solitary wave-like solution. Such an equation can have important applications in describing nonlinear wave propagation in a dusty plasma.

1. Introduction

The study of nonlinear wave propagation in plasmas forms an important part of theoretical research in plasma physics. Perhaps the initiation of such a study was the pioneering paper of Washimi and Taniuti (1986). After that various modifications have been incorporated to explain different kinds of wave phenomena taking place in a plasma. Some noteworthy attempts are those of Mojølhus and Wyller (1988), Ichikawa and Watanabe (1977), Mukherhjee and Roy Chowdhury (1995) and many more (see e.g. Das and Paul 1985; Rizzato 1988; Yu and Luo 1992). In this communication we show that by adopting a modified form of reductive perturbation we can deduce a new form for a set of coupled nonlinear equations for a nonrelativistic plasma with negative ions. It can easily be demonstrated that this new set of equations does have cnoidal type and also solitary wave-like solutions.

The present set of equations can be thought of as an extended version of the Davey–Stewarston (1974) equation. It is actually a multicomponent generalisation which considers the negative ion to be important on various accounts. Firstly, in the ionospheric plasma, the presence of negative ions is an established fact. Furthermore, there is now widespread interest in the study of dusty plasmas. A dusty plasma (de Angelis 1991) can be modelled in various ways. The simplest is to consider the presence of a typical dust grain, either with positive or negative charge, though in nature the fluctuation of a dust charge does occur due to the collision of these particles with the streaming electrons and ions. Furthermore, a dust particle is usually considered to have a mass greater than or equal to the ion. So we can think of the third species as being either a dust particle or negative ion.

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2. Formulation

Let us consider a nonrelativistic plasma consisting of electrons and both positive and negative ions. We also assume that a hydrodynamic description is possible for our plasma. Then the equations of motion describing our plasma can be written as

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}(n_\alpha u_\alpha) + \frac{\partial}{\partial y}(n_\alpha v_\alpha) = 0\,, \tag{1a}$$

$$n_\alpha\left(\frac{\partial u_\alpha}{\partial t} + u_\alpha\frac{\partial u_\alpha}{\partial x} + v_\alpha\frac{\partial u_\alpha}{\partial y}\right) + n_\alpha\frac{\partial \phi}{\partial x} + T_\alpha\frac{\partial n_\alpha}{\partial x} = 0\,, \tag{1b}$$

$$n_\alpha\left(\frac{\partial v_\alpha}{\partial t} + u_\alpha\frac{\partial v_\alpha}{\partial x} + c v_\alpha\frac{\partial v_\alpha}{\partial y}\right) + n_\alpha\frac{\partial \phi}{\partial y} + T_\alpha\frac{\partial n_\alpha}{\partial y} = 0\,, \tag{1c}$$

$$\frac{\partial n_\beta}{\partial t} + \frac{\partial}{\partial x}(n_\beta u_\beta) + \frac{\partial}{\partial y}(n_\beta u_\beta) = 0\,, \tag{1d}$$

$$n_\beta\left(\frac{\partial u_\beta}{\partial t} + u_\beta\frac{\partial u_\beta}{\partial x} + v_\beta\frac{\partial u_\beta}{\partial y}\right) - \frac{n_\beta}{Q}\frac{\partial \phi}{\partial x} + \frac{T_\beta}{Q}\frac{\partial n_\beta}{\partial x} = 0\,, \tag{1e}$$

$$n_\beta\left(\frac{\partial v_\beta}{\partial t} + u_\beta\frac{\partial v_\beta}{\partial x} + v_\beta\frac{\partial v_\beta}{\partial y}\right) - \frac{n_\beta}{Q}\frac{\partial \phi}{\partial y} + \frac{T_\beta}{Q}\frac{\partial n_\beta}{\partial y} = 0\,, \tag{1f}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \mathrm{e}^{\phi} + n_\beta - n_\alpha\,, \tag{1g}$$

where the subscript α stands for a positive ion and β for a negative ion, (u_α, v_α) are the (x, y) components of the velocity of the positive ion and (u_β, v_β) those of the negative ion, T_α, T_β are respectively their temperatures, and the electrons are assumed to form the background. The velocities (u_α, u_β) and densities (n_α, n_β) are all properly normalised. Here ϕ denotes the electrostatic potential. The explicit form of the normalisation for the various physical quantities is as follows:

