Aust. J. Phys., 1996, **49**, 765–75

Reflection and Diffraction of Atomic de Broglie Waves by Evanescent Laser Waves—Bare-state Method*

Xiao-Ping Feng,[A] *N. S. Witte, Lloyd C. L. Hollenberg and Geoffrey I. Opat*

School of Physics, University of Melbourne,
Parkville, Vic. 3052, Australia.
[A]Present address: Institute of Industrial Science, University of Tokyo,
7-22-1 Roppongi, Minoto-ku, Tokyo 153, Japan.

Abstract

We present two results in an investigation of reflection and diffraction of atoms by gratings formed either by standing or travelling evanescent laser waves. Both results use the bare-state rather than dressed-state picture. One is based on the Born series, whereas the other is based on the Laplace transformation of the coupled differential equations. The two solutions yield the same theoretical expressions for reflected and diffracted atomic waves in the whole space, including the interaction and the asymptotic regions.

1. Introduction

The reflection and diffraction of atomic de Broglie waves by laser radiation have attracted increasing attention in the developing field of atom optics (Cook and Hill 1982; Balykin *et al.* 1988; Hajnal and Opat 1989*a*, 1989*b*; Deutschmann *et al.* 1993; Zhang *et al.* 1992; Murphy *et al.* 1993; Stenlake *et al.* 1994). Atomic mirrors and gratings formed by evanescent laser waves are expected to be used as optical elements in highly sensitive atom interferometers (Hajnal and Opat 1989*a*, 1989*b*). Experimental and theoretical studies of the characteristics of such optical elements have been carried out (Hajnal and Opat 1989*a*, 1989*b*; Deutschmann *et al.* 1993; Murphy *et al.* 1993; Murphy *et al.* 1994). A schematic of such a grating is shown in Fig. 1.

The original treatment by Cook (Cook and Hill 1982) was semiclassical. Fully quantum mechanical analysis of the reflection and diffraction of atomic de Broglie waves has been undertaken by several authors (Hajnal and Opat 1989*a*; Deutschmann *et al.* 1993; Murphy *et al.* 1993, 1994). The dressed-state approach (Deutschmann *et al.* 1993) gives great insight into the mechanisms of reflection and diffraction and, with the aid of multi-slice techniques (Murphy *et al.* 1994), has yielded results consistent with experiment (Stenlake *et al.* 1994). The successful dressed-state method may be understood as follows. To reflect atoms, a strong laser beam is required to form a high potential barrier for the incoming atomic de Broglie waves. However, the strong laser field also causes a large amount of Rabi oscillations at a frequency proportional to the amplitude of the laser field. As Rabi oscillations represent energy exchanges between the

* Refereed paper based on a series of lectures presented to the Atom Optics Workshop, held at the Institute for Theoretical Physics, University of Adelaide, in September 1995.

0004-9506/96/040765$05.00

atoms and the laser field, it is also accompanied by high-speed atomic population changes between different diffraction orders (defined by the atomic momentum states, i.e. bare states). The behaviour of the atomic populations is complicated and thus makes direct numerical calculations using the bare-state description difficult. In the dressed-state description, the atomic states are redefined to include the high-frequency Rabi oscillations. As a result, the populations of such 'dressed atoms' will show less complicated variations in the interaction region and the numerical calculations are much easier than in the bare-state case.

