Aust. J. Phys., 1994, **47**, 245–52

An Experiment to Test the Gravitational Aharonov–Bohm Effect

Vu B. Ho and Michael J. Morgan

Department of Physics, Monash University,
Clayton, Vic. 3168, Australia.

Abstract

The gravitational Aharonov–Bohm (AB) effect is examined in the weak-field approximation to general relativity. In analogy with the electromagnetic AB effect, we find that a gravitoelectromagnetic 4-vector potential gives rise to interference effects. A matter wave interferometry experiment, based on a modification of the gravity-induced quantum interference experiment of Colella, Overhauser and Werner (COW), is proposed to explicitly test the gravitoelectric version of the AB effect in a uniform gravitational field.

1. Introduction

The electromagnetic Aharonov–Bohm (AB) effect (Aharonov and Bohm 1959) has been widely studied theoretically (see e.g. Peshkin and Tonomura 1989), and confirmed experimentally, most recently in the elegant experiments of Tonomura *et al.* (1983, 1986). The AB effect represents a global anholonomy associated with the electromagnetic gauge potentials; it is one example among a plethora of general phenomena known as topological phases (see e.g. Shapere and Wilczek 1989). The gravitational analogue of the electromagnetic AB effect has also received attention in the literature (Ford and Vilenkin 1981; Bezerra 1990, 1991). When particles are constrained to move in a region where the Riemann curvature tensor vanishes, a gravitational AB effect arises due to the global influence of a region of nonzero curvature from which the particles are excluded. The gravitational AB effect viewed in this way is a manifestation of the non-trivial topology of spacetime. It is also known that the gravitational field of a rotating mass distribution gives rise to effects, such as the Sagnac effect, that are analogous to the AB effect (Ashtekar and Magnon 1975; Cohen and Mashhoon 1993). More recently, gravitational analogues of the AB effect have been studied in static cylindrically symmetric cosmic string models (Aliev and Gal'tsov 1989) and in the Safko–Witten (1972) model of spacetime, wherein a tubular matter source is established with an axial interior magnetic field and vanishing exterior field.

In this paper the equivalence principle is invoked to establish a simple realisation of the gravitational AB effect. Section 2 discusses the gravitational AB effect by considering the quantum-mechanical behaviour of a point particle in a weak homogeneous gravitational field. In Section 3 we propose several interferometry experiments utilising matter waves to explicitly test the electric version of the gravitational AB effect. The most promising possibility is to exploit neutral atom

0004-9506/94/030245$05.00

interferometry in a modification of the classic experiment of Colella, Overhauser and Werner (COW) (1975).

2. Gravitational Aharonov–Bohm Effect

The motion of a particle of mass m in a gravitational field is governed by the geodesic equation

$$\frac{\mathrm{d}^2 x^\mu}{\mathrm{d}\tau^2} + \Gamma^\mu{}_{\alpha\beta}\,\frac{\mathrm{d}x^\alpha}{\mathrm{d}\tau}\,\frac{\mathrm{d}x^\beta}{\mathrm{d}\tau} = 0\,, \tag{1}$$

where $x^\mu = x^\mu(\tau)$ is the world line of the particle in spacetime, and $\Gamma^\mu{}_{\alpha\beta}$ are the components of the affine connection defined by

$$\Gamma^\mu{}_{\alpha\beta} = \tfrac{1}{2} g^{\mu\lambda}(\partial_\alpha\, g_{\lambda\beta} + \partial_\beta\, g_{\alpha\lambda} - \partial_\lambda\, g_{\alpha\beta})\,. \tag{2}$$

