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Variations of Helicon Wave-induced Radial Plasma Transport in Different Experimental Conditions

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Abstract

Variations of the helicon wave-induced radial plasma transport are presented depending on values of the plasma radius, magnetostatic field, plasma density and the frequency of the helicon wave. It is shown that the value of the helicon wave-induced transport may be significant for plasma confinement; this is demonstrated for the experiments BASIL and SHEILA. Whereas $m = +1$ helicons induce an inward-directed transport and thus improve the confinement, $m = -1$ helicons induce an outward-directed transport velocity.

1. Introduction

The purpose here is to estimate variations of the helicon wave-induced radial transport of plasma due to variations in the radius of the plasma cylinder, imposed magnetostatic field, plasma density, frequency of the transport driving helicon wave and the composition of the gas, i.e. the ion charge.

First estimates of such helicon wave driven radial plasma transport were presented by Petržílka (1993*b*) together with radial profiles of the radio frequency (RF) driven transport, assuming that the collision frequency is much lower than the wave frequency. The same assumption has been used by Petržílka (1993*a*), which allowed analytical expressions for ponderomotive forces in the equation for RF induced transport to be obtained and a connection with the wave helicity to be established.

The possibility of influencing the radial transport by RF waves was suggested by Klíma (1980); this idea has been further developed for a cylindrical two-fluid plasma model by Klíma and Petržílka (1980). The radio-frequency flux control has also been studied by Inoue and Itoh (1980) and Fukuyama *et al.* (1982). An enhanced transport associated with high-power low-hybrid heating has been found by Sperling (1978), while specifically toroidal effects have been accounted for by Antonsen and Yoshioka (1986). The transport influenced by RF fields has been found in experiments by Demirkhanov *et al.* (1981) and Kauschke (1992).

We use two-fluid time-averaged magnetohydrodynamic equations (Klíma 1980) and consider here again a cylindrical model of plasma (Petržílka 1993*a*). However,

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the assumption that the collision frequency is much smaller than the wave frequency is not used here, which is important for the numerical estimates of RF-induced transport in experiments, namely in the BASIL (Schneider *et al.* 1993) and SHEILA (Blackwell *et al.* 1989) devices of the Australian National University. The results are relevant for helicon wave plasma sources and also in other cases when helicon waves are excited (Chen 1991, 1992; Loewenhardt *et al.* 1991; Boswell 1984).

2. Wave-induced Radial Transport Velocity

We assume the presence of a quasi-steady field $\boldsymbol{E}_0$, $\boldsymbol{B}_0$ and an RF field $\boldsymbol{E}$, $\boldsymbol{B}$. The plasma consists of singly charged ions, electrons and possibly neutral particles. Time-averaged (over RF oscillations) distribution functions of charged particles, but not necessarily their derivatives with respect to particle velocities, do not differ strongly from Maxwellian distributions. The energy of the oscillating motion of particles is much less than their thermal energy; this condition is not necessary and may be weakened—see the papers by Petržílka *et al.* (1991) and Petržílka (1991*a*, 1991*b*). The frequency of Coulomb collisions of electrons with ions is much less than the electron cyclotron frequency. Ionisation and recombination processes do not effect the oscillating motions. Under these assumptions, the quasi-steady plasma state is governed by time-averaged hydrodynamic equations, which were derived from the averaged kinetic equation (Klíma 1980). These equations have been used for a study of effects of RF fields on diffusion and convection in a plasma column by Klíma and Petržílka (1980), assuming that all time-averaged quantities are constant along the z-axis, which coincides with the plasma column axis; that the magnetic surfaces of the magnetostatic field are concentric cylinders; and that values of the electron and ion temperatures and plasma density do not depend strongly on the poloidal angle; these assumptions are used also in this paper. Then, the radial component of the time-averaged plasma mass velocity $\boldsymbol{V}$ is given by the following equation, where all quantities are averaged over the magnetic surface [see equation (5.2) of Klíma and Petržílka (1980)]:

$$V_{\mathrm{r}} = -\frac{B_{0\theta}}{B_0}E_{\mathrm{d}} + \frac{1}{eB_0n_0}\left(\frac{B_{0\mathrm{z}}}{B_0}F_{\mathrm{i}\theta} - \frac{B_{0\theta}}{B_0}F_{\mathrm{iz}} + \frac{3n_0}{2\omega_{\mathrm{ce}}\tau_{\mathrm{e}}}\frac{\partial T_{\mathrm{e}}}{\partial r}\right) - \frac{\eta_\perp}{B_0^2}\frac{\partial p}{\partial r}\,. \tag{1}$$