For each n_α, n_β

$$n_\alpha/n_{\alpha 0} \longrightarrow n_\alpha, \qquad n_\beta/n_{\beta 0} \longrightarrow n_\beta\,;$$

for the electrostatic potential ϕ

$$e\phi/KT_\mathrm{e} \longrightarrow \phi\,;$$

for the coordinates (x, y)

$$(x,\, y)/\lambda_{\mathrm{de}} \longrightarrow (x,\, y)$$

where $\lambda_{\mathrm{de}}^2 = kT_\mathrm{e}/4\pi n_0\, e^2$; for the time coordinate t

$$t\Omega_\mathrm{i} \longrightarrow t\,,$$

with $\Omega_i^2 = 4\pi n_0\, e^2/M_{\rm i}$, and with $M_{\rm i}$ the ion mass. Lastly the velocity v is normalised as

$$v/C_0 \longrightarrow v, \qquad C_0 = \lambda_{\rm de}\,\Omega_{\rm i}\,.$$

We consider next the time evolution of the wave packets of a perturbation and use the following stretched variables:

$$\xi = \epsilon(x - V_{\rm g}t), \quad \eta = \epsilon y, \quad \tau = \epsilon^2 t\,, \tag{2a, b, c}$$

where $V_{\rm g}$ is the group velocity. At this point we may add some comments regarding the form of this stretching. Note that if we forget the y coordinate then we have a two-dimensional problem. In that case to deduce a nonlinear Schrödinger equation in two dimensions one uses stretching similar to that given in equations (2a) and (2c) (Verheast 1988; Tagare and Das 1975; Watanabe 1984), along with a Fourier-like expansion of the dependent variable. We have modified it slightly by adding the stretching of the y coordinate, but have kept intact the expansion of the variables n_α, n_β etc. (Sato *et al.* 1990). We also expand the physical variables as follows (Mukhopadhyay *et al.* 1994; Mukhopadhyay and Roy Chowdhury 1995):

$$n_\alpha = n_{\alpha 0} + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} n_\alpha^m(l)(\epsilon,\,\eta,\,\tau)\exp({\rm i}\,X)\,, \tag{3}$$

$$n_\beta = n_{\beta 0} + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} n_\beta^m(l)(\epsilon,\,\eta,\,\tau)\exp({\rm i}\,X)\,,$$

$$\begin{pmatrix} u_\alpha \\ v_\alpha \end{pmatrix} = \begin{pmatrix} u_{\alpha 0} \\ v_{\alpha 0} \end{pmatrix} + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} \begin{pmatrix} u_\alpha^m(l)(\epsilon,\,\eta,\,\tau) \\ v_\alpha^m(l)(\epsilon,\,\eta,\,\tau) \end{pmatrix} \exp({\rm i}\,X)\,,$$

$$\begin{pmatrix} u_\beta \\ v_\beta \end{pmatrix} = \begin{pmatrix} u_{\beta 0} \\ v_{\beta 0} \end{pmatrix} + \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} \begin{pmatrix} u_\beta^m(l)(\epsilon,\,\eta,\,\tau) \\ v_\beta^m(l)(\epsilon,\,\eta,\,\tau) \end{pmatrix} \exp({\rm i}\,X)\,,$$

where $X = l(kx-\omega t)$. The following relations are imposed:

$$v_1^{(m)*} = -v_{-1}^{(m)}; \quad n_{\alpha,\beta}^{(m)*}(l) = n_{\alpha,\beta}^{(m)}(-l); \quad \phi^{(m)*}(l) = \phi^{(m)}(-l), \tag{4}$$

where the asterisk denotes complex conjugation, to ensure reality of the physical variables. Also we have set

$$\phi = \sum_{m=1}^{\infty} \epsilon^m \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\epsilon,\,\eta,\,\tau)\exp({\rm i}\,X)\,.$$