Here Greek indices take on the values $0, 1, 2, 3$, and are used to denote the spacetime components (ct, x, y, z), while latin indices $(i, j = 1, 2, 3)$ will be used to refer to spatial components only. The Minkowski metric $\eta_{\alpha\beta} = \mathrm{diag}(-1, +1, +1, +1)$ is used throughout. Indices are raised and lowered with $\eta^{\alpha\beta}$ and $\eta_{\alpha\beta}$ respectively. We will assume that the gravitational field is weak ($|\,\Delta\Phi\,|/c^2 \ll 1$) and that particles move with velocities that are small compared to that of light ($v^2/c^2 \ll 1$), for which the magnitudes of the components of the energy–momentum tensor are limited by $\rho_\mathrm{m}\, c^2 = |\,T^{00}\,| \gg |\,T^{i0}\,| \gg |\,T^{ij}\,|$. In the weak-field approximation the metric tensor $g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}$ deviates only slightly from a flat Minkowski metric $\eta_{\alpha\beta}$, so that $|\,h_{\alpha\beta}\,| \ll 1$. With these assumptions the linearised field equations of general relativity assume a form similar to Maxwell's equations (Forward 1961; Braginsky *et al.* 1977; Ross 1983; Li and Torr 1991), and the motion of a particle moving under the influence of a time-dependent gravitoelectromagnetic field is identical in form to that of a charged particle in an electromagnetic field. Writing the components of $h_{\alpha\beta}$ as $h_{00} = -\Phi/c^2$, $h_{ij} = -\delta_{ij}\,\Phi/c^2$ and $h_{0j} = h_{j0} = A_j/c$, we can define

$$2\boldsymbol{g} = -\,\nabla\Phi - \frac{\partial \boldsymbol{A}}{\partial t}\,, \tag{3}$$

$$\boldsymbol{H} = \nabla \times \boldsymbol{A}\,, \tag{4}$$

where $\boldsymbol{g}$ is referred to as the gravitoelectric field and $\boldsymbol{H}$ is the gravitomagnetic field. A dimensional analysis (using SI units) reveals that $\boldsymbol{g}$ has the dimensions of acceleration, $[\mathrm{L}][\mathrm{T}]^{-2}$. The gravitomagnetic field has the dimensions of angular velocity, $[\mathrm{T}]^{-1}$; consequently, it can be rewritten in the alternative form $\boldsymbol{H} = (c/\hbar)\boldsymbol{\mathcal{P}}$, where $|\,\boldsymbol{\mathcal{P}}\,| = (\hbar/c)\omega$ has the dimensions of linear momentum. This latter expression defines the relationship for the momentum of a particle of zero rest mass, as expected for the exchange quanta of the long-range gravitoelectromagnetic field. The gravitoelectromagnetic Maxwell equations can be simplified further by utilising the Fock–de Donder gauge condition, $\partial_\alpha\, h^\alpha{}_\beta - \frac{1}{2}\partial_\beta\, h^\mu{}_\mu = 0$. In this gauge $\boldsymbol{H}$ is constant, which results in the homogeneous Maxwell equation: $\nabla \times \boldsymbol{g} = 0$. For later convenience we define a gravitoelectromagnetic 4-vector potential $\Phi^\alpha = (\Phi/c, \boldsymbol{A})$. Neglecting terms of order v^2/c^2 in equation (1), the nonrelativistic equation of motion reduces to the Lorentz-type expression (Harris 1991)

$$m\,\frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = m(\boldsymbol{g} + \boldsymbol{v} \times \boldsymbol{H})\,. \tag{5}$$

In order to describe the quantum-mechanical behaviour of a particle moving in a gravitational field it is necessary to use the 4-vector potential, rather than the gravitoelectromagnetic fields. The gravitoelectromagnetic potentials have a physical significance, and can affect the quantum-mechanical behaviour of a particle even when it is constrained to regions of spacetime where the Riemann curvature vanishes. The relativistic Lagrangian of a test particle of mass m in a gravitoelectromagnetic field is

$$L = -mc^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}} + \frac{mc}{\hbar}\,\boldsymbol{v}\,.\,\boldsymbol{\mathcal{L}} - m\Phi\,, \tag{6}$$

where $\boldsymbol{\mathcal{L}}$ is a quantity having dimensions of angular momentum, defined by $\boldsymbol{\mathcal{L}} = (\hbar/c)\boldsymbol{A}$ so that $\boldsymbol{\mathcal{P}} = \nabla \times \boldsymbol{\mathcal{L}}$. In the nonrelativistic limit the Lagrangian of the system reduces to

$$L = \tfrac{1}{2}mv^2 - m\left(\Phi - \frac{c}{\hbar}\,\boldsymbol{v}\,.\,\boldsymbol{\mathcal{L}}\right). \tag{7}$$