We use cylindrical coordinates r, θ, z, the symbol of averaging over the magnetic surface is omitted, $\boldsymbol{B}_0$ is the magnetostatic field, n_0 the time-averaged plasma density, E_{d} is the toroidal component (equal to zero for SHEILA and BASIL) of the electrostatic field, $\boldsymbol{F}_{\mathrm{i(e)}}$ is the force acting on ions (electrons) given by the RF field, by collisional friction with neutral particles and by ionisation and recombination [see equation (2.5) of (Klíma and Petržílka (1980)], $\omega_{\mathrm{ce(i)}} > 0$ is the electron (ion) cyclotron frequency, $\nu_{\mathrm{e(i)}} = \tau_{\mathrm{e(i)}}^{-1}$ is the electron (ion) collision frequency, T_{e} is the electron temperature, $\eta_{\perp(\parallel)}$ is the perpendicular (parallel) resistivity and p is the plasma pressure. Alternatively, the coordinate system with unit vectors $\boldsymbol{e}_{\mathrm{r}}, \boldsymbol{e}_\chi = \boldsymbol{e}_\parallel \times \boldsymbol{e}_{\mathrm{r}}, \boldsymbol{e}_\parallel = \boldsymbol{B}_0/B_0$ will also be used.

We note that the terms proportional to $\partial T_{\mathrm{e}}/\partial r$ and $\partial p/\partial r$ in equation (1) are well known ones corresponding to the Braginskii (1963) thermoforce and to the classical diffusion respectively. We will not present estimates of these terms in this paper. Only the velocity of the classical diffusion will be given in some of the figures, using simplifying assumptions for a better understanding of the real meaning of the computed RF wave-induced transport velocities for experiments.

As we study a stationary state, the forces $\boldsymbol{F}_{\mathrm{i}}$ and $\boldsymbol{F}_{\mathrm{e}}$ fulfill the equation

$$F_{\mathrm{i}\|} - \boldsymbol{e}_{\|} \cdot (\nabla \cdot \boldsymbol{P}_{\mathrm{i}}^{\mathrm{T}}) = \boldsymbol{e}_{\|} \cdot (\nabla \cdot \boldsymbol{P}_{\mathrm{e}}^{\mathrm{T}}) - F_{\mathrm{e}\|}\,, \tag{2}$$

where

$$\boldsymbol{P}_{\alpha}^{\mathrm{T}} = \boldsymbol{P}_{\alpha} + m_{\alpha} n_{\alpha} \boldsymbol{U}_{\alpha} \boldsymbol{U}_{\alpha}\,, \tag{3}$$

$\alpha = \mathrm{e,i}$, the superscript T means total, $\boldsymbol{P}_{\alpha}$ is essentially the thermal pressure tensor and $n_0 \boldsymbol{U}_{\alpha}$ is the time-averaged particle flux density.

The ponderomotive forces $\boldsymbol{F}_{\mathrm{i(e)}}^{\mathrm{P}}$, which are parts of the forces $\boldsymbol{F}_{\mathrm{i(e)}}$, are given by equations (3.5) and (3.6) of Klíma and Petržílka (1980),

$$F_{\alpha,z}^{\mathrm{P}} = \sum \frac{k_z}{\omega} A_{\alpha}(r,m,k_z) - \tfrac{1}{2}\mathrm{Re}\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r j_{\alpha,\mathrm{r}}\left(\frac{i}{\omega}E_{\mathrm{z}}^{*} + \frac{j_{\alpha,z}^{*}}{\epsilon_0 \omega_{\mathrm{p}\alpha}^2}\right)\right]\right\}, \tag{4}$$

$$F_{\alpha,\theta}^{\mathrm{P}} = \sum \frac{m}{r\omega} A_{\alpha}(r,m,k_z) - \frac{1}{2r^2}\mathrm{Re}\left\{\frac{\partial}{\partial r}\left[r^2 j_{\alpha,\mathrm{r}}\left(\frac{i}{\omega}E_{\theta}^{*} + \frac{j_{\alpha,\theta}^{*}}{\epsilon_0 \omega_{\mathrm{p}\alpha}^2}\right)\right]\right\}, \tag{5}$$

where $\alpha = \mathrm{i,e}$, symbols denoting averaging over the polar angle θ are omitted, A_{α} is the RF power density absorbed by the given particle type from the m, k_z mode, $\omega_{\mathrm{p}\alpha}$ is the Langmuir frequency, $j_{\alpha,\mathrm{i}}$ are the oscillating currents and E_{i} denotes the oscillating electric field of the wave.

In the following, we consider the case of wave propagation governed by the cold plasma dielectric tensor (Ginzburg 1960); we *do not* assume that the electron–ion collision frequency ν is smaller than the wave angular frequency ω. An eventual Landau damping may be represented by an effective collision frequency ν_{LD} (Chen 1991).