Substituting these expressions in the basic equations (1a) to (1g) and transforming the independent variables as in equations (2), we equate various powers of ϵ and coefficients $\exp({\rm i}\,X)$ for different l.

From the terms which are first order in ϵ we get

$$
\begin{aligned}
v_\alpha^{(1)}(l) &= v_\beta^1(l) = 0\,,\\
n_\alpha^{(1)}(l) &= \frac{k^2 n_{\alpha 0}}{p^2 - k^2 T_\alpha}\,\phi_1^{(1)}\,,\\
n_\beta^{(1)}(l) &= \frac{k^2 n_{\beta 0}}{k^2 T_\beta - p^2 Q}\,\phi_1^{(1)}\,,\\
u_\alpha^{(1)}(l) &= \frac{k(kv_x - w)}{k^2 T_\alpha - (kv_x - w)^2}\,\phi_1^{(1)}\,,\\
u_\beta^{(1)}(l) &= \frac{k(kv_x - w)}{(kv_x - w)^2 Q - k^2 T_\beta}\,\phi_1^{(1)}\,,
\end{aligned}
\tag{5}
$$

along with the dispersion relation

$$
1 + k^2 - \frac{k^2 n_{\alpha 0}}{p^2 - k^2 T_\alpha} + \frac{k^2 n_{\beta 0}}{k^2 T_\beta - p^2 Q} = 0\,,
\tag{6}
$$

where $p = kv_x - \omega$.

For second order in ϵ we can proceed in the same fashion and obtain explicit expressions for $n_\alpha^2(1)$, $n_\beta^2(1)$, $u_\beta^{(2)}(1)$, $v_\alpha^{(2)}(1)$, $v_\beta^{(2)}(1)$, $n_\alpha^{(2)}(2)$, $u_\alpha^{(2)}(2)$, $n_\beta^{(2)}(2)$, $u_\beta^{(2)}(2)$, $v_\beta^{(2)}(2)$, etc. in terms of $\phi^{(1)}$, $\phi^{(2)}$, $\phi_2^{(2)}$, $n_\alpha^{(1)}$, $n_\beta^{(1)}(1)$. Since these expressions are quite lengthy we do not reproduce them here.