The corresponding Hamiltonian has the form identical to that of a charged particle moving in an external electromagnetic field:

$$H = \frac{1}{2m}\left(\boldsymbol{\pi} - \frac{mc}{\hbar}\,\boldsymbol{\mathcal{L}}\right)^2 + m\Phi\,, \tag{8}$$

where the canonical momentum $\boldsymbol{\pi}$ is defined via the Lagrangian (7) as

$$\pi_j = \frac{\partial L}{\partial v_j} = mv_j + \frac{mc}{\hbar}\,\mathcal{L}_j\,. \tag{9}$$

To implement the quantisation ansatz we introduce the gauge-covariant derivative D_α defined by

$$D_\alpha = \partial_\alpha - \frac{\mathrm{i}\,m}{\hbar}\,\Phi_\alpha\,. \tag{10}$$

This is equivalent to the minimal-coupling procedure in quantum mechanics. Using minimal coupling (10) we write the nonrelativistic Schrödinger equation for a particle of rest mass m as

$$-\frac{\hbar^2}{2m}\,D_j\,D^j\,\Psi(\boldsymbol{x},t) = \mathrm{i}\,\hbar D_0\,\Psi(\boldsymbol{x},t)\,. \tag{11}$$

The solution to this equation is of the form

$$\Psi(\boldsymbol{x},t)=\Psi_0(\boldsymbol{x},t)\,\exp\left(\frac{\mathrm{i}}{\hbar}\,S(\Gamma)\right), \tag{12}$$

where $\Psi_0(\boldsymbol{x},t)$ is the solution to the Schrödinger equation in the absence of a gravitoelectromagnetic 4-vector potential and $S(\Gamma)$ is the line integral evaluated over a path Γ with endpoints $(\boldsymbol{x}_1,t_1)$ and $(\boldsymbol{x}_2,t_2)$:

$$S(\Gamma)=m\int_\Gamma \Phi_\alpha\,\mathrm{d}x^\alpha\,. \tag{13}$$

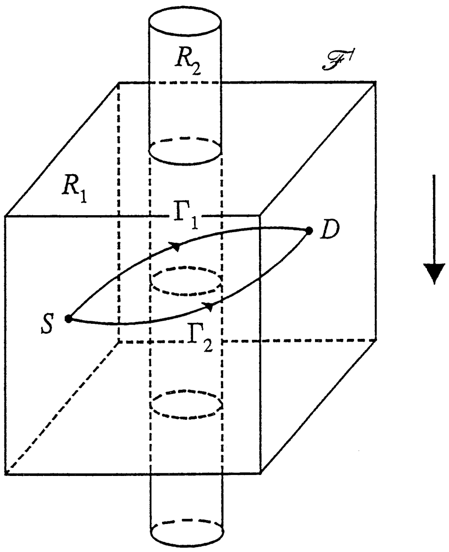

Fig. 1. A hypothetical interference experiment carried out in a freely falling (indicated by arrow), non-rotating reference frame $\mathcal{F}'$. Here S denotes a beam of nonrelativistic coherent particles that is split into two parts which travel over the paths Γ_1 and Γ_2 around the cylindrical region R_2. The particles are prevented from entering region R_2. When the particles are brought together an interference pattern is produced at D. In the freely falling reference frame $\mathcal{F}'$ the region R_1 is not simply connected. An interference experiment performed in this frame will produce a phase shift, $\Delta\theta$, relative to a frame at rest in the uniform gravitational field; this phase shift originates from a gravitoelectromagnetic AB effect as discussed in the text.

