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# Cross-field Current Closure below the Solar Photosphere

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*Abstract*

A simple model is developed to describe how an externally imposed current closes as a function of time below the photosphere. A vertical current density is assumed to turn on at the photospheric boundary. The model implies that the subsequent closure of the current in the sub-photosphere depends only on the ratio $R_A/R$, where $R_A = \mu_0 v_A$ is the Alfvénic impedance of the photosphere and $R = 1/\sigma_P l$ is the resistance corresponding to the conductivity $\sigma_P$ and a characteristic length $l$. For $R_A/R \gg 1$, current closure occurs at a front, propagating with the Alfvén speed. For $R_A/R \ll 1$, current closure is a diffusive process ahead and behind a slowly propagating Alfvénic front. The first case is the relevant one for the Sun, where $R_A/R \sim 10^8/v_A$, for $v_A$ in kilometres per second.

## 1. Introduction

The question of what happens to currents which flow in the corona as they reach the photosphere is a controversial one in solar physics. Vector magnetogram data typically show regions of strong current ($\sim 10^{12}$ A) flowing into the photosphere at one footpoint of a coronal loop and out at the other footpoint (Moreton and Severny 1968; Hagyard 1989; Romanov and Tsap 1990). This is interpreted as a current flowing along the field lines of the coronal loop. The behaviour of this current below the photosphere—in particular whether it closes across field lines there—is open to question. Some authors (Hudson 1987; McClymont and Fisher 1989; Melrose 1991) claimed that the observed currents flow through the sub-photosphere along field lines, coupling the corona to the deep interior of the Sun. Other authors (e.g. Kan *et al.* 1983) argued that the large scale currents (which are associated with solar flares) are generated at the photosphere by fluid motions: the so-called photospheric dynamo. A third possibility is implicit in a wide class of models for coronal structures in which the boundary condition at the photosphere is taken to be the 'line-tying' assumption (Van Tend and Kuperus 1978; Priest 1982). As demonstrated below, line-tying implies that coronal currents close in the photospheric boundary, independent of the field lines.

In this paper we consider a simple model for the dynamic response of the sub-photospheric plasma to a current imposed from above. The objective is to determine the conditions under which the current closes across field lines locally in the photosphere (as implied by the line-tying assumption) and when closure

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occurs far below the photosphere (as argued by Hudson 1987). Our approach is to assume a two-dimensional, constant density, isotropically conducting photosphere threaded by a vertical, uniform magnetic field. The photosphere occupies the plane $z \geq 0$ (no variation is permitted in $y$) and a vertical current density $J_z$ is assumed to turn on in the photospheric boundary at a particular time ($t = 0$), introducing a total current $I_0$ into the sub-photosphere. This current must close across field lines in the model sub-photosphere for all subsequent times. The problem posed is the specification of $J_x$ and $J_z$ for all $x$, $z$ and $t > 0$, which completely describes how the current closes.

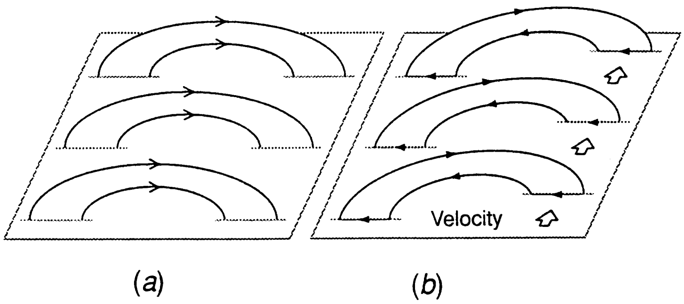


**Fig. 1.** Effect of shearing one side of a coronal arcade. Diagram ($a$) is the arcade prior to any shearing: the open arrow heads denote the direction of the field. The photospheric boundary is the plane shown. In ($b$) the arcade is subject to a shearing velocity field in the photosphere at the right-hand row of footprints, as shown. This plasma motion drives a current in the photospheric boundary which flows along the magnetic field over the arch of the arcade. The current is denoted by the closed arrowheads of the figure. The line-tying boundary condition implies that this current closes across field lines at the passive row of footprints, as shown.