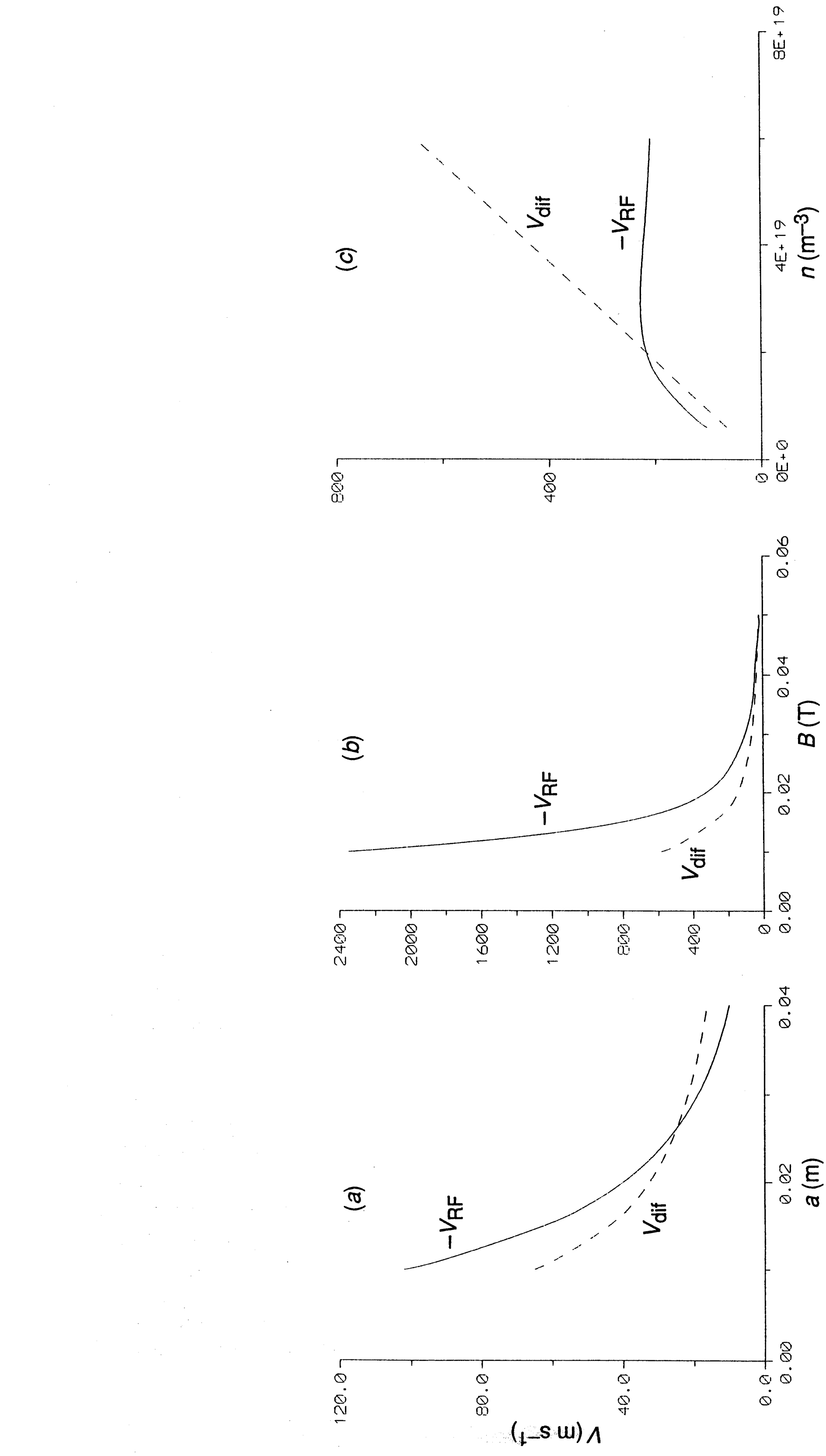
We further note that when

$$F_{\mathrm{i}\|} \simeq -F_{\mathrm{e}\|}\,, \tag{6}$$

which may easily be the case [cf. equation (5.8) in Klíma and Petržílka (1980)], we may express V_{r} in terms of $F_{\mathrm{e}\|}$ from equation (1) simply by substitution according to equation (6). Again, symbols of averaging over the polar angle θ are omitted in (6). In case the momentum imparted to the plasma by the RF field is balanced by ions, e.g., by collisions of neutrals with ions, the computation of V_{r} by means of $F_{\mathrm{e}\|}$ may be more straightforward. In this case, to evaluate the RF field-induced radial transport velocity as given by equation (1), we have to evaluate the ponderomotive forces in equations (4) and (5). For this purpose, currents $j_{\mathrm{e},\beta}$ ($\beta = r, \theta, z$) are expressed in terms of oscillating electric fields for an arbitrary value of the collision frequency with respect to the wave frequency (see the Appendix).