To understand how a gravitational AB effect can arise, consider a hypothetical interference experiment carried out in a homogeneous gravitational field. A reference frame $\mathcal{F}$ is at rest in this gravitational field. The region inside this reference frame is divided into two sub-regions, denoted by R_1 and R_2, wherein the gravitoelectromagnetic field is nonzero. An interference experiment is performed in $\mathcal{F}$ using a beam of nonrelativistic coherent particles that is split into two parts which travel over paths Γ_1 and Γ_2 around the region R_2, from which the particles are excluded. When the particles are brought together an interference pattern is observed. The reference frame $\mathcal{F}$ is now allowed to fall freely, without rotation, in the uniform gravitational field; this freely falling frame is denoted by $\mathcal{F}'$ (see Fig. 1). The arrow in Fig. 1 indicates a geodesic along which the reference frame $\mathcal{F}'$ is freely falling. In the region R_1 the particles are co-moving with the frame $\mathcal{F}'$. In this situation the equivalence principle (Ohanian 1977) may be invoked, and it is possible to transform to local geodesic coordinates (Anderson and Gautreau 1969) in which the results of the experiment are independent of the gravitational field surrounding the system. The freely falling frame is locally inertial and the connection coefficients vanish in this frame. The gravitational field inside the region R_2 is nonzero, and in the freely falling reference frame the region R_1 is multiply connected. An interference experiment that is performed in the freely falling frame $\mathcal{F}'$ will produce an observable shift in the interference pattern relative to a frame at rest in the uniform gravitational field; this phase shift arises through a gravitoelectromagnetic AB effect.

The fringe shift is calculated from the solution to the Schrödinger equation (12), which gives the phase difference between the two paths as

$$\begin{aligned}\Delta\theta &= \frac{m}{\hbar}\int_{\Gamma_1}\Phi_\alpha \mathrm{d}x^\alpha - \frac{m}{\hbar}\int_{\Gamma_2}\Phi_\alpha \mathrm{d}x^\alpha \\ &= \frac{m}{\hbar}\oint_{\Gamma=\partial\Sigma}\Phi_\alpha\,\mathrm{d}x^\alpha\,. \end{aligned}\tag{14}$$

The gravitoelectromagnetic flux is non-vanishing inside the region R_2, and the gravitoelectromagnetic 1-form $\mathbf{\Phi} = \Phi_\alpha\,\mathrm{d}x^\alpha$ does not vanish on any closed path Γ which encompasses the region Σ. Utilising the generalised Stokes theorem we obtain

$$\Delta\theta = \frac{m}{\hbar}\oint_{\Gamma=\partial\Sigma}\Phi_\alpha\,\mathrm{d}x^\alpha = \frac{m}{\hbar}\iint_\Sigma \Phi_{\alpha,\beta}\,\mathrm{d}x^\beta\wedge\mathrm{d}x^\alpha\,. \tag{15}$$

The phase difference between the two paths depends only on the flux of the gravitoelectromagnetic field through the region Σ bounded by the closed path Γ.

To realise the gravitational analogue of the magnetic version of the electromagnetic AB effect, it is necessary to allow a test particle to fall freely in the direction of the gravitomagnetic force $m\boldsymbol{v}\times\boldsymbol{H}$. Under these circumstances we reproduce the situation depicted in Fig. 1, in which case the phase shift is given by

$$\Delta\theta = \frac{m}{\hbar}\oint_{\Gamma=\partial\Sigma} A_j\,\mathrm{d}x^j\,. \tag{16}$$