The assumption that coronal magnetic structures are line-tied at the solar photosphere implies that magnetic field lines joining the photosphere to the corona are frozen-in to the sub-photospheric plasma and so are fixed immovably there by the inertia of the denser plasma below. Line-tying implicitly requires that coronal currents close across field lines, as surface currents in the photospheric boundary. For example, consider a coronal arcade, subject to a shear. If the magnetic field at one line of footpoints of the arcade is assumed to be initially vertical at the photosphere (of magnitude $B_0$, say), then line-tying implies that after shearing, a kink appears in the field lines there. This kink implies a nonzero $\nabla \times \mathbf{B}$ at the photosphere, and hence a nonzero surface current there. Specifically, if the kink is a departure $\theta$ from the vertical, then a surface current $B_0 \tan\theta/\mu_0$ must flow, perpendicular to the plane of the kinked field. This surface current must be set up by a sequence of events in which a coronal current is imposed on the photospheric boundary. To understand this point, consider Fig. 1. A shear imposed at one row of footpoints of a coronal arcade drives a current along the coronal field lines of the arcade. If the photospheric motion is slow compared with the Alfvén propagation time in the corona, the shearing of the arcade may be considered as a sequence of magnetostatic equilibrium states in which the footpoints of the arcade are successively displaced (Priest 1982). Each equilibrium state in the sequence has a greater field-aligned current flowing in the coronal part of the arcade, and closing (as a consequence of line-tying) in the photospheric boundary at the passive footpoint. This cross-field boundary current must be set up as the magnetostatic response of the passive footpoint to an increased current density arriving from above, along the field lines.

A second example of the line-tying assumption is provided by the Kuperus and Raadu (1974) model for the support of prominences. In the model, a current-carrying filament is introduced into the corona and induces surface currents in the photospheric boundary, as a consequence of line-tying. The induced currents support the filament through current–current interaction. The initial (pre-filament) magnetic field is predominantly vertical at the photosphere and so current closure there must occur perpendicular to the magnetic field. The surface current induced at the photospheric boundary is the closure current for the coronal filament current, set up after the filament is introduced (e.g. Martens 1986). Variants of the model have been proposed (Van Tend and Kuperus 1978; Lerche and Low 1980; Martens 1986; Martens and Kuin 1989; Priest and Forbes 1990), and all rely on the same assumption about the photosphere. The model developed here tests the validity of the line-tying assumption at the photosphere by considering the response of the sub-photosphere to currents imposed from above.

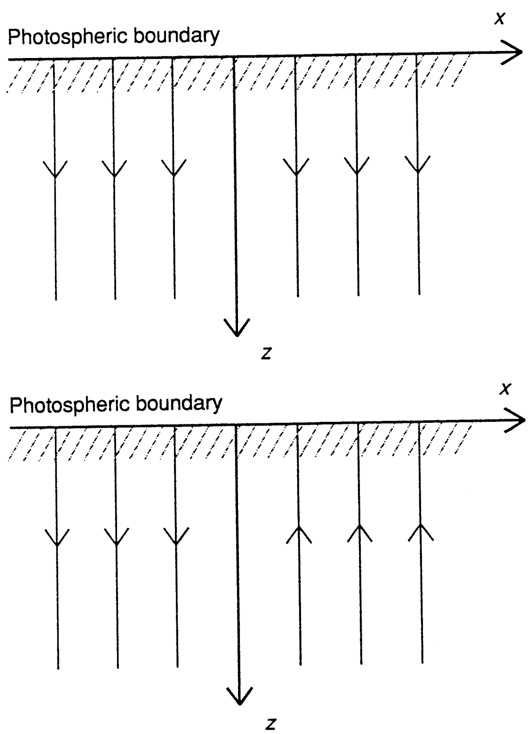


**Fig. 2.** Geometry of the simple model photosphere considered, with two possible field configurations. In the upper picture the background field is uniform and in the lower it is oppositely directed in $x > 0$ and $x < 0$.

The sections of the paper are divided as follows. In Section 2 a simple description of current closure in a model photosphere is presented, which becomes the basis in Section 3 for a model of the response of the sub-photosphere to a current applied from above. The various limiting cases in which analytical solutions to the model exist are investigated in Section 4. The general case requires numerical solution, which is discussed in Section 5, together with the consequences of the model for solar parameters. The main results of the model are discussed in Section 6.