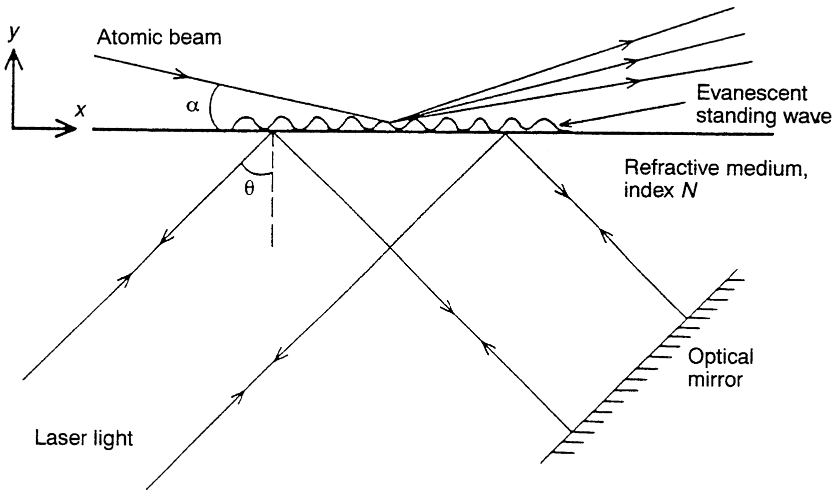

Fig. 1. Schematic diagram of the atomic reflection grating. The grating consists of an evanescent wave produced by total internal reflection of two counter-propagating laser beams.

On the other hand, in the dressed-state description the populations of each diffraction order are not explicitly expressed in the interaction region. Instead, a mixture of them is included in the dressed states, and the reflected and diffracted atoms of each diffraction order are obtained by the asymptotic solutions of the dressed states in the region where the atom–field interaction disappears. If we go to the inside of the interaction region, populations of each diffraction order can only be obtained under a bare-state description.

The dressed-state and bare-state descriptions are complementary. In the asymptotic region, each description has states corresponding to the various diffraction orders. However, in the interaction region they are quite different; this fact allows us to study the diffraction mechanism from different points of view. In the dressed-state description, quasi-potentials are defined for each state, enabling us to study the trajectories of the atom along the quasi-potential curves within the interaction region. On the other hand, in the bare-state description each state is defined by a particular momentum (or wave number) in the x direction (Fig. 1), providing us with a way to study the problem in the momentum space.

In this paper we base our studies on the bare-state picture. The traditional Born series (Hajnal and Opat 1989a) is used to solve both the standing and travelling wave gratings. A solution for the travelling wave grating problem, based on a Laplace transformation method, is also given and is compared directly to the Born-series calculation.

2. Atom–Field Interaction Model and Coupled Differential Equations

Fig. 1 shows schematically the atomic mirror considered in this paper. A laser beam inside a quartz block is totally reflected at the quartz–vacuum interface with an angle q larger than the critical angle. Thus on the surface of the quartz block an evanescent travelling wave is produced. Two such counter-propagating laser beams produce an evanescent standing wave. In the case of an evanescent standing wave, the wave vector is given by $Q_x \hat{i} + iq\hat{j}$ with

$$Q_x = (\omega/c)\, N\sin\theta, \qquad q = (\omega/c)\,(N^2\sin^2\theta - 1)^{\frac{1}{2}}\,,$$

and the electric field can be written as

$$E(t,x,y) = \exp(-qy)\{E_0\exp[-i(wt - Q_x x)] + E_0^*\exp[i(wt - Q_x x)]$$
$$+E_0\exp[-i(wt + Q_x x)] + E_0^*\exp[i(wt + Q_x x)]\}\,.$$

Here, the electric field $\mathbf{E}$ of the laser beam is assumed to be linearly polarised in the z direction with $\mathbf{E} = \hat{k}E$. We consider the de Broglie wave of a two-level atom whose level spacing is near the laser frequency. Its incoming momentum is $P_0 = (\hbar k_x, \hbar k_y)$. We ignore the motion in the z direction where the atomic momentum is constant. The wavefunction in the Schrödinger picture may be written in the rotating wave approximation as

$$\Psi(t,x,y) = \sum_{n\,\mathrm{even}} \exp(-i\Omega_0 t)\exp[(i(k_x + nQ_x)x)]\phi_n(y)\,\psi_{\mathrm{g}}$$
$$+\sum_{n\,\mathrm{odd}} \exp[-i(\Omega_0+\omega)t]\exp[i(k_x + nQ_x)x]\phi_n(y)\,\psi_{\mathrm{e}}\,, \tag{1}$$

where the initial energy $\hbar\Omega_0 = (\hbar^2\ 2M)(k_x^2 + k_y^2)$, and ω is the angular frequency of the laser beam. Here, ψ_{g} and ψ_{e} are the internal atomic ground and excited states. The function $\phi_n(y)$ for even n in the first term represents the spatial dependence of atomic waves with the atom in the ground state, and for odd n in the second term, that of atomic waves with the atom in the excited state. To solve for $\phi_n(y)$ for various orders n forms the main work of this paper.