In practice a test particle will fall in the direction of the resultant force (5), and it will not be possible to realise experimentally the gravitomagnetic AB effect. However, the magnitude of the gravitomagnetostatic field of the Earth is approximately six orders of magnitude smaller than that of the gravitoelectric field (Harris 1991). If the test particles move at speeds that are small compared to that of light ($v/c \ll 1$), the magnitude of the gravitomagnetic force on the particles is smaller than that of the gravitoelectric force by a factor of $(v/c)(u_\oplus/c)$, where $u_\oplus$ is the equatorial speed of the Earth about its axis. For thermal neutrons ($v \sim 10^3\ \mathrm{m\,s^{-1}}$) this factor is $(v/c)(u_\oplus/c) \sim 10^{-11}$. Consequently, it is possible to realise the gravitational analogue of the electric version of the electromagnetic AB effect. Fig. 2 shows an idealised experimental configuration. The phase shift is calculated from

$$\Delta\theta = \frac{m}{\hbar}\oint_{\Gamma=\partial\Sigma}\Phi\,\mathrm{d}t\,. \tag{17}$$

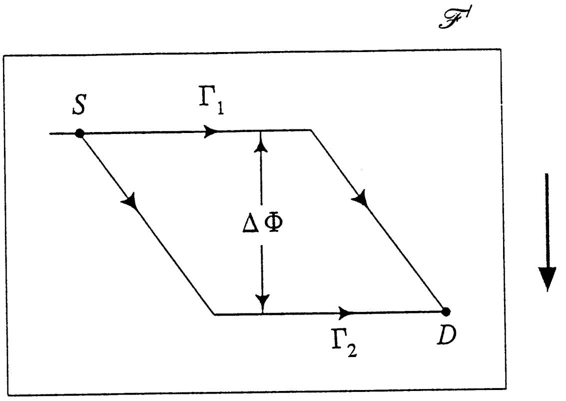

Fig. 2. Experiment to measure the gravitoelectric AB effect. An interference experiment is performed in a freely falling (indicated by arrow), non-rotating frame, $\mathcal{F}'$, near the surface of the Earth. The beam of coherent particles is split at point S, travels over the paths Γ_1 and Γ_2, and particles are detected at point D. An interference experiment performed in this frame exhibits a phase shift, $\Delta\theta \sim (m/\hbar)\Delta\Phi\tau$, relative to a frame at rest in the uniform gravitational field.

3. Matter Wave Interferometry

It should be possible to observe a gravitoelectric AB effect by performing a matter wave interferometry experiment with thermal neutrons (Greenberger 1983; Werner *et al.* 1988), electrons (Tonomura *et al.* 1989; Hasselbach and Nicklaus 1993) or neutral atoms (Carnal and Mlynek 1991; Audretsch *et al.* 1992) near the surface of the Earth, where the gravitational field is approximately uniform. We now discuss these experimental tests.

In principle the gravitoelectric AB effect could be tested by flying an interferometer in an aircraft, and recording an interference pattern under conditions in which the aircraft maintains a straight and level flight path at constant speed. The interference pattern recorded in the inertial frame is then compared with the results of an interferometry experiment carried out when the aircraft follows a parabolic flight path. A particle moving in an inertial frame in a gravitational field will experience the Lorentz force (5). In a two-slit interference experiment this results in a shift in the diffraction envelope. However, in the freely falling reference frame the particles exhibit force-free motion; nevertheless, the gravitoelectromagnetic potential is nonzero, which results in a shift of the interference pattern within the diffraction envelope (Kobe 1979). To calculate the expected phase shift we need to know the linear dimensions of the interferometer. For example, with the Laue-type neutron interferometer used by Greenberger and Overhauser (1979) the height difference of the interferometer is $0\cdot02$ m, corresponding to a potential difference of $\Delta\Phi \sim 0\cdot2\ \mathrm{J\,kg^{-1}}$, and the path length is $0\cdot1$ m. For thermal neutrons ($v \sim 10^3\ \mathrm{m\,s^{-1}}$) the traversal time is $\tau \sim 10^{-4}$ s. The phase shift calculated from equation (17) is $\Delta\theta \sim (m/\hbar)\Delta\Phi\tau \sim 100$ rad. Although this shift is large and should be amenable to measurement, laboratory neutron sources produce small count rates, so that typically only one neutron per second passes through the interferometer, resulting in poor fringe visibility. Clearly it is not a practicable suggestion to increase the neutron flux by using an airborne high-flux reactor!