## 2. Description of Cross-field Currents in the Sub-photosphere

Consider the following two-dimensional ($\partial/\partial y = 0$) forms for fluid velocity and magnetic field, respectively, in a model photosphere (Scholer 1970):

$$\mathbf{v} = [0, v_y(x,z,t), 0], \qquad \mathbf{B} = [0, B_y(x,z,t), B_z(x)]\,. \tag{1}$$

The photosphere is assumed to occupy the half-space $z \geq 0$, threaded by a background field $B_z(x)$. For the purposes of the model we consider only those cases where $B_z^2$ is constant. Two specific cases of interest are $B_z = B_0$ and also the simple form

$$B_z(x) = \begin{cases} B_0 & \text{if } x > 0\,, \\ -B_0 & \text{if } x < 0\,, \end{cases} \tag{2}$$

representing an idealised photospheric field with a neutral (or inversion) line. These two choices are shown in Fig. 2. The geometry adopted implies cross-field and parallel current densities

$$J_x = -\frac{1}{\mu_0}\frac{\partial B_y}{\partial z} \quad \text{and} \quad J_z = \frac{1}{\mu_0}\frac{\partial B_y}{\partial x}\,. \tag{3}$$

The photosphere is modelled here as a single resistive fluid with isotropic conductivity $\sigma_P$. The simple Ohm law

$$\mathbf{J} = \sigma_P(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \tag{4}$$

is assumed to apply, where $\mathbf{E}$ is the electric field present.

Assuming also a constant density photosphere, Maxwell's equations and the fluid equation of motion imply the pressure balance

$$p = p_0 - B_y^2/2\mu_0 \tag{5}$$

and the partial differential equation for the magnetic field

$$\frac{\partial^2 B_y}{\partial t^2} - v_A^2\frac{\partial^2 B_y}{\partial z^2} - \frac{1}{\gamma}\frac{\partial}{\partial t}\left(\frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial z^2}\right) = 0\,, \tag{6}$$

where $v_A = B_0/(\mu_0\rho_0)^{\frac{1}{2}}$ is the Alfvén speed and $\gamma = \mu_0\sigma_P$. The components of the current density obey the same partial differential equation and may be obtained from equation (3). Equation (6) has been discussed by Alfvén and Fälthammar (1963).

Several restrictive assumptions are adopted in the derivation of equation (6) which require comment. The restrictive geometry adopted (equation 1) allows only transverse perturbations of the sub-photosphere. Together with the assumption of constant density, this means that only transverse Alfvén waves are considered. These provide the mechanism of dynamic current closure. The role of magneto-acoustic waves is ignored. The stratification of the solar atmosphere is also neglected for simplicity: this is also not expected to influence the basic results presented. Inhomogeneities in the sub-photospheric magnetic field are also neglected. A more sophisticated treatment would consider the role of variations in the magnetic field. The sub-photosphere is assumed to be isotropically conducting. In fact, the sub-photosphere is a partially ionised gas, with a significantly anisotropic conductivity (Khan 1987). The simplest magnetohydrodynamic (MHD) description of the conductivity of the sub-photosphere must involve a Hall current—with corresponding conductivity $\sigma_H$—in addition to the Pederson current described here by $\sigma_P$ (Krall and Trivelpiece 1973). The neglect of the Hall term is consistent with the neglect of $J_y$. Both would need to be considered in a more general model.

## 3. Model for Response of the Sub-photosphere to a Current imposed from above

A solution to equation (6) in the half space $z \geq 0$ is presented below, subject to the boundary condition

$$J_z(x, z{=}0, t) = f(x)\theta(t)\,, \tag{7}$$

where $\theta(t)$ is the step function. The initial conditions assumed are those appropriate to Laplace transform problems,

$$J_z(x, z, t{=}0) = \left.\frac{\partial J_z}{\partial t}\right|_{t=0} = 0\,. \tag{8}$$

Equation (7) prescribes a vertical current density $f(x)$ in the photospheric boundary for $t > 0$. This function is assumed to be odd in $x$, so that current enters the half-space $z \geq 0$ in the half-plane $z = 0$, $x > 0$ and leaves in $z = 0$, $x < 0$. A total current $I_0$ flows in the photosphere for $t > 0$, defined by

$$I_0 \equiv \int_0^\infty dx' f(x') = -\int_{-\infty}^0 dx' f(x')\,. \tag{9}$$

This current must close across field lines in the photosphere.