The Hamiltonian governing the motion of the atom is given by

$$\hat{H} = \hat{H}_a + \frac{\hat{\mathbf{P}}^2}{2M} - \hat{\mu}E\,, \tag{2}$$

where $\hat{H}_{\mathrm{a}}$ is the Hamiltonian of the atomic internal energy, $\hat{\mathbf{P}}$ is the atomic momentum, and $\hat{\mu}$ is the electric dipole moment operator.

The Schrödinger equation $i\hbar(\partial/\partial t)\Psi = H\Psi$ leads to the following differential equations coupling the wave functions $\phi_n(y)$ of different diffraction orders (Hajnal and Opat 1989a):

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + k_{yn}^2\right)\phi_n(y) = -\frac{2M\mu E_0}{\hbar^2}\exp(-qy)[\phi_{n+1}(y) + \phi_{n-1}(y)]\,, \tag{3}$$

where

$$k_{yn}^2 = k_x^2 + k_y^2 - (k_x + nQ_x)^2, \qquad (n \text{ even})$$

$$k_{yn}^2 = k_x^2 + k_y^2 - (k_x + nQ_x)^2 + \frac{2M}{\hbar}(\omega - \omega_{\rm a}), \qquad (n \text{ odd})$$

and we set $\mu = \langle a|\hat{\mu}|b\rangle = \langle b|\hat{\mu}|a\rangle$, and $E_0 = E_0^*$. Further, $\omega_{\rm a}$ is the atomic transition frequency, and M is the atomic mass.

3. Solution Based on Born Series—Standing Wave Case

With the Green function

$$G_n(y, y_0) = \frac{1}{2ik_{yn}} \exp(ik_{yn}|y - y_0|)\,,$$

which satisfies an outgoing wave boundary condition, the differential equations (3) can be transformed to the following integral equations (Hajnal and Opat 1989*a*):

$$\begin{aligned}\phi_n(y) &= \delta_{n0} \exp(-ik_{y0}y) \\ &\quad - \frac{2M\mu E_0}{\hbar^2} \frac{1}{2ik_{yn}} \int_0^\infty \exp(ik_{yn}|y - y_0|) \exp(-qy_0)\,[\phi_{n+1}(y_0) + \phi_{n-1}(y_0)]\, dy_0\,.\end{aligned} \tag{4}$$

The first term represents the incoming de Broglie wave, and the second term represents the outgoing scattered waves. We solve equation (4) with the Born series method, which starts with a weak-field assumption. Here the incoming wave is in the ground state $\phi_0(y) = \exp(-ik_{y0}y)$. When the field E_0 is weak enough, the scattered zero-order wave contributes much less to the whole zero-order wave compared to the original incoming wave. Therefore, one can assume that the zero-order wave is approximately given by $\exp(-ik_{y0}y)$, and use it as a source to generate scattered waves $\phi_{+1}(y)$ and $\phi_{-1}(y)$. These scattered waves become new sources for the generation of waves $\phi_{+2}(y)$ and $\phi_{-2}(y)$, as well as for higher-order contributions to the zeroth-order wave $\phi_0(y)$. The newly generated $\phi_0(y)$ includes both an outgoing wave component $\exp(ik_{y0}y)$ and an incoming wave component $\exp(-ik_{y0}y)$. Repeating such processes, we obtain $\phi_n(y)$ in the form of Born series:

$$\phi_n(y > 0) = \sum_{b=0}^{N_b} \beta_{nb} \exp[(-bq - ik_{y0})y] + \sum_{a=-N_a}^{N_a} \sum_{b=0}^{N_b} \alpha_{nab} \exp[(-bq + ik_{ya})y]\,. \tag{5}$$

Here the index a corresponds to the wave number k_{ya} in the y direction which contributes to the nth-order wave $\phi_n(y)$ through scattering in the interaction region. These waves do not contribute uniformly throughout the interaction region. The damping term $\exp(-bqy)$ becomes sharper with increasing b. We show in the next section that for the case of a simple travelling evanescent waves the coefficients β_{nb} and α_{nab} converge at large b. Hence we set an upper limit

N_b for the index b. We also set a lower limit $-N_a$ and an upper limit $+N_a$ for the index a (i.e. we limit the waves considered in a given calculation).