The problem of low neutron flux can be circumvented by performing an interferometry experiment with electrons, where the count rate can be increased by a factor of 10^4. In this case, however, the phase shift is expected to be much smaller. For example, using the parameters ($\Delta\Phi \sim 10^{-4}\,\mathrm{J\,kg^{-1}}$ and $\tau \sim 10$ ns) reported in the double-slit electron interference experiment of Tonomura *et al.* (1989), the phase shift calculated from equation (17) is $\Delta\theta \sim 10^{-8}$ rad. It is possible to increase this small phase shift by an order of magnitude by using less energetic electrons ($E \sim 1$ keV) (see e.g. Hasselbach and Nicklaus 1993); however, it will be very difficult to increase the experimental sensitivity to approach the threshold of detection. Since the gravitoelectric AB phase shift increases with mass it should be possible to exploit atomic beam interferometry (see e.g. Audretsch *et al.* 1992), which represents a compromise between high intensity and large phase shifts. For purposes of illustration we use the parameters ($\Delta\Phi \sim 10^{-4}\,\mathrm{J\,kg^{-1}}$ and $\tau \sim 1$ ms) reported by Carnal and Mlynek (1991) in their double-slit experiment with helium atoms ($v \sim 500\,\mathrm{m\,s^{-1}}$), in which case the calculated gravitoelectric AB phase shift is $\Delta\theta \sim 6$ rad. It should be possible to increase the magnitude of this phase shift by a factor of 10^4 by using laser-cooled atomic beams (see e.g. Kasevich and Chu 1991).

In practice extraneous phase shifts will complicate the interpretation of any matter wave interferometry experiment. Perturbing forces due to tidal effects, the effects of the Earth's rotation and higher order corrections in the Earth's gravitational field (Anandan 1984) will produce additional phase shifts. For atomic beam interferometry the magnitude of these phase shifts is estimated to be 10^{-7}, $0\cdot05$ and 10^{-9} rad respectively. In addition, centrifugal and Coriolis forces can play a significant role in the proposed experiment. The effects of centrifugal acceleration can be reduced to less than $0\cdot1$ rad by flying a trajectory with a large ($\gtrsim$10 km) radius of curvature. Coriolis forces can be eliminated entirely by choosing an appropriate orientation for the particle beam. Regardless of their origin, the effect of these additional forces is to shift the interference fringes and the diffraction envelope equally, whereas the gravitoelectric AB effect shifts the interference fringes within the diffraction envelope, thereby producing an observable asymmetric fringe pattern.

The most promising possibility for detecting a gravitoelectric AB effect would be to locate an atomic beam interferometer in a near-Earth circular orbit. This would allow any effects that depend on the gravitoelectric field $\boldsymbol{g}$ to be eliminated. To be more precise the effective $\boldsymbol{g}$ is zero only at the centre of mass of the satellite, however, the linear dimensions of a typical atomic beam interferometer are sufficiently small that tidal effects on the phase shift can be neglected in comparison with the geometric phase contribution. In this environment we propose an indirect test of the gravitoelectric AB effect by carrying out a variant of the classic experiment of Colella, Overhauser and Werner (COW) (1975) with neutral atoms. In the original COW experiment a Laue-type interferometer was used to split a beam of thermal neutrons into two beams, which travel over two paths around a parallelogram in the presence of a gravitational field. Gravity-induced quantum interference, in the form of a series of maxima and minima in the neutron intensity, is observed when the interferometer is rotated about the beam of incident neutrons. In a similar manner we suggest that by rotating an atomic beam interferometer about the incident beam of atoms, a phase shift due to a gravitoelectric AB effect will be produced. It should be emphasised that in this modified COW experiment both the interferometer and the particle beam will be freely falling; consequently, unlike the gravity-induced phase measured in the conventional COW experiment, there will be an observable shift of the interference pattern within the diffraction envelope, whose magnitude for an atomic beam interferometry experiment with helium atoms will be $\Delta\theta \sim 6$ rad. Despite the numerous difficulties envisaged in doing these experiments, it may be possible to realise an empirical test of the gravitational AB effect in the near future.