The equivalent boundary and initial conditions on $B_y$ are, respectively,

$$B_y(x, z{=}0, t) = \mu_0 \phi(x)\theta(t)\,, \tag{10}$$

$$B_y(x, z, t{=}0) = \left.\frac{\partial B_y}{\partial t}\right|_{t=0} = 0\,, \tag{11}$$

with

$$\phi(x) = \int_{-\infty}^{x} dx' f(x') . \tag{12}$$

The lower limit in equation (12) is determined by the requirement that $B_y$ approaches zero for large $x$. A simple choice of $f(x)$ is the delta function profile

$$f(x) = I_0\{\delta(x - x_0) - \delta(x + x_0)\} , \tag{13}$$

which implies

$$\phi(x) = \begin{cases} -I_0 & \text{if } x < x_0 , \\ 0 & \text{otherwise} . \end{cases} \tag{14}$$

This choice corresponds to a localised current density directed into the model photosphere at $x = x_0$ and out at $x = -x_0$. The delta function choice is considered here, along with the general case.

It follows that the Laplace transform of the solution to equation (6) subject to these boundary and initial conditions is

$$\tilde{B}_y(x, z, s) = \frac{\mu_0 \alpha z}{\pi} \left(\frac{\gamma}{s}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} d\xi \, \phi(\xi) \frac{K_1([\gamma s\{\alpha^2 z^2 + (x - \xi)^2\}]^{\frac{1}{2}})}{\{\alpha^2 z^2 + (x - \xi)^2\}^{\frac{1}{2}}} , \tag{15}$$

with

$$\alpha^2 = \frac{s}{s + \gamma v_A^2} \tag{16}$$

and where $K_1$ is the modified Bessel function of the second kind (Abramowitz and Stegun 1965). Equation (15) in general does not correspond to a tabulated Laplace transform (e.g. Erdelyi *et al.* 1954). Various limiting cases permit inversion of this transform and these are investigated below. Consider first, however, the total current crossing the plane $x = 0$. It is straightforward to verify from equation (15) that this quantity is $-I_0$, as is required by continuity of current. To describe where cross-field current closure occurs in this model photosphere for $t > 0$, it is only necessary to obtain the current density $J_x$ in the plane $x = 0$, rather than considering $J_x$ everywhere in $z \geq 0$.

An alternative way to characterise how the current closes in general is to consider the total current closing in the plane $x = 0$ above a height $z = h$:

$$I_\perp(h, t) = -I_0 - \frac{1}{\mu_0} B_y(x{=}0, z{=}h, t) . \tag{17}$$

This quantity decreases from zero at $h = 0$ to asymptotically approach $-I_0$ as $h$ tends to infinity.

## 4. Limiting Cases

Various limiting cases provide insight into the general behaviour of the model:

*(4a) Infinite Conductivity* ($\gamma \to \infty$)

In the limit $\gamma \to \infty$, equation (6) becomes the one-dimensional wave equation. It can be shown from equation (15) that

$$\lim_{\gamma \to \infty} \tilde{B}_y(x,z,s) = \frac{\mu_0}{s}\phi(x)\exp(-sz/v_A)\,, \tag{18}$$

which yields the solution to the wave equation

$$\lim_{\gamma \to \infty} B_y(x,z,t) = \mu_0\phi(x)\theta(t - z/v_A)\,. \tag{19}$$

Denoting this limit '$P$' (for propagating), the corresponding current densities are

$$J_x^P = \frac{\phi(x)}{v_A}\delta(t - z/v_A)\,, \tag{20}$$

$$J_z^P = f(x)\theta(t - z/v_A)\,. \tag{21}$$

The interpretation of equations (20) and (21) is that current closure occurs at an Alfvénic front propagating into the photosphere. In the plane $x = 0$, the cross-field current is

$$J_x^P(x{=}0,z,t) = -\frac{I_0}{v_A}\delta(t - z/v_A)\,, \tag{22}$$

so in the absence of diffusion, a propagating delta function of cross-field current is seen in the plane of symmetry of the model.

*(4b) Zero Alfvén Speed* ($v_A \to 0$)

In the limit of zero Alfvén speed, equation (6) becomes the two-dimensional diffusion equation. Denoting this limit '$D$' (for diffusive), equation (15) becomes

$$\begin{aligned}\tilde{B}_y^D(x,z,s) &= \lim_{v_A \to 0} \tilde{B}_y(x,z,s) \\ &= \frac{\mu_0 z}{\pi}\left(\frac{\gamma}{s}\right)^{\frac{1}{2}} \int_{-\infty}^{\infty} d\xi\, \phi(\xi)\frac{K_1\left([\gamma s\{z^2 + (x-\xi)^2\}]^{\frac{1}{2}}\right)}{\{z^2 + (x-\xi)^2\}^{\frac{1}{2}}}\,,\end{aligned} \tag{23}$$

which can be inverted (Erdelyi *et al.* 1954) to give the classical Green function solution to the diffusion equation

$$B_y^D(x,z,t) = \frac{\mu_0 z}{\pi}\int_{-\infty}^{\infty} d\xi \frac{\phi(\xi)}{z^2 + (x-\xi)^2}\exp\left(-\frac{\gamma}{4t}\{z^2 + (x-\xi)^2\}\right). \tag{24}$$