Instead of performing the iteration as stated above (which was carried out in Hajnal and Opat 1989a), we can calculate the coefficients β_{nb} and α_{nab} by solving the following two sets of coupled linear equations, which satisfy the integral equations (4) or the differential equations (3),

$$\begin{aligned} \beta_{nb} &= B_{nb}(\beta_{n+1,b+1} + \beta_{n-1,b-1}), \qquad (b = 1, \ldots, N_b), \\ \beta_{n0} &= \delta_{n0}, \qquad (b = 0), \end{aligned} \tag{6}$$

and

$$\begin{aligned} \alpha_{nab} &= A_{nab}(\alpha_{n+1,a,b-1} + \alpha_{n-1,a,b-1}) \qquad (b = 1, \ldots, N_b, a = -N_a, \ldots, N_a), \\ \alpha_{na0} &= \alpha_n\, \delta_{na} \qquad (b = 0, a = -N_a, \ldots, N_a), \\ \alpha_n &= \sum_{b=1}^{N_b} X_{nb}\, \beta_{nb} + \sum_{a=-N_a}^{N_a} \sum_{b=1}^{N_b} Y_{nab}\, \alpha_{nab}, \qquad (b = 0, a = 0), \end{aligned} \tag{7}$$

where $n = -N_a, \ldots, N_a$, $\eta = 2M\mu E_0/\hbar^2$ and

$$\begin{aligned} B_{nb} &= \frac{-\eta^2}{k_{yn}^2 + (-ik_{y0} - bq)^2}, \qquad & A_{nab} &= \frac{-\eta^2}{k_{yn}^2 + (ik_{ya} - bq)^2}, \\ X_{nb} &= \frac{bq + ik_{y0} - ik_{yn}}{2ik_{yn}}, \qquad & Y_{nab} &= \frac{bq - ik_{ya} - ik_{yn}}{2ik_{yn}}. \end{aligned} \tag{8}$$

Equations (6) for β_{n0} and (7) for α_{na0} arise from the boundary conditions.

As y approaches infinity, the damping terms disappear, and equation (5) becomes

$$\lim_{y\to\infty} \phi_n(y) = \delta_{n0} \exp(-ik_{y0}y) + \alpha_n \exp(ik_{yn}y). \tag{9}$$

Thus, α_n gives the amplitude of the nth-order reflected wave.

We notice that the coefficients α_{nab} are not involved in equations (6), so we can solve them in two steps. First, on solving the set of $(N_b+1)(2N_a+1)$ equations in (6), we get the solution for all the $(N_b+1)(2N_a+1)$ coefficients of β_{nb}. This can be done by calculating an inverse matrix of the size $[(N_b+1)(2N_a+1)]^2$ numerically. With the result for β_{nb}, we can then solve all the α_{nab} in equation (7) in the same way but for a larger matrix of the size $[(N_b+1)(2N_a+1)^2]^2$.

Finally, disregarding the presence of the quartz block for a moment, we may also calculate the outgoing waves in the region $y < 0$, produced by scattering in the interaction region $0 < y < 1/q$. We suppose that such waves go into the quartz block and are completely absorbed without any reflection into the region $y > 0$. Such an assumption was made in equation (4) (Hajnal and Opat 1989a). From the original integral (4) and equation (5), we find

$$\phi_n(y) = \delta_{n0} \exp(-ik_{y0}y) + \beta_n \exp(-ik_{yn}y) \qquad (y < 0), \tag{10}$$

where

$$\beta_n = \sum_{b=1}^{N_b} X_{nb}^{-}\beta_{nb} + \sum_{a=-N_a}^{N_a}\sum_{b=1}^{N_b} Y_{nab}^{-}\,\alpha_{nab}\,, \tag{11}$$

$$X_{nb}^{-} = \frac{bq + ik_{y0} + ik_{yn}}{2ik_{yn}}\,,$$

$$Y_{nab}^{-} = \frac{bq - ik_{ya} + ik_{yn}}{2ik_{yn}}\,, \tag{12}$$

and β_{nb} and α_{nab} are those in equations (6) and (7).