We now consider $l = 0$ terms in second order of ϵ which leads to

$$
\begin{aligned}
&(v_x - v_{\rm g})\frac{\partial n_\alpha^{(2)}(0)}{\partial \xi} + n_{\alpha 0}\,\frac{\partial u_\alpha^{(2)}(0)}{\partial \xi} + n_{\alpha 0}\,\frac{\partial v_\alpha^{(2)}(0)}{\partial \eta} + a\,\frac{\partial}{\partial \xi}\,|\phi_1^{(1)}|^2 = 0\,,\\
&n_{\alpha 0}(v_x - v_{\rm g})\frac{\partial u_\alpha^{(2)}(0)}{\partial \xi} + (T_\alpha + n_{\alpha 0})\frac{\partial n_\alpha^{(2)}(0)}{\partial \xi} - n_{\alpha 0}\,\frac{\partial n_\beta^{(2)}(0)}{\partial \xi} + b\,\frac{\partial}{\partial \xi}\,|\phi_1^{(1)}|^2 = 0\,,\\
&n_{\alpha 0}(v_x - v_{\rm g})\frac{\partial v_\alpha^{(2)}(0)}{\partial \xi} + (n_{\alpha 0} + T_\alpha)\frac{\partial n_\alpha^{(2)}(0)}{\partial \eta} - n_{\alpha 0}\,\frac{\partial n_\beta^{(2)}(0)}{\partial \eta} + c\,\frac{\partial}{\partial \eta}\,|\phi_1^{(1)}|^2 = 0\,,\\
&(v_x - v_{\rm g})\frac{\partial n_\beta^{(2)}(0)}{\partial \xi} + n_{\beta 0}\,\frac{\partial u_\beta^{(2)}(0)}{\partial \xi} + n_{\beta 0}\,\frac{\partial v_\beta^{(2)}(0)}{\partial \eta} + d\,\frac{\partial}{\partial \xi}\,|\phi_1^{(1)}|^2 = 0\,,\\
&n_{\beta 0}(v_x - v_{\rm g})\frac{\partial u_\beta^{(2)}(0)}{\partial \xi} - \frac{n_{\beta 0}}{Q}\,\frac{\partial n_\alpha^2(0)}{\partial \xi} + \frac{n_{\beta 0} + T_\beta}{Q}\,\frac{\partial n_\beta^{(2)}(0)}{\partial \xi} + g\,\frac{\partial}{\partial \xi}\,|\phi_1^{(1)}|^2 = 0\,,\\
&n_{\beta 0}(v_x - v_{\rm g})\frac{\partial v_\beta^{(2)}(0)}{\partial \xi} - \frac{\beta_0}{Q}\,\frac{\partial n_\alpha^2(0)}{\partial \eta} + \frac{n_{\beta 0} + T_\alpha}{Q}\,\frac{\partial n_\beta^{(2)}(0)}{\partial \eta} + h\,\frac{\partial}{\partial \xi}\,|\phi_1^{(1)}|^2 = 0\,,
\end{aligned}
\tag{7}
$$

where a, b, c etc. are constants given in the Appendix. On the other hand equating coefficients of terms of second order in ϵ and with $l = k$ we get

$$(kv_x - \omega)n_\alpha^3(1) + \mathrm{i}\,kn_{\alpha 0}\,u_\alpha^3(1) = -(v_x - v_\mathrm{g})\frac{\partial n_\alpha^{(2)}(1)}{\partial \xi} - \frac{\partial n_\alpha^{(1)}(1)}{\partial \tau} - n_{\alpha 0}\left(\frac{\partial u_\alpha^{(2)}(1)}{\partial \xi} + \frac{\partial v_\alpha^{(2)}(1)}{\partial \eta}\right) + \mathrm{i}\,k[n_\alpha^{(2)}(2) \times u_\alpha^{(1)*}(1) + u_\alpha^{(2)}(2)\,n_\alpha^{(1)*}(1)] - \mathrm{i}k[u_\alpha^{(2)}(0)\,n_\alpha^{(1)}(1) + n_\alpha^{(2)}(0)\,u_\alpha^{(1)}(1)]\,, \tag{8a}$$

$$\mathrm{i}\,n_{\alpha 0}(kv_x - \omega)u_\alpha^{(3)}(1) + \mathrm{i}\,kn_\alpha(0)\,\phi_1^{(3)} + \mathrm{i}\,kT_\alpha\,n_\alpha^{(3)}(1) = -\,n_{\alpha 0}(v_x - v_\mathrm{g})\frac{\partial u_\alpha^{(2)}(1)}{\partial \xi} + \mathrm{i}\,n^{(2)}(2)\,u_\alpha^{(1)*}(1)(kv_x - \omega) + \mathrm{i}\,k[n_\alpha^{(1)}(2)\,\phi_1^{(1)*} - n_\alpha^{(2)}(0)\,\phi_1^{(1)*}] - n_\alpha(0)\,\frac{\partial \phi_1(2)}{\partial \xi} - \mathrm{i}\,kn_{\alpha 0} \times u_\alpha^{(2)}(0)\,u_\alpha^{(1)}(1) - T_\alpha\,\frac{\partial n_\alpha^{(2)}(1)}{\partial \xi} - n_{\alpha 0}\,\frac{\partial u_\alpha^{(1)}(1)}{\partial \tau} - \mathrm{i}(kv_x - \omega) \times n_\alpha^{(2)}(0)\,u_\alpha^{(1)}(1) + \mathrm{i}\,kn_\alpha(0)\,u_\alpha^{(2)}(2)\,u_\alpha^{(1)*}(1)\,, \tag{8b}$$