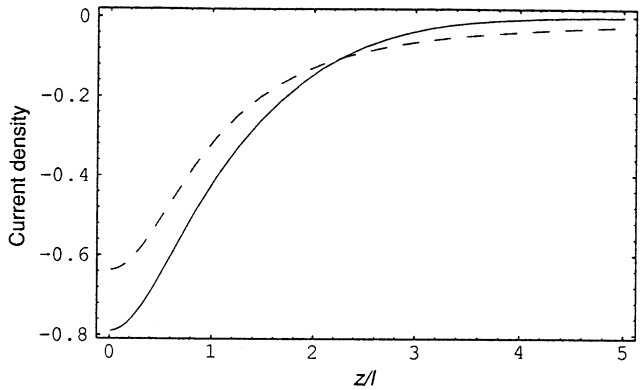


**Fig. 3.** Scaled current density $J_x^D(x{=}0,z,t)/(I_0/l)$ as a function of scaled distance into the photosphere for one diffusive timescale, and in the steady state limit of an infinite number of such timescales. The dashed curve is the asymptotic case.

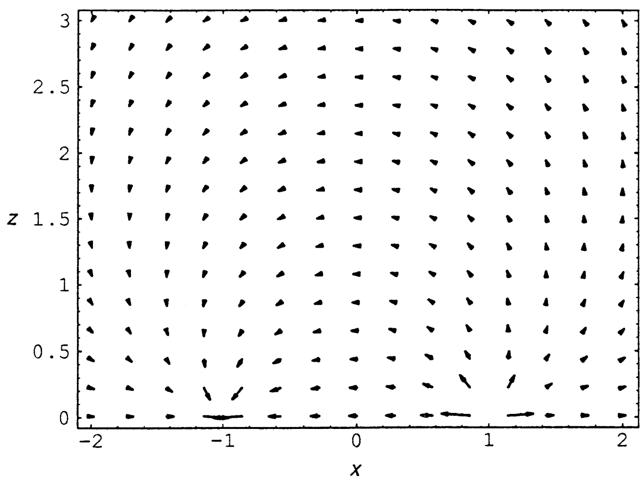


**Fig. 4.** Vector field of the current density in the asymptotic limit of the diffusive case. For simplicity, $x_0 = 1$ has been chosen.

It is straightforward to write down $J_x^D$ and $J_z^D$ from equation (24), but here note only that the cross-field current in the plane $x = 0$ is

$$J_x^D(x{=}0,z,t) = \frac{2}{\pi}\int_0^\infty d\xi\,\phi(\xi)\left(\frac{\gamma z^2/2t}{z^2+\xi^2} - \frac{-z^2+\xi^2}{(z^2+\xi^2)^2}\right)\exp\left(-\frac{\gamma}{4t}(z^2+\xi^2)\right). \tag{25}$$

Fig. 3 illustrates this current density for the delta function choice of $f(x)$, equation (13). As $t\to\infty$ it approaches the asymptotic form

$$J_x^D(x{=}0,z,t{\to}\infty) = -\frac{2I_0x_0}{\pi(x_0^2+z^2)}. \tag{26}$$

In this limit, cross-field current closure occurs for all relevant timescales in a region just below the photospheric boundary. Fig. 4 shows the vector field of current density $(J_x^D, J_z^D)$ in the limit $t \to \infty$. This behaviour is consistent with the Kuperus and Raadu (1974) model for support of current carrying filaments. However, as discussed below, the solar behaviour is much closer to the limit of infinite conductivity than that of zero Alfvén speed.

### (4c) *Steady State* $(t \to \infty)$

The asymptotic time behaviour of equation (6) can be obtained directly by setting $\partial/\partial t$ to zero or more formally by evaluating $\lim_{s\to 0} s\tilde{B}_y(x,z,s)$ using equation (15). The behaviour depends only on whether the Alfvén speed is zero or not.