The above analysis reduces to the travelling wave case if we limit the waves to only the zeroth and first orders by making the change

$$\sum_{a=-N_a}^{N_a} \rightarrow \sum_{a=0}^{1}\,.$$

This Born series method has the weak-field assumption built in from the outset. It is questionable whether it can be extended to the strong-field case (as is required for the reflection of the atom) by including terms of sufficiently high order in the Born series of equation (5) (i.e. sufficiently large N_b). We will address this question in the next section by solving the original equation (5) analytically for the simple travelling evanescent laser wave case. It will be shown that the same wavefunction of (5) is also obtained from the Laplace transformation method (which does not depend on the weak-field assumption). Therefore, we might expect that the Born series method is applicable also to the strong-field standing wave case if a sufficiently large number of terms are included in equation (5).

4. Solution with Laplace Transformation—Travelling Wave Case

In this section we present the results of a Laplace transformation method; for the derivation and further mathematical properties of the solution the reader should consult Witte (1994). For a mirror formed by evanescent travelling waves, we have only two diffraction orders, 0 and +1 (or −1 depending on the relation between the directions of the atomic and laser beams) and equation (4) becomes

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + k_0^2\right)\phi_0(y) = -\,\eta^2\exp(-qy)\,\phi_1(y)\,,$$

$$\left(\frac{\mathrm{d}^2}{\mathrm{d}y^2} + k_1^2\right)\phi_1(y) = -\,\eta^2\exp(-qy)\,\phi_0(y)\,, \tag{13}$$

where $k_0 = k_{y0}$ and $k_1 = k_{y1}$.

In the interests of simplicity the following scalings have been made:

$$\psi(z) = \frac{1}{2q}\phi(y), \qquad z = 2qy, \qquad \mu_0 = \frac{k_0}{2q}, \qquad \mu_1 = \frac{k_1}{2q}, \qquad V = \frac{\eta}{2q}\,.$$

The solution for $\psi_0(z)$ (the other component is simply related to this one by symmetric interchange of arguments) was found to be (Witte 1994)

$$\psi_0(z) = \sum_{m=0}^{\infty}\sum_{l=0}^{m}\Big\{F_{1\,ml}\exp[(-l-i\mu_0)z] + F_{2\,ml}\exp[(-l+i\mu_0)z]$$
$$+F_{3\,ml}\exp[(-l-\tfrac{1}{2}-i\mu_1)z] + F_{4\,ml}\exp[(-l-\tfrac{1}{2}+i\mu_1)z]\Big\}, \tag{14}$$

where

$$F_{1\,ml} \equiv F_{1\,ml}(\mu_0,\mu_1) = V^{4m}\frac{(-1)^l}{l!(m-l)!}$$
$$\times\left\{\frac{\psi_0(0)}{(-l-2i\mu_0)_m\,(-l+\frac{1}{2}-i[\mu_0-\mu_1])_m\,(-l+\frac{1}{2}-i[\mu_0+\mu_1])_m}\right.$$
$$\left.-\frac{V^2\,\psi_1(0)}{(-l-2i\mu_0)_{m+1}\,(-l+\frac{1}{2}-i[\mu_0-\mu_1])_{m+1}\,(-l+\frac{1}{2}-i[\mu_0+\mu_1])_m}\right\},$$

$$F_{2\,ml} \equiv F_{2\,ml}(\mu_0,\mu_1) = V^{4m}\frac{(-1)^l}{l!(m-l)!}$$
$$\times\left\{\frac{\psi_0(0)(m-l)}{(-l+2i\mu_0)_{m+1}\,(-l+\frac{1}{2}+i[\mu_0-\mu_1])_m\,(-l+\frac{1}{2}+i[\mu_0+\mu_1])_m}\right.$$
$$\left.-\frac{V^2\,\psi_1(0)}{(-l+2i\mu_0)_{m+1}\,(-l+\frac{1}{2}+i[\mu_0-\mu_1])_m\,(-l+\frac{1}{2}+i[\mu_0+\mu_1])_{m+1}}\right\},$$