Acknowledgments

The authors would like to thank a referee for his constructive comments. One of the authors (VBH) acknowledges the support of an APRA scholarship.

References

Aharonov, Y., and Bohm, D. (1959). *Phys. Rev.* **115**, 485.
Aliev, A. N., and Gal'tsov, D. V. (1989). *Ann. Phys. (NY)* **193**, 142.
Anandan, J. (1984). *Phys. Rev.* **30**, 1615.
Anderson, J. L., and Gautreau, R. (1969). *Phys. Rev.* **185**, 1656.
Ashtekar, A., and Magnon, A. (1975). *J. Math. Phys.* **16**, 34.
Audretsch, J., Hedl, F. W., and Lämmerzahl, C. (1992). *In* Lecture Notes in Physics, Vol. 410, 'Relativistic Gravity Research' (Eds J. Ehlers and G. Schäfer), p. 368 (Springer: Berlin).
Bezerra, V. B. (1990). *Ann. Phys. (NY)* **203**, 392.
Bezerra, V. B. (1991). *Class. Quantum Grav.* **8**, 1939.
Braginsky, V. B., Caves, C. M., and Thorne, K. S. (1977). *Phys. Rev.* D **15**, 2047.
Carnal, O., and Mlynek, J. (1991). *Phys. Rev. Lett.* **66**, 2689.
Cohen, J. M., and Mashhoon, B. (1993). *Phys. Lett.* A **181**, 353.
Colella, R., Overhauser, A. W., and Werner, S. A. (1975). *Phys. Rev. Lett.* **34**, 1472.
Ford, L. H., and Vilenkin, A. (1981). *J. Phys.* A **14**, 2353.
Forward, R. L. (1961). *Proc. IRE* **49**, 892.
Greenberger, D. M. (1983). *Rev. Mod. Phys.* **55**, 875.
Greenberger, D. M., and Overhauser, A. W. (1979). *Rev. Mod. Phys.* **51**, 43.
Harris, E. G. (1991). *Am. J. Phys.* **59**, 421.
Hasselbach, F., and Nicklaus, M. (1993). *Phys. Rev.* A **48**, 143.
Kasevich, M., and Chu, S. (1991). *Phys. Rev. Lett.* **67**, 181.
Kobe, D. H. (1979). *Ann. Phys. (NY)* **123**, 381.
Li, N., and Torr, D. G. (1991). *Phys. Rev.* D **43**, 457.
Ohanian, H. C. (1977). *Am. J. Phys.* **45**, 903.
Peshkin, M., and Tonomura, A. (1989). *In* Lecture Notes in Physics, Vol. 340, 'The Aharonov–Bohm Effect' (Springer: Berlin).
Ross, D. K. (1983). *J. Phys.* A **16**, 1331.
Safko, J. L., and Witten, L. (1972). *Phys. Rev.* D **5**, 293.
Shapere, A., and Wilczek, F. (Eds) (1989). 'Geometric Phases in Physics' (World Scientific: Singapore).
Tonomura, A., Endo, J., Matsuda, T., Kawasaki, T., and Ezawa, H. (1989). *Am. J. Phys.* **57**, 117.
Tonomura, A., Osakabe, N., Matsuda, T., Kawasaki, T., Endo, J., Yano, S., and Yamada, H. (1986). *Phys. Rev. Lett.* **56**, 792.
Tonomura, A., Umezaki, H., Matsuda, T., Osakabe, N., Endo, J., and Sugita, Y. (1983). *Phys. Rev. Lett.* **51**, 331.
Werner, S. A., Kaiser, H., Arif, M., and Clothier, R. (1988). *Physica* B **151**, 22.

Manuscript received 10 February, accepted 12 April 1994