If $v_A = 0$, the behaviour is described by equation (24) in the limit $t \to \infty$, i.e.

$$B_y^D(x,z,t\to\infty) = \frac{\mu_0 z}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\phi(\xi)}{z^2 + (x-\xi)^2} \tag{27}$$

and equation (26) gives the cross-field current in the plane $x = 0$.

If $v_A \neq 0$ then taking $\lim_{s\to 0} s\tilde{B}_y(x,z,s)$ using equation (15) gives

$$\lim_{t\to\infty} B_y(x,z,t) = f(x)\,, \tag{28}$$

implying

$$\lim_{t\to\infty} J_x(x,z,t) = 0\,, \tag{29}$$

$$\lim_{t\to\infty} J_z = f(x)\,. \tag{30}$$

Equations (29) and (30) imply that, for nonzero Alfvén speed, there are no cross-field currents in the model photosphere in the static limit. The physical interpretation of this result is that whenever the Alfvén speed is nonzero, a cross-field current density $\mathbf{J}_\perp$ implies a force density $\mathbf{J}_\perp \times \mathbf{B}$ which launches an Alfvén wave to propagate the stress away. So in the static limit in any layer of the atmosphere, the cross-field current density must be zero. Zero Alfvén speed corresponds to an unmagnetised plasma, or an infinitely dense plasma. In either case, Alfvén waves cannot propagate to remove the stress implied by a cross-field current density.

Two other limits are of physical interest and are mentioned briefly here. The first is that of zero conductivity, $\gamma \to 0$. In this limit the $v_A = 0$ steady state (equation 27) is achieved immediately at $t = 0$, because the diffusive timescale $\gamma l^2$ is zero. Similarly, in the limit of infinite Alfvén speed ($v_A \to \infty$), the $v_A \neq 0$ steady state, i.e. equation (28), is achieved instantly because the Alfvén timescale is zero in this limit.

## 5. The General Case

In the general case, equation (15) must be inverted numerically, since expansions in $\gamma$ and $v_A$ about the limits previously considered (Section 4) are not possible. It is appropriate at this point to discuss a formal scaling procedure.

By introducing the scaled variables

$$\bar{t} = t/\tau, \quad \bar{z} = z/l, \quad \bar{x} = x/l, \quad \bar{B}_y = B_y/B_0\,, \tag{31}$$

where $\tau$ and $l$ are as yet undefined scale parameters, equation (6) becomes

$$\frac{\partial^2 \bar{B}_y}{\partial \bar{t}^2} - c_P \frac{\partial^2 \bar{B}_y}{\partial \bar{z}^2} - c_D \frac{\partial}{\partial \bar{t}}\left(\frac{\partial^2 \bar{B}_y}{\partial \bar{x}^2} + \frac{\partial^2 \bar{B}_y}{\partial \bar{z}^2}\right) = 0\,. \tag{32}$$

Here the coefficients are

$$c_P = \frac{v_A^2 \tau^2}{l^2} \quad \text{and} \quad c_D = \frac{\tau}{\gamma l^2}\,. \tag{33}$$

The relative sizes of $c_P$ and $c_D$ determine whether equation (6) behaves like the wave equation or the diffusion equation. In particular, the ratio $c_P/c_D$ determines the qualitative behaviour. Taking (without loss of generality) $\tau$ equal to the Alfvén transit time for the characteristic length $l$ gives $c_P = 1$ and $c_D = 1/\gamma v_A l$. Then the ratio of coefficients is

$$\frac{c_P}{c_D} = \gamma v_A l = \frac{R_A}{R}\,, \tag{34}$$

where $R_A = \mu_0 v_A$ is the Alfvénic impedance and $R = 1/\sigma_P l$ is the resistance corresponding to the conductivity $\sigma_P$. This demonstrates that the behaviour of the model depends only on the ratio $R_A/R$.