$$\mathrm{i}(kv_x - \omega)n_{\alpha 0}\,v_\alpha^{(3)}(1) = -\mathrm{i}\,n_{\alpha 0}(v_x - v_\mathrm{g})\frac{\partial v_\alpha^{(2)}(1)}{\partial \xi} - \mathrm{i}\,n_{\alpha 0} \times \frac{\partial \phi_1^{(2)}}{\partial \eta} + T_\alpha\,\frac{\partial n_\alpha^{(2)}(1)}{\partial \eta}\,, \tag{8c}$$

$$\mathrm{i}(kv_x - \omega)n_{\beta 0}\,v_\beta^{(3)}(1) = -(v_x - v_\mathrm{g})n_{\beta 0}\,\frac{\partial v_\beta^{(2)}(1)}{\partial \xi} - \frac{T_\beta}{Q}\,\frac{\partial n_\beta^{(2)}(1)}{\partial \eta} + \frac{n_{\beta 0}}{Q}\,\frac{\partial \phi_1}{\partial \eta}\,, \tag{8d}$$

$$(1 + k^2)\phi_1^{(3)} + n_\beta^{(3)}(1) - n_\alpha^{(3)}(1) = \mathrm{i}\,k\frac{\partial \phi_1^{(2)}}{\partial \xi} + \frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + \frac{\partial^2 \phi_1^{(1)}}{\partial \eta^2}\,, \tag{8e}$$

$$\mathrm{i}(kv_x - \omega)n_{\beta 1}^{(3)} + \mathrm{i}\,kn_{\beta 0}\,u_{\beta 1}^{(3)} = -(v_x - cv_\mathrm{g})\frac{\partial n_\alpha^{(2)}(1)}{\partial \xi} - \frac{\partial n_\beta^{(1)}(1)}{\partial \tau} - n_{\beta 0}\left(\frac{\partial u_\beta^{(2)}(1)}{\partial \xi} + \frac{\partial v_\alpha^{(2)}(1)}{\partial \eta}\right) + \mathrm{i}\,k[n_\beta^{(2)}\,u_\beta^{(1)*}(1) + u_\beta^{(2)}(2)\,n_\beta^{(1)*}(1)] - \mathrm{i}k[u_\beta^{(2)}(0)\,n_\beta^{(1)}(1)(1) + n_\beta^{(2)}(0)\,u_\beta^{(1)}(1)]\,, \tag{8f}$$

$$\mathrm{i}(kv_x - \omega)n_{\beta 0}\,u_\beta^{(3)}(1) - (\mathrm{i}k/Q)n_{\beta 0}\,Q_1^{(3)} + \mathrm{i}\,k(k_\beta/Q)n_\beta^{(3)}(1)$$