$$F_{3\,ml} \equiv F_{3\,ml}(\mu_0,\mu_1) = V^{4m}\frac{(-1)^l}{l!(m-l)!}$$
$$\times\left\{\frac{\psi_0(0)(m-l)}{(-l-2i\mu_1)_m\,(-l-\frac{1}{2}+i[\mu_0-\mu_1])_{m+1}\,(-l-\frac{1}{2}-i[\mu_0+\mu_1])_m}\right.$$
$$\left.-\frac{V^2\,\psi_1(0)}{(-l-2i\mu_1)_m\,(-l-\frac{1}{2}+i[\mu_0-\mu_1])_{m+1}\,(-l-\frac{1}{2}-i[\mu_0+\mu_1])_{m+1}}\right\},$$

$$F_{4\,ml} \equiv F_{4\,ml}(\mu_0,\mu_1) = V^{4m}\frac{(-1)^l}{l!(m-l)!}$$
$$\times\left\{\frac{\psi_0(0)(m-l)}{(-l+2i\mu_1)_m\,(-l-\frac{1}{2}-i[\mu_0-\mu_1])_m\,(-l-\frac{1}{2}+i[\mu_0+\mu_1])_{m+1}}\right.$$
$$\left.-\frac{V^2\,\psi_1(0)(m-l)}{(-l+2i\mu_1)_{m+1}\,(-l-\frac{1}{2}-i[\mu_0-\mu_1])_{m+1}\,(-l-\frac{1}{2}+i[\mu_0+\mu_1])_{m+1}}\right\}, \tag{15}$$

and $(x)_m \equiv x(x+1)(x+2)\ldots(x+m-1)$.

In order to compare equation (14) with equation (5), we change the order of the summations over m and l. Thus (14) can be written in a similar form to equation (5) as

$$\frac{1}{2q}\phi_0(y>0) \equiv \psi_0(z)$$

$$= \sum_{l=0}^{\infty}\Big\{ F_{1l}\exp[(-2ql - ik_0)y] + F_{2l}\exp[(-2ql + ik_0)y] \tag{16}$$

$$+F_{3l}\exp[(-q(2l+1) - ik_1)y] + F_{4l}\exp[(-q(2l+1) + ik_1)y]\Big\},$$

with

$$F_{kl} \equiv F_{kl}(\mu_0, \mu_1) = \sum_{m=l}^{\infty} F_{kml}(\mu_0, \mu_1), \qquad k = 1\ldots 4. \tag{17}$$

The summations in equation (17) should converge for fixed μ_0, μ_1, V and l at large values of m, i.e. $(m-l) > V$. This is ensured by the coefficient $V^m/(m-l)!$ appearing in F_{kml} and by the denominators, which grow factorially. Furthermore, $l!$ appearing in the denominators in (15) ensures the convergence of the coefficients F_{1l} in (16) at large values of l.

Exchanging μ_0 and μ_1 (or k_0 and k_1) in $\phi_0(y)$, and the positions of $\phi_0(0)$ and $\phi_1(0)$, in equation (15) we get the solution for $\phi_1(y)$ as

$$\frac{1}{2q}\phi_1(y>0) = \sum_{l=0}^{\infty}\Big\{G_{1l}\exp[(-2ql - ik_1)y] + G_{2l}\exp[(-2ql + ik_1)y] \tag{18}$$

$$+G_{3l}\exp[(-q(2l+1) - ik_0)y] + G_{4l}\exp[(-q(2l+1) + ik_0)y]\Big\},$$

with

$$G_{kl} = F_{kl}(0 \leftrightarrow 1), \qquad k = 1\ldots 4. \tag{19}$$

In equations (16) and (18), the numbers $2l$ and $2l+1$ correspond to the number b in (5). We notice that in (16) and (18) there are additional terms for the wave $\exp(-ik_1y)$ in comparison with (5). The existence of such higher-order waves propagating toward the quartz surface seems to be reasonable. Indeed, if we add terms $\beta_{n1b}\exp[(-bq - ik_1)y]$, $(n = 0, 1)$ to (5), it still satisfies the original integral equation (4). However, it is interesting that we find that the initial condition $\beta_{n10} = 0$ leads to $\beta_{n1b} = 0$ (for $b \geq 1$), which means that terms like $\beta_{n1b}\exp[(-bq - ik_1)y]$ exist neither in $\phi_0(y)$ nor in $\phi_1(y)$. Thus, we can say that in equation (16)