The delta function choice of $f(x)$ is adopted in this section. The current density is scaled so that $\bar{J}_i = J_i/(I_0/l)$ and also the distance $x_0$ is taken to be $l$ for simplicity. Davies (1978) provided the method for numerical inversion of the Laplace transform used here. The quantity to be inverted is

$$\mathcal{L}\{\bar{J}_x(\bar{x}{=}0,\bar{z},\bar{t})\} = \frac{2\bar{\alpha}}{\pi}\left(\frac{r}{s}\right)^{\frac{1}{2}} \int_0^1 d\xi\, \frac{\partial}{\partial \bar{z}}\left(\bar{z}\frac{K_1\left[\{rs(\bar{\alpha}^2\bar{z}^2+\xi^2)\}^{\frac{1}{2}}\right]}{(\bar{\alpha}^2\bar{z}^2+\xi^2)^{\frac{1}{2}}}\right), \tag{35}$$

where

$$\bar{\alpha} = \frac{s}{s+r}, \quad r \equiv \frac{R_A}{R}\,. \tag{36}$$

Fig. 5*a* is a plot of the cross-field current density at one and two Alfvén transit times when $R_A/R = 10$. The perpendicular current propagates as a spreading pulse symmetrically around $z = v_A t$. This example is close to the propagating, or infinite conductivity limit of Section 4*a*.

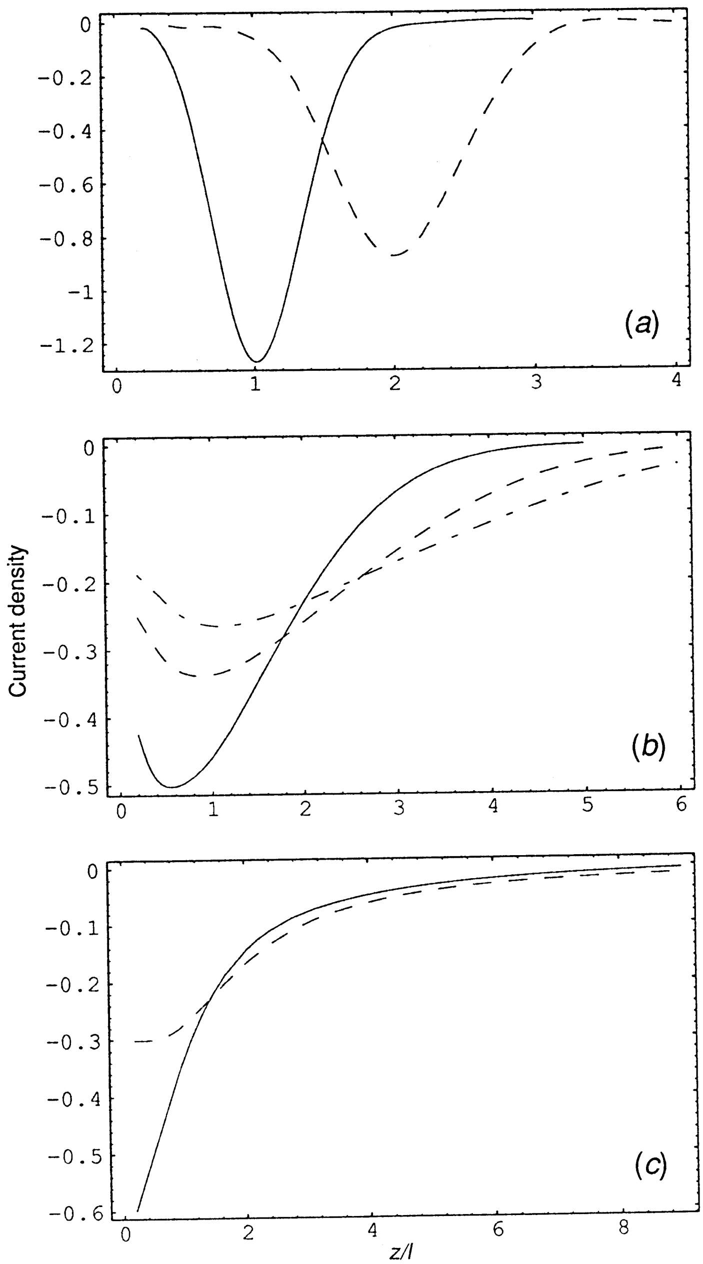


**Fig. 5.** Scaled current density $J_x(x{=}0, z, t)/(I_0/l)$ as a function of scaled distance into the photosphere: ($a$) For the case $R_A/R = 10$ when $\bar{t} = 1$ (solid curve) and $\bar{t} = 2$ (dashed curve), i.e. at one and two Alfvén transit times. The propagating term of equation (6) is dominant in this case. ($b$) For the case $R_A/R = 1$ at one (solid), two (dashed) and three (dot–dash) Alfvén transit times. ($c$) For the case $R_A/R = 0{\cdot}1$ at one (solid) and twenty (dashed) Alfvén transit times. The diffusive term of equation (6) is dominant in this instance.