$$
\begin{aligned}
= & - n_{\beta 0}(v_x - v_{\mathrm{g}}) \frac{\partial u_\beta^{(2)}(1)}{\partial \xi} - n_{\beta 0} \frac{\partial u_\beta^{(1)}(1)}{\partial \tau} + \mathrm{i}(kv_x - \omega) n_\beta^{(2)}(2) \\
& \times u_\beta^{(1)*}(1) - (\mathrm{i}k/Q)[n_\beta^{(2)}(2)\, \phi_1^{(1)} - n_\beta^{(2)}(0)\, \phi_1^{(1)*}] - \mathrm{i}(kv_x - \omega) \\
& \times n_\beta^{(2)}(0)\, u_\beta^{(1)} + \mathrm{i}\, k n_{\beta 0}[u_\beta^{(2)}(2)\, u_\beta^{(1)*}(1) - u_\beta^{(2)}(0)\, u_\beta^{(1)*}(1)] \\
& - \frac{T_\beta}{Q} \frac{\partial n_\beta^{(2)}(1)}{\partial \xi} + \frac{n_{\beta 0}}{Q} \frac{\partial \phi_1^{(2)}}{\partial \xi} .
\end{aligned} \tag{8g}
$$

It is interesting to note that due to the existence of the dispersion relation it is possible to eliminate the unwanted quantities and we obtain the following equations (by using equations 7):

$$
s_1 \frac{\partial^2 n_\alpha^{(2)}(0)}{\partial \xi^2} + n_{\alpha 0} \frac{\partial^2 n_\beta^{(2)}(0)}{\partial \xi^2} - (n_{\alpha 0} + T_\alpha) \frac{\partial^2 n_\alpha^{(2)}(0)}{\partial \eta^2} + n_{\alpha 0} \frac{\partial^2 n_\beta^{(2)}(0)}{\partial \eta^2}
\left(- c \frac{\partial^2}{\partial \eta^2} + s_2 \frac{\partial^2}{\partial \xi^2} \right) |\phi_1^{(1)}|^2 = 0 , \tag{9a}
$$

$$
s_3 \frac{\partial^2 n_\beta^{(2)}(0)}{\partial \xi^2} + \frac{n_{\beta 0}}{Q} \frac{\partial^2 n_\beta^{(2)}(0)}{\partial \xi^2} - \frac{\eta_{\beta 0} + T_\beta}{Q} \frac{\partial^2 n_\beta^{(2)}(0)}{\partial \eta^2}
+ \frac{n_{\beta 0}}{Q} \frac{\partial^2 n_\alpha^{(2)}(0)}{\partial \eta^2} - \left(h \frac{\partial^2}{\partial \eta^2} + s_4 \frac{\partial^2}{\partial \xi^2} \right) |\phi_1^{(1)}|^2 = 0 , \tag{9b}
$$

$$
\mathrm{i} \frac{\partial}{\partial \tau} \phi_1^{(1)} + p_1 \phi_1^{(1)} + \rho \frac{\partial^2 \phi_1^{(1)}}{\partial \xi^2} + \gamma \frac{\partial^2 \phi_1^{(1)}}{\partial \eta^2} + R |\phi_1^{(1)}|^2 \phi_1^1 + s \phi_1^{(1)}\, n_\alpha^{(2)}(0)
+ T \phi_1^{(1)}\, n_\beta^{(2)}(0) = 0 , \tag{9c}
$$

where we have set

$$
\begin{aligned}
s_1 &= (v_x - v_{\mathrm{g}})^2 - (T_\alpha + n_{\alpha 0}) , \\
s_2 &= a(v_x - v_{\mathrm{g}}) - b , \\
s_3 &= (v_x - v_{\mathrm{g}})^2 - \frac{T_\beta + n_{\beta 0}}{Q} , \\
s_4 &= g - d(v_x - v_{\mathrm{g}}) ,
\end{aligned}
$$

describing the evolution of the nonlinear wave inside the plasma. The constants in these equations are given as

$$\nu = -\frac{p^2}{2k^2},$$

$$p_1 = \frac{(kv_g-\omega)^2}{2k^2p} - \frac{kv_g-\omega}{k^2}\left(\frac{n_{\beta 0}/Q}{a^4-k^4T_{\beta^2}/Q^2} + \frac{n_{\alpha 0}}{a^4-k^4T_\alpha^2}\right)\Big/ D,$$