$$F_{3l} \equiv 0 \quad \text{and} \quad G_{1l} \equiv 0, \tag{20}$$

for all $l \geq 0$.

Equation (20) may also be proved by substituting the wavefunctions of (16) and (18) into the original differential equations of (13). Also, corresponding to the index b given by equation (5), we have either an even number $2l$ or an odd number $2l+1$ in (16) and (18). It may be easily proved that the number b in (5) (in which $a = 0, 1$ only, for a mirror formed by travelling waves, as opposed to the standing-wave case where $a \in [-N_a, +N_a]$) will also be either even or odd for a fixed value of the index a. Therefore, the wavefunctions of (16) and (18) derived from the Laplace transformation are identified with those of equation (5) based on the Born series.

In the asymptotic region $y \to \infty$, only the non-evanescent terms survive. Thus, as $y \to \infty$, equations (16) and (18) become

$$\psi_0(z) = F_{10}\exp(-ik_0 y) + F_{20}\exp(+ik_0 y)\,,$$

$$\psi_1(z) = G_{10}\exp(-ik_1 y) + G_{20}\exp(+ik_1 y)\,. \tag{21}$$

We notice that there are two unknown coefficients $\psi_0(0)$ and $\psi_1(0)$ still involved in F_{10}, F_{20}, G_{10} and G_{20} in equation (21), as can be seen from (15) and (17). These can be determined by the boundary conditions that we have only an incoming wave in the ground state:

$$F_{10} = 1, \quad \text{and} \quad G_{10} = 0\,. \tag{22}$$

Here the second one is also included in equation (20). In (21) the terms $\exp(+ik_0 y)$ and $\exp(+ik_1 y)$ represent the reflected de Broglie waves, and the amplitudes R_0 of zeroth-order and R_1 of first-order reflected waves are given by

$$F_{20} = R_0, \qquad G_{20} = R_1\,. \tag{23}$$

After simple algebraic calculations for (22) and (23), it was found (Witte 1994) that the reflection coefficients are

$$R_0 = \frac{S_{+0}W_{-1} - S_{+1}W_{-0}}{S_{-0}W_{-1} - S_{-1}W_{-0}}\,,$$

$$R_1 = \frac{W_{+0}W_{-1} - W_{+1}W_{-0}}{S_{-0}W_{-1} - S_{-1}W_{-0}}\,, \tag{24}$$

where in our notation we have set

$$F_{10} = S_{-0}\,\psi_0(0) + S_{-1}\,\psi_1(0)\,,$$

$$F_{20} = S_{+0}\,\psi_0(0) + S_{+1}\,\psi_1(0)\,,$$

$$G_{10} = W_{-0}\,\psi_0(0) + W_{-1}\,\psi_1(0)\,,$$

$$G_{20} = W_{+0}\,\psi_0(0) + W_{+1}\,\psi_1(0)\,. \tag{25}$$

Definitions (15) and (17) give

$$\begin{aligned}
S_{-0} &\equiv S_{-0}(\mu_0, \mu_1) \\
&= \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{1}{(-2i\mu_0)_m \left(\frac{1}{2} - i[\mu_0 - \mu_1]\right)_m \left(\frac{1}{2} - i[\mu_0 + \mu_1]\right)_m}, \\
S_{-1} &\equiv S_{-1}(\mu_0, \mu_1) \\
&= \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{-V^2}{(-2i\mu_0)_{m+1} \left(\frac{1}{2} - i[\mu_0 - \mu_1]\right)_{m+1} \left(\frac{1}{2} - i[\mu_0 + \mu_1]\right)_m}, \\
S_{+0} &\equiv S_{+0}(\mu_0, \mu_1) \\
&= \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{m}{(2i\mu_0)_{m+1} \left(\frac{1}{2} + i[\mu_0 - \mu_1]\right)_m \left(\frac{1}{2} + i[\mu_0 + \mu_1]\right)_m}, \\
S_{+1} &\equiv S_{+1}(\mu_0, \mu_1) \\
&= \sum_{m=0}^{\infty} \frac{V^{4m}}{m!} \frac{-V^2}{(2i\mu_0)_{m+1} \left(\frac{1}{2} + i[\mu_0 - \mu_1]\right)_m \left(\frac{1}{2} + i[\mu_0 + \mu_1]\right)_{m+1}}, \qquad (26)
\end{aligned}$$