The second example considered is the intermediate case $R_A/R = 1$, where both the propagating and diffusive terms of equation (6) are influential. Fig. 5*b* indicates that a peak in current density propagates but lags behind $z = v_A t$, and the behaviour has more in common with the diffusive limit of Section 4*b* than the previous example.

Fig. 5*c* is for $R_A/R = 1/10$, close to the diffusive limit of Section 4*b*. Current closure occurs in the first few characteristic lengths below the photosphere for many Alfvén transit times. For times less than an Alfvén transit time, but greater than the diffusive timescale, the asymptotic diffusive behaviour of equation (26) is established ahead of $z = v_A t$.

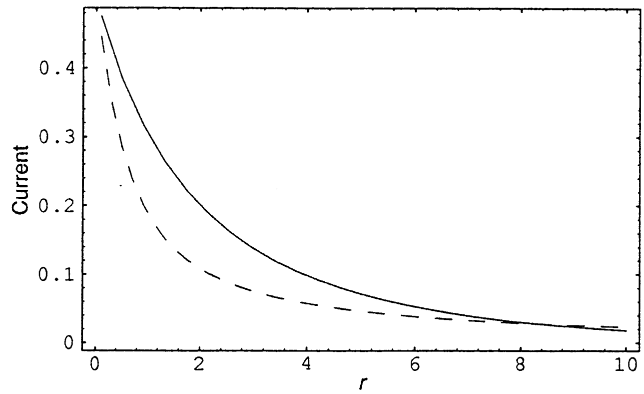


**Fig. 6.** Fraction of the total current $I_0$ closing in the first characteristic length $l$ below the photospheric boundary as a function of $r \equiv R_A/R$ at two (solid curve) and four (dashed curve) Alfvén transit times of $l$.

In Fig. 6, the fraction of current closing in the first characteristic length below the photosphere is shown, at two and four Alfvén transit times as a function of the ratio $R_A/R$. This quantity is defined by equation (17). For small $R_A/R$, the behaviour is diffusive and substantial cross-field closure occurs just below the photospheric boundary. [It follows from equation (24) that the value $\frac{1}{2}$ is expected when $R_A/R = 0$.] For large $R_A/R$ the behaviour is propagating and after a number of Alfvén times there is no appreciable current closing in the region of interest.

The typical solar value taken here is $R_A/R \sim 10^8/v_A$, where $v_A$ is in kilometres per second, corresponding to a characteristic length of $\simeq$1 Mm (Khan 1987). For reasonable values of the photospheric Alfvén speed, this implies a behaviour close to the infinite conductivity limit of Section 4*a*. Imposed currents which turn on in the photosphere close essentially at an Alfvénic front propagating into the sub-photosphere, with only weak diffusion of cross-field current density about $z = v_A t$.

For solar parameters ($R_A/R \sim 10^8$), equation (17) (see also Fig. 6) implies that after two Alfvén transit times, the fraction of current closing in the first characteristic length below the photospheric boundary is of the order of several per cent. So the cross-field current has propagated out of the region of interest in a brief time.

## 6. Conclusions

A simple model is formulated to describe the time-dependent response of the sub-photospheric plasma to a current imposed from above. The main results are as follows:

(1) Cross-field current closure in the photosphere is a dynamic process. The stress implied by a cross-field current density $\mathbf{J}_\perp$ is propagated away by Alfvén waves.
(2) In the static limit, currents imposed along field lines at the photospheric boundary flow along field lines below the photosphere.
(3) The time evolution of cross-field current closure in the model depends only on the ratio $R_A/R$, where $R_A = \mu_0 v_A$ is the Alfvénic impedance and $R = 1/\sigma_P l$ is the resistance corresponding to the conductivity $\sigma_P$ and the characteristic length $l$. For $R_A/R \gg 1$, current closure occurs at an Alfvénic front propagating into the photosphere, with only weak diffusion about the front. For $R_A/R \ll 1$, current closure is a diffusive process, with cross-field currents flowing just below the photosphere for many Alfvén times.
(4) Solar parameters imply $R_A/R \gg 1$. So after a number of Alfvén times, current closure occurs deep in the photosphere.

These results imply that the line-tying assumption is valid only on a timescale of the order of the Alfvén transit time of a given layer of the sub-photosphere. Models of coronal magnetic structures based on line-tying at the photosphere need to be reconsidered in this light.

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