$$D = \frac{n_{\beta 0}/Q}{(a^2-k^2T_\beta/Q)^2} + \frac{n_{\alpha 0}}{(a^2-k^2T_\alpha)^2},$$

$$DS = \frac{2n_{\beta 0}k^3p}{Q^2(v_x-v_g)(a^2-k^2T_\beta)/Q^2} + \frac{k^2}{a^2-k^2T_\alpha} + \frac{T_\alpha+n_{\alpha 0}}{v_x-v_g} \times \frac{2k^3p}{(a^2-k^2T_\alpha)^2},$$

$$DT = -\frac{2k^3n_{\alpha 0}p}{(v_x-v_g)(a^2-k^2T_\alpha)^2} + \frac{k^2/Q}{a^2-k^2T_\beta/Q} + \frac{T_\beta+n_{\beta 0}}{v_x-v_g}\frac{1}{Q} \times \frac{2k^3p}{(a^2-k^2T_\beta/Q)^2},$$

$$\rho = \frac{1}{v_x-v_g}\frac{1}{n_{\beta 0}/Q(a^2-k^2T_\alpha)^2} + n_{\alpha 0}(a^2-k^2T_\beta/Q)^2,$$

$$DR = \left(\frac{g}{Q(v_x-v_g)}\frac{2k^3p}{a^2-(k^2T_\beta/Q)^2} + \frac{b}{v_x-v_g}\frac{2k^3p}{(a^2-k^2T_\alpha)^2}\right) + \left(\frac{k^2\delta a^2}{3}E + \frac{k^2\delta}{6}E_1 - \frac{k^6}{2}E_2 + a^3k^7E_3\right), \tag{10}$$

with

$$E = \frac{n_{\alpha 0}}{(a^2-k^2T_\alpha)^3} + \frac{n_{\beta 0}/Q^2}{(a^2-k^2T_\beta/Q)^3},$$

and similar expressions for E_1, E_2 and E_3. It may be pointed out that if there was no dependence on η (that is the second space variable), then equations (9a) and (9b) could be integrated at once to obtain $n_\alpha^{(2)}(0)$ and $n_\beta^{(2)}(0)$ in terms of $|\phi_1^{(1)}|^2$, and (9c) would lead to a single nonlinear Schrödinger equation describing the evolution of an envelope soliton. Such an NLS equation has been widely discussed in the literature. On the other right hand, if one of the species of ions is absent, that is n_α or n_β is zero, then one of (9a) or (9b) drops out and we get back the Davey–Stewartson (1974) equation. Thus, our equation is a generalisation of such a system.

3. Solution

In our analysis we have deduced a new set of coupled nonlinear evolution equations for a plasma with negative and positive ions at different temperatures. This set of equation seems to be a multicomponent generalisation of a Davey–

Stewartson type equation, which is already well-known. In the following we try to find an explicit form of the wave sustained by such an equation.

Let us assume that

$$\begin{aligned} \phi_1^{(1)} &= \Psi(p_1\,\xi + k_1\,\eta + n_1\,\tau)\mathrm{e}^{\mathrm{i}\phi(p\xi+k\eta+n\tau)}\,, \\ n_\alpha^{(2)}(0) &= Y_\alpha(p_1\,\xi + k_1\,\eta + n_1\,\tau)\,, \\ n_\beta^{(2)}(0) &= Y_\beta(p_1\,\xi + k_1\eta + n_1\,\tau)\,, \end{aligned} \tag{11}$$

where $\sigma = p\xi + k\eta + n\tau$ is the wave front in three-dimensional space–time and ϕ is a phase factor. It is then easy to observe that (9a) and (9b) reduce to

$$\begin{aligned} x_1\,Y_\alpha + y_1\,Y_\beta - z_1\,|\Psi|^2 &= 0\,, \\ x_2\,Y_\alpha + y_2\,Y_\beta - z_2\,|\Psi|^2 &= 0\,, \end{aligned} \tag{12}$$