3. Computational Results

The part of the radial transport velocity in equation (1) induced by RF fields has been computed for various sets of parameters, assuming homogeneous plasma


(a)
$-V_{RF}$
V_{dif}
V (m s^{-1})
120.0
80.0
40.0
0.0
0.00
0.02
0.04
a (m)
(b)
$-V_{RF}$
V_{dif}
2400
2000
1600
1200
800
400
0
0.00
0.02
0.04
0.06
B (T)
(c)
V_{dif}
$-V_{RF}$
800
400
0
0E+0
4E+19
8E+19
n (m^{-3})

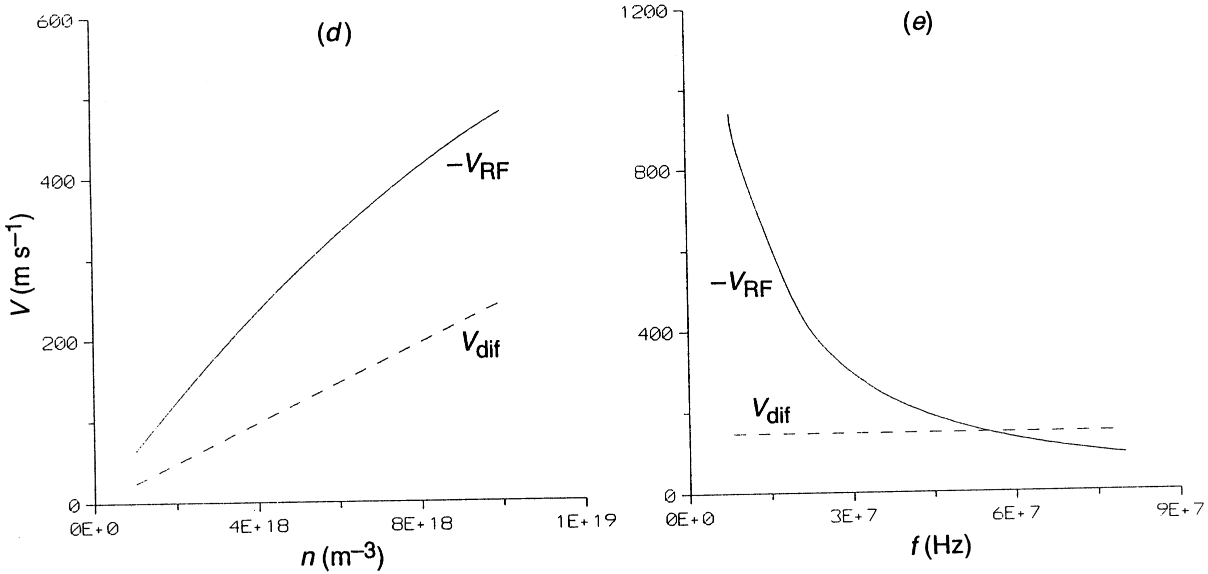
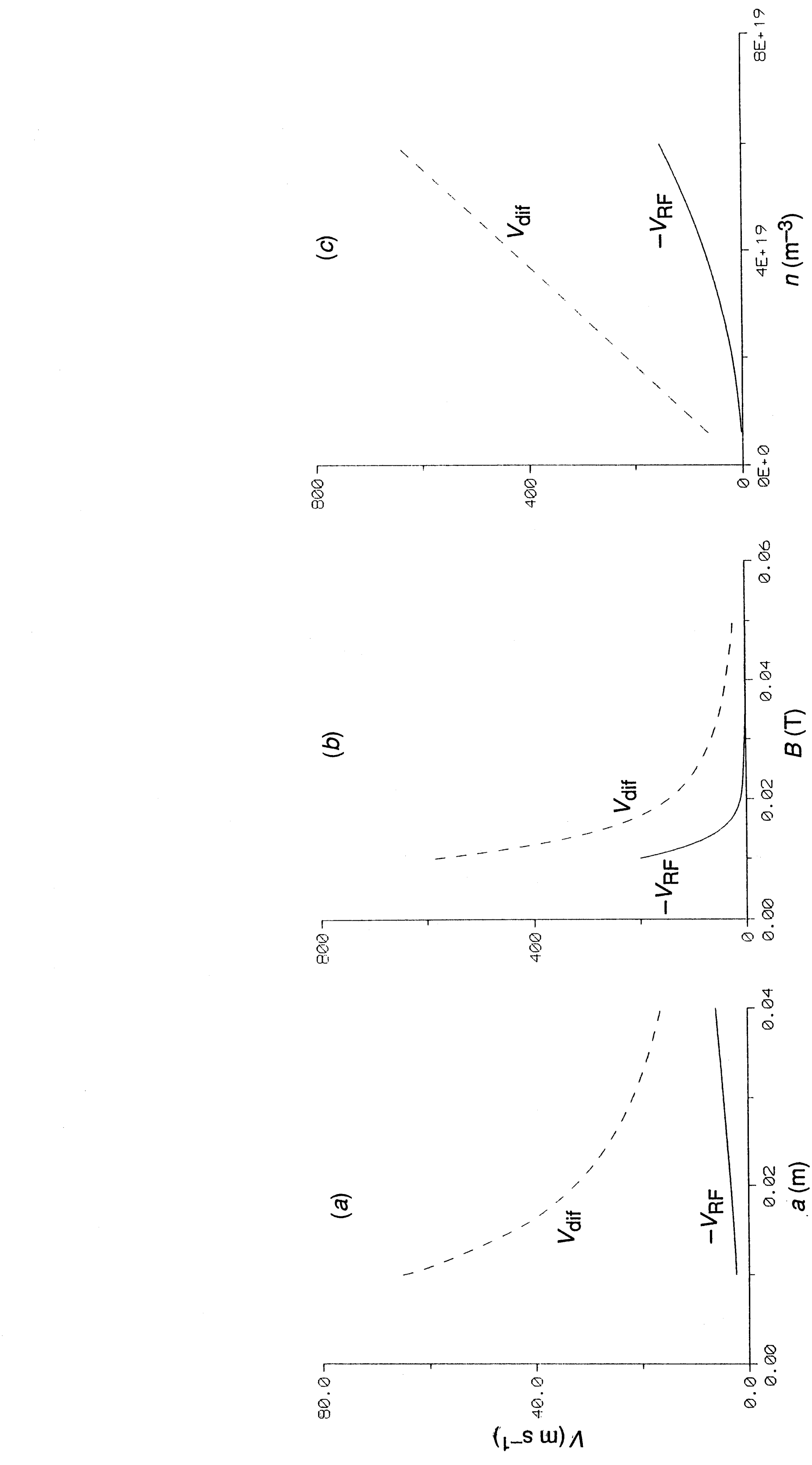