and

$$W_{-0} = S_{-1}(\mu_1, \mu_0), \qquad W_{-1} = S_{-0}(\mu_1, \mu_0),$$

$$W_{+0} = S_{+1}(\mu_1, \mu_0), \qquad W_{+1} = S_{+0}(\mu_1, \mu_0). \qquad (27)$$

And the solution, as expressed by equation (14), is completed by giving explicit forms for the set of amplitudes that arise in equation (15), namely

$$\begin{aligned}
\psi_0(0) &= \frac{W_{-1}}{S_{-0}W_{-1} - S_{-1}W_{-0}}, \\
\psi_1(0) &= \frac{-W_{-0}}{S_{-0}W_{-1} - S_{-1}W_{-0}}. \qquad (28)
\end{aligned}$$

5. Conclusion

The results obtained using two different bare-state methods for the investigation of reflection and diffraction of atomic de Broglie waves by evanescent laser waves have been presented. The Born series for the diffraction and reflection of atomic beams from a standing evanescent wave was derived and expressions for the solutions in the bare-state picture were obtained for the whole space including the interaction region and the asymptotic region. Of particular interest is the question of convergence of the series in the strong-field case. The simpler case of a travelling wave grating was considered in both the Born series approach and a direct solution by Laplace transformation, which is valid for all field strengths. The two methods give the same form of the wavefunction, indicating that the

Born series approach can be extended to strong fields, for the travelling wave case. We foresee no reasons why this observation will not be valid for the standing wave case.

To test the convergence of the Born series for the standing wave case a great deal of numerical work must be undertaken. Such calculations will supplement numerical calculations based on the multi-slice method, in which the interaction region is divided into slices and the coupled differential equations (3) are solved with the assumption that the laser light intensity is constant inside each of the slices (Murphy *et al.* 1993, 1994). Thus the quantum mechanical analysis of the standing evanescent wave grating is nearing completion.

Acknowledgments

The authors would like to thank P. Goodman and J. Murphy for stimulating discussions. This work was supported by the Australian Research Council.

References

Balykin, V. I., Letokhov, V. S., Ovchinnikov, Yu. B., and Sidorov, A. I. (1988). *Phys. Rev. Lett.* **60**, 2137.

Born, M., and Wolf, E. (1980). 'Principles of Optics', 6th edn (Pergamon: Oxford).

Cook, R. J., and Hill, R. K. (1982). *Opt. Commun.* **43**, 258.

Deutschmann, R., Ertmer, W., and Wallis, H. (1993). *Phys. Rev.* A **47**, 2169.

Hajnal, J. V., and Opat, G. I. (1989*a*). *Opt. Commun.* **71**, 119.

Hajnal, J. V., and Opat, G. I. (1989*b*). *Opt. Commun.* **73**, 331.

Murphy, J. E., Goodman, P., and Smith, A. (1993). *J. Phys.* D **5**, 4665.

Murphy, J. E., Hollenberg, L. C. L., and Smith, A. (1994). *Phys. Rev.* A **49**, 3100.

Stenlake, B. W., Littler, I. C. M., Bachor, H.-A., Baldwin, K. G. H., and Fisk, P. T. H. (1994). *Phys. Rev.* A **49**, R16.

Witte, N. S. (1994). To be published.

Zhang, W., and Walls, D. F. (1992). *Phys. Rev. Lett.* **68**, 3287.

Manuscript received 30 October 1995, accepted 13 March 1996