leading to

$$Y_\alpha = \lambda_1\,|\Psi|^2, \qquad Y_\beta = \lambda_2|\Psi|^2\,, \tag{13}$$

where λ_1, λ_2 are constants. On the other hand, the imaginary and real parts of (9c) lead to respectively

$$\theta = -\frac{n}{\alpha_1\,p^2 + \alpha_2\,k^2}, \qquad z = +\theta_0\,z \quad \text{(say)}\,,$$

$$\begin{aligned} (\alpha_1\,p^2 + \alpha_2\,k^2)\Psi_{zz} - \theta_0(n + p^2\alpha_1 + k^2\alpha_2)\Psi \\ + (\alpha_3 + \lambda_1\,\alpha_4 + \lambda_2\,\alpha_5)\Psi^3 = 0\,, \end{aligned} \tag{14}$$

$$\text{or} \quad \mathrm{u}\Psi_{zz} + \mathrm{v}\Psi + \omega\Psi^3 = 0\,.$$

Upon integration this yields

$$\frac{z}{\sqrt{u}} + c = \int \frac{\mathrm{d}\Psi}{(k - \alpha\Psi^2 - \omega/2\Psi^4)^{1/2}}\,. \tag{15}$$

The integral on the right hand side can be expressed in terms of inverse elliptic functions through the formulae

$$\begin{aligned} \int_0^u \frac{\mathrm{d}u}{(x^2 + a^2)(x^2 + b^2)} &= (1/a)\mathrm{sn}^{-1}(u,\,\alpha)\,, \\ \alpha &= \tan^{-1}(b/a)\,, \end{aligned} \tag{16}$$

whence we observe that Ψ can be expressed as a cnoidal wave in (2+1) dimensions and, hence, also Y_α and Y_β through formulae (13). On the other hand, if $k=0$ in equation (15), v and ω are both negative, then the integral can be evaluated in terms of the hyperbolic inverse functions

$$\Psi = A\,\mathrm{sech}[2\sqrt{v_1}(z/\sqrt{u}+c)]\,,$$

which is simply the desired solitary wave.

4. Discussion

In our computations we have shown that, by taking recourse to a modified form of a reductive perturbation technique, a new set of nonlinear equations can be derived for a plasma with more than one ion species. This extra ion species could also be a dust grain present in the ionospheric plasma or elsewhere. Already the three component model has been used to simulate the phenomenon of dusty plasmas. Of course we have not used the exact features pertaining to a dusty plasma, but even so equations of the form (9) can be deduced.

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Appendix

Here we give the expressions for the parameters which we require in the dispersion relation:

$$a = -\frac{k^3 n_{\alpha 0}}{\lambda^2}(kv_x-\omega)\,,$$

$$b = -\tfrac{1}{2}(v_x-v_{\mathrm{g}})\frac{k^3 n_{\alpha 0}(kv_x-\omega)}{\lambda^2}+\frac{k^2 n_{\alpha 0}(kv_x-\omega)^2}{2\lambda^2}+\frac{k^2 n_{\alpha 0}}{2\lambda}\,,$$

$$c = \frac{k^2 n_{\alpha 0}}{2\lambda}\,,$$

$$
\begin{aligned}
d &= \frac{k^3 n_{\beta 0}(kv_x - \omega)}{\gamma^2}\,,\\
g &= -\tfrac{1}{2}(v_x - v_{\mathrm{g}})\frac{k^3 n_{\beta 0}(kv_x - \omega)}{\gamma^2} + \frac{n_{\beta 0}k^2(kv_x - \omega)^2}{2\gamma^2} + \frac{k^2 n_{\beta 0}}{2Q\gamma}\,,\\
h &= \frac{k^2 n_{\beta 0}}{2\gamma}; \qquad \gamma = Q\left((kv_x - \omega)^2 - \frac{k^2 T_\beta}{Q}\right).
\end{aligned}
$$

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