Fig. 1. Variations in the helicon $m = +1$ wave-induced radial transport velocity for a plasma, where $a = 0 \cdot 01$ m, $n = 6 \times 10^{18}$ m^{-3}, $f = 27$ MHz, $T = 3$ eV, $E_{\max} = 2$ kV m^{-1}, $Z = 1$ and $B_0 = B_z = 0 \cdot 02$ or $0 \cdot 03$ T: (*a*) Variation of the plasma radius a of the whole plasma column (experimental device); (*b*) variation of the magnetostatic field B_0; (*c*) variation of the plasma density n (for $B_0 = B_z = 0 \cdot 03$ T); (*d*) variation in the plasma density n (for $B_0 = B_z = 0 \cdot 02$ T); and (*e*) variation in the wave frequency f.


(a)
(b)
(c)
V (m s⁻¹)
80.0
40.0
0.0
0.00
0.02
0.04
a (m)
800
400
0
0.06
B (T)
0E+0
4E+19
8E+19
n (m⁻³)
Vdif
−VRF

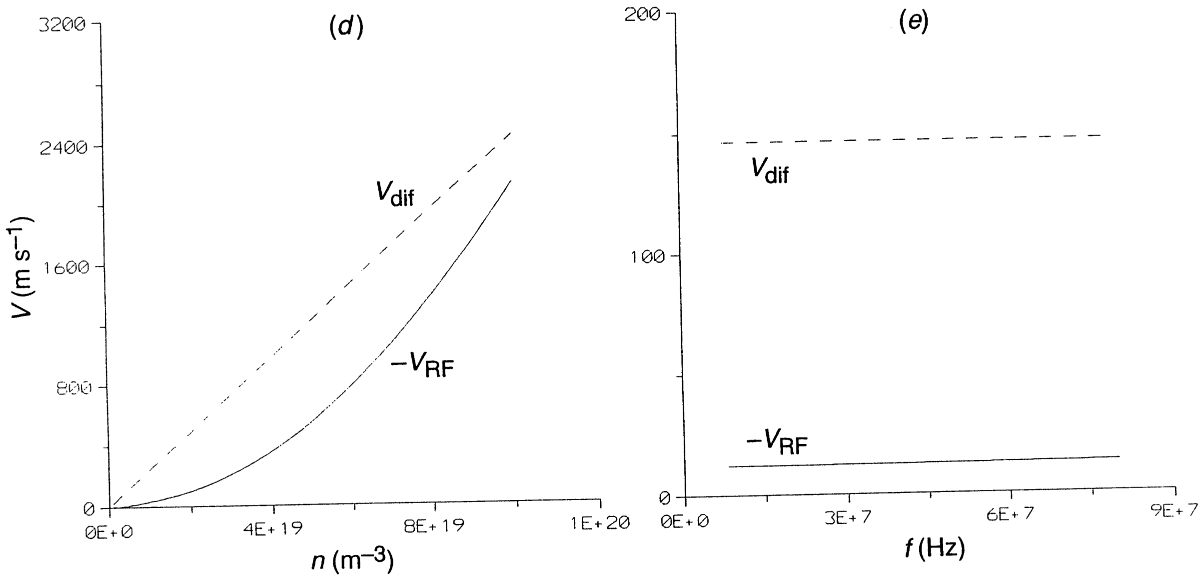


Fig. 2. Variations in the helicon $m = 0$ wave-induced radial transport velocity for a plasma. The plasma parameters are the same as in Fig. 1: (a) variation of the plasma radius a of the whole plasma column (experimental device); (b) variation of the magnetostatic field B_0; (c) variation of the plasma density n (for $B_0 = B_z = 0{\cdot}03$ T); (d) variation in the plasma density n (for $B_0 = B_z = 0{\cdot}02$ T); and (e) variation in the wave frequency f.

density and temperature profiles $n = \text{const.}$ and $T = \text{const.}$ The corresponding curves are in Figs 1–4. The radial profile of the electric fields has been taken according to Chen (1991), where a homogeneous plasma density profile is assumed.

Our task is to estimate the RF induced transport velocity, so that the assumption $T = \text{const.}$ means that we neglect gradients of the collision frequency and corresponding changes in the wave dissipation along the plasma column radius. The plasma density gradient neglected in the computations could change the wave profiles used, according to Chen (1991), in the computations. According to our estimates, in which we used the measured plasma temperature, density and wave field profiles (G. Borg, D. Schneider and B. Zhang, personal communication, 1993), both these effects should not change the main conclusions of the paper concerning the relative value and the direction of the RF induced transport velocity.

It is further assumed that the influence of mutual collisions between electrons and neutral particles may be neglected in comparison to the effect of electron–ion collisions. The degree of ionisation in SHEILA and BASIL experiments is usually high enough that this condition is not violated. The effective collision frequency ν_{LD} representing the Landau damping was put equal to zero for simplicity. This condition could possibly be violated for the case of plasmas with lower densities, where the wave dissipation would be thus higher than that assumed here. This would result in higher absolute values of the RF induced transport velocity.

Results of the computations presented in what follows indicate that the RF induced plasma diffusion may compete with the experimentally estimated plasma diffusion velocity (Boswell *et al.* 1982).

For the azimuthal wave number $m = +1$, estimates of the helicon wave-induced radial transport velocity V_{RF} in hydrogen are presented in Fig. 1. The value of V_{RF} is taken always at half the plasma radius $r = 0{\cdot}5a$. This gives only first-order information on the value of V_{RF}: the RF induced transport velocity profiles are given in Fig. 3. For $m = +1$ helicons, the velocity V_{RF} is negative with the exception of a very small region near the plasma axis, cf. the velocity profile in Fig. 3a ($a = 1$ cm, the other parameters are the same as in Fig. 1). The opposite may be said for $m = -1$ helicons, cf. the velocity profile in Fig. 3c.

For a better orientation as far the value of V_{RF} is concerned, the transport velocity V_{dif} corresponding to the classical diffusion (assuming a linear ramp plasma density profile and $T = \text{const.}$) is also depicted (dashed curves). As the diffusion transport velocity is outward directed and therefore positive, the inward directed (negative) RF field induced transport velocity is denoted in Figs 1 and 2 by $-V_{\mathrm{RF}}$.

In Fig. 1a, the plasma radius a of the whole plasma column (of the experimental device) is varied from 1 to 4 cm, the other parameters being fixed: the magnetostatic field $B_0 = B_z = 0{\cdot}03$ T, the helicon wave frequency $f = 27$ MHz, the electron temperature $T = 3$ eV, the maximum helicon wave electric field E_{r} amplitude is $E_{\max} = 2\,\mathrm{kV\,m^{-1}}$, the ion charge $Z = 1$ (hydrogen) and the plasma density $n = 6 \times 10^{18}$ m^{-3} is assumed to be constant across the plasma cylinder. For singly ionised heavier gases, like argon, the RF induced transport velocity is almost the same for sets of parameters studied here. For doubly ionised gases, we have a higher Z and therefore a higher collision frequency, and the RF induced

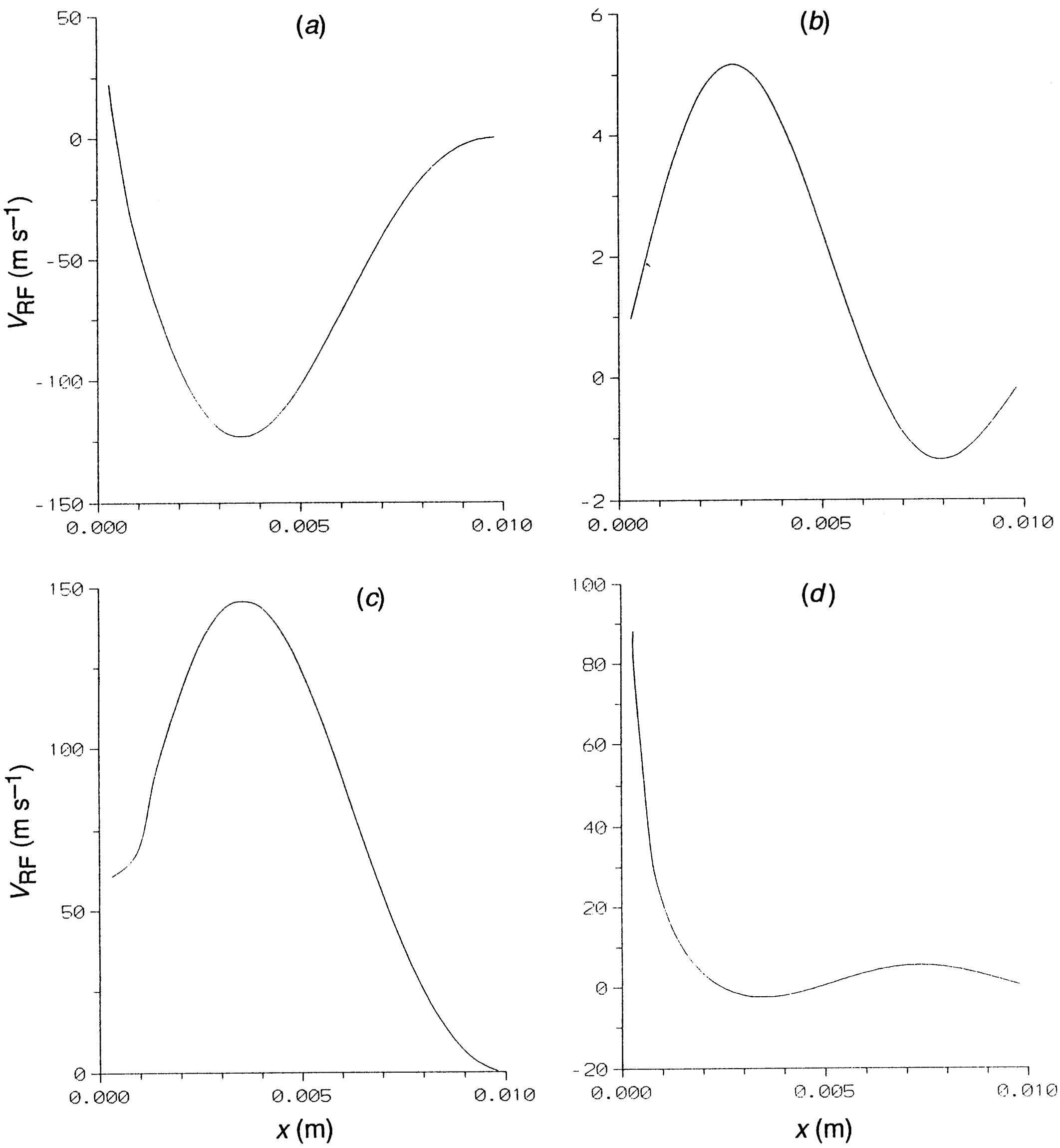


Fig. 3. Radial profile of the RF induced transport velocity where x is the plasma column radius: (a) $m = +1$ helicons; (b) $m = 0$ helicons; (c) $m = -1$ helicons; and (d) $m = +1$ and -1 helicons.

radial transport velocity V_{RF} has less opportunity of competing with the velocity of the classical diffusion.

In Fig. 1b, the magnetostatic field is varied, $0 \cdot 01 < B_0 < 0 \cdot 05$ T; it can be seen that for smaller magnetostatic fields the RF field-induced diffusion may more easily prevail over the classical one.

In Figs 1c and 1d, the plasma density n is varied. We see that for lower densities, V_{RF} may more easily reach greater values than V_{dif}. Fig. 1e shows that V_{RF} may be of importance for lower helicon wave frequencies f.

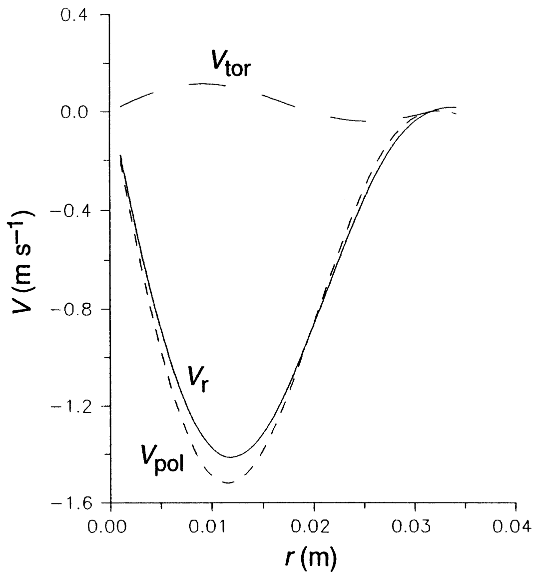


Fig. 4. Transport velocities V_{tor} and V_{pol} driven by the toroidal and poloidal components of the ponderomotive force respectively. The plasma parameters are $B_0 = 0{\cdot}1$ T, $B_{0z} = 0{\cdot}08$ T, $B_{0\theta} = 0{\cdot}05$ T, $E_r = 2000$ V m^{-1}, $m = +1$ and $n_{0e} = 1{\times}10^{19}$ m^{-3}.

Results in Fig. 1 for $m = +1$ helicons imply that the RF induced transport velocity is negative, directed inwards the plasma column, and that it is higher for lower magnetostatic fields, lower wave frequencies, lower plasma densities and in devices with lower radii a. In other words, the effects of $m = +1$ helicons on plasma confinement are beneficial.

The $m = -1$ helicons induce an RF transport velocity of approximately the same absolute value but of opposite sign, so that the prevailing sign of the RF induced transport velocity for these helicons is positive, cf. Figs. 3a and 3c for the velocity profiles.

For $m = 0$ helicons, the dependence of V_{RF} on various parameters is depicted in Fig. 2, analogously to Fig. 1. We see that V_{RF} for $m = 0$ helicons is smaller in its absolute value than V_{RF} induced by $m = +1$ or by $m = -1$ helicons.

The radial profile of V_{RF} for $m = +1$ helicons is depicted in Fig. 3a. Again, similar to Fig. 3b for an $m = 0$ helicon, in Fig. 3c for an $m = -1$ helicon and in Fig. 3d for both $m = +1$ and $m = -1$ helicons excited, the plasma radius is $a = 1$ cm, the other parameters being the same as in Fig. 1.

The influence of the poloidal magnetostatic field B_θ on the RF induced transport is illustrated in Fig. 4. The part of the RF induced transport velocity connected with the poloidal component of the ponderomotive force, equation (1), is denoted by V_{pol}; the part connected with the toroidal component of the ponderomotive force we denote by V_{tor}. This velocity V_{tor} is nonzero due to the presence of the poloidal magnetostatic field. The RF induced transport velocity V_{RF} is denoted simply as V_r in Fig. 4; of course, $V_r = V_{RF} = V_{pol} + V_{tor}$. We see that the velocity V_{pol} is the dominant one for the input data considered in Fig. 4.

4. Discussion and Conclusion

As the first term in the poloidal component of the ponderomotive force given by equation (5) is proportional to the azimuthal wavenumber m, the RF induced transport velocity is different for various values and signs of m.

It has been shown that $m = +1$ helicons contribute to plasma confinement positively almost in the whole plasma volume with the exception of a very small region near the plasma cylinder axis. The opposite may be said as far as $m = -1$ helicons are concerned. Helicons with an azimuthal wave number $m = 0$ induce a much smaller $V_{\rm RF}$, this velocity being positive in the plasma interior and negative near the boundary, where the effects are beneficial for confinement. These helicons may support the creation of hollow discharges. When both $m = +1$ and $m = -1$ helicons are excited together, the confinement should deteriorate in the plasma interior; again, hollow discharges might perhaps be created or their creation supported.

The beneficial influence of the $m = +1$ helicons found here is in accordance with what has been found in experiments by Sato *et al.* (1983), Okamura *et al.* (1986) and Blackwell *et al.* (1993). An alternative explanation was presented in terms of plasma density inhomogeneity effects on the wave propagation by Chen and Light (1993); however, this theory encountered difficulties in measurements of helicon wave field patterns (Light and Chen 1993).

According to our results presented in Figs 1–4, magnetostatic field values for which the RF induced transport may compete more easily with the outward directed diffusion transport velocity and therefore be observed more easily are the lower ones used in the BASIL and SHEILA experiments at the Australian National University in Canberra, assuming that the maximum helicon wave electric field amplitude is in the range 1–2 $\mathrm{kV\,m^{-1}}$.

We note that due to the simplifying assumption of $n =$ const. and $T =$ const. in the computations of $V_{\rm RF}$, the profiles of the RF induced transport velocity should be considered as a first approximation only. The important result of this work is that sets of parameters have been found where the RF induced transport may be important.

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Appendix

The electric current density $\boldsymbol{j}_\mathrm{e}$ is expressed by means of the oscillating electric field $\boldsymbol{E}$,

$$j_{\mathrm{e},i} = \sigma_{ik} E_k \,, \tag{A1}$$

where

$$\sigma_{xx} = h_1(h_2 b + h_3 a)\,, \qquad \sigma_{xy} = h_1(h_3 b - h_2 a)\,,$$

$$\sigma_{zz} = i\epsilon_0 \omega \omega_{\mathrm{pe}}^2 \Big/ \omega^2 \left(1 - \frac{\nu + \nu_{\mathrm{LD}}}{i\omega}\right), \tag{A2}$$

and where

$$h_1 = \frac{\epsilon_0 \omega_{\mathrm{pe}}^2}{a^2 + b^2}\,,$$

$$h_2 = x_3 x_4 \omega_{\rm ci}\,, \qquad h_3 = 1 + i x_3 x_4 \omega_\nu\,,$$

$$a = -i\omega + x_1 \nu + i x_2 x_4 \nu \omega_\nu\,, \qquad b = -\omega_{\rm ce} + x_2 x_4 \nu \omega_\nu\,,$$

$$x_1 = 1840A/(1+1840A)\,, \qquad x_2 = Z x_1^2/(1840A)\,,$$

$$x_3 = Z/(1+1840A)\,, \qquad x_4 = \nu/(\omega_{\rm ci}^2 - \omega_\nu^2)\,,$$

$$\omega_{\rm pe} = (ne^2/m_{\rm e}\epsilon_0)^{0\cdot5}\,, \qquad \omega_{\rm pi} = \omega_{\rm pe}(Z/1840A)^{0\cdot5}\,,$$

$$\omega_{\rm ce} = eB_0/m_{\rm e}\,, \qquad \omega_{\rm ci} = Z\omega_{\rm ce}/1840A\,, \tag{A3}$$

and further, Z is the ion charge, A is the atomic number, $m_{\rm e}$ is the electron mass and $e > 0$ is the absolute value of the electron charge.

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