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Consistent Treatment of Pion Exchange Force and Meson versus Quark Dynamics in the Nucleon–Nucleon Interaction

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Abstract

To distinguish explicit quark effects from meson exchange in the NN interaction, it is necessary to splice the long-range meson exchange forces and short-distance dynamics due to quarks. However, in most quark model studies the short-range part of the pion exchange is usually treated differently, which makes it difficult to get a uniform picture of the short-range dynamics. We make a comparison between meson exchange and quark–gluon dynamics using the same pion exchange potential based on a quark–pion coupling model. The roles of vector meson exchange and gluon exchange in the NN interaction are compared by calculating NN phase parameters. It is shown that, with this consistent one-pion exchange force, the vector meson exchange gives a better fit to the data. This suggests that non-perturbative mechanisms responsible for meson exchange may need more careful handling to supplement the usual one-gluon exchange mechanism in describing the NN interaction.

1. Introduction

The extent to which the constituent quark model can be used to describe the NN interaction is still under debate. This is manifested by the various quark-based models that provide different pictures for the short-range NN force (for recent reviews see Myhrer and Wroldsen 1988; Shimizu 1989). It is now commonly held that the short-range NN force is dominated by one-gluon exchange (OGE) between quarks, which, when coupled with the requirement of the Pauli principle, yields the well known short-range repulsion. While this is consistent with some models for hadron spectroscopy, there exist drastically different models which present a very different picture (Neudatchin *et al.* 1991, and references therein; Kukulin 1991), but claim to have a similar basis in studies of hadron structure.

This diversity of models obviously indicates the inadequacy of our understanding of the NN interaction process. It seems that one of the reasons for this plethora of models is our lack of understanding and inconsistent treatment of the short-range part of the meson exchange force. In recent quark model investigations of the NN interaction, the pion exchanges (including one-pion exchange and two-pion exchange parametrised by σ exchange) are usually added to the quark–gluon forces so that the known long-range NN force is produced. But the method of handling pion exchange at short range varies from group to group (Neudatchin *et al.* 1991; Kukulin 1991; Faessler *et al.* 1983; Faessler and Fernandez 1983; Fernandez 1987;

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Braúer *et al.* 1985, 1990; Takeuchi *et al.* 1989). RGM calculations by the Tokyo (Takeuchi *et al.* 1989; Morimatsu *et al.* 1984) and Tübingen (Faessler *et al.* 1983; Faessler and Fernandez 1983; Fernandez 1987; Braúer *et al.* 1990) groups show that in the presence of a quark–quark force such as OGE, meson exchange contributions still play a crucial role for low partial waves. For example, to obtain the correct P-wave splitting, Morimatsu *et al.* (1984) found that an effective meson exchange potential (EMEP) was necessary, while Bräuer *et al.* (1985, 1990) achieved a fit by enhancing the calculated OGE spin–orbit force eight times. Therefore, to find a consistent picture for the short-distance dynamics, there seems to be a need to find a consistent model for extending the pion exchange force to short range.

Similar to the case of interatomic forces, quark model studies of the NN interaction rely heavily on information from hadron spectroscopy—e.g. the use of the OGE potential is justified mostly from hadron structure studies. Though there are many similarities, it should also be noted that the exchange of collective degrees of freedom (mesons) in the NN system is a distinct feature that has no counterpart in the case of interatomic forces. This makes the dynamics more complicated and indicates an important difference in the interaction processes. In other words, while single-hadron models provide a test mostly for perturbative quark–gluon dynamics, which is believed to affect only the short-range part of the NN force, non-perturbative effects play a very important role and need to be treated carefully in hadron–hadron interactions.

Because of the importance of one-pion exchange in the NN interaction, its short-range extension is of particular relevance. In a recent work (Liu *et al.* 1993) an attempt was made to extend the one-pion exchange force to short distance by including the explicit coupling of the pion field to the quarks. The basis for pion–quark coupling is chiral symmetry and its spontaneous breaking, as is known in theories like chiral bag models (Thomas 1981, 1983) and recent investigations of effective chiral theories (Manohar and Georgi 1984; Weinberg 1979, 1990; Diakonov *et al.* 1988). Some groups (Faessler *et al.* 1983; Faessler and Fernandez 1983; Fernandez 1987; Braúer *et al.* 1985, 1990; Obukhovsky and Kusainov 1990) have used quark–pion coupling in the study of the NN interaction. Since those calculations all involve phenomenological modifications of the short-range part in order to fit the NN data, inconsistency and ambiguity may arise. In some cases, the RGM method is used. Though more accurate, its non-local nature and computational complications render it harder to get a clear intuitive picture like that in meson exchange or well known phenomenological models (Reid 1968). Our purpose is to try to extend the pion exchange mechanism to short range in a unique and parameter-free manner, rather than providing a phenomenological means to fit the NN data. This puts a strong theoretical constraint on the short-range one-pion exchange force. We use the adiabatic approximation to find a local potential to be compared with known meson exchange theories.

In this paper we report our study of the NN interaction with this specific and consistent short-range pion force. We combine the pion potential as derived in Liu *et al.* (1993) with those of vector meson exchange and one-gluon exchange to calculate the NN scattering parameters. Various combinations involving vector meson exchange and OGE are considered. A comparison suggests that vector meson exchanges contain dynamics not accounted for by the OGE mechanism.

The effect of quark–gluon dynamics on the ω–nucleon coupling constant is also discussed.

2. Potentials in Quark Model and Meson Exchange Frameworks

The main purpose of this work is to explore any connections in short-range dynamics between meson exchange and quark model pictures using a consistent one-pion exchange force. The procedure is to add a specific short-range NN force to the background quark–pion potential of Liu *et al.* (1993) to obtain an NN potential, which is then inserted into the Schrödinger equation to calculate the NN scattering parameters. By using the same force for the one-pion exchange (OPE) but different models for the short-range NN force, we can get a reliable and consistent comparison of different short-range mechanisms. In this paper, four alternative short-range forces are considered, as given by equations (1)–(4). The first, V_1, consists of ρ, ω and σ exchanges, which are the main contributors in conventional meson exchange models. In V_2, δ and η exchanges are included as well. In V_3, we use the short-range force from one-gluon exchange, which has been quite successful in hadron spectroscopy calculations. Since OGE only provides the short-range repulsion, we also added the sigma exchange. In V_4 we include all five mesons and the OGE as the short-range component. More details about various potential terms are discussed later.

In order to highlight differences between quark dynamics and traditional parametrisations, these models are compared with a conventional meson exchange calculation in which quark dynamics are not involved. The five potential models are represented as:

Potentials with meson exchange only:

$$V_1 = V_{q\pi} + V_\sigma + V_\rho + V_\omega \,, \tag{1}$$

$$V_2 = V_{q\pi} + V_\sigma + V_\rho + V_\omega + V_\eta + V_\delta \,. \tag{2}$$

One-gluon exchange plus pion exchanges:

$$V_3 = V_{q\pi} + V_\sigma + V_{\mathrm{OGE}} \,. \tag{3}$$

OGE and vector mesons both present:

$$V_4 = V_{q\pi} + V_\sigma + V_\rho + V_\omega + V_\eta + V_\delta + V_{\mathrm{OGE}} \,. \tag{4}$$

A conventional meson exchange potential:

$$V_5 = V_\pi + V_\sigma + V_\rho + V_\omega + V_\eta + V_\delta \,. \tag{5}$$

In this study the potential $V_{q\pi}$ is the main starting point. It is an NN potential derived from coupling the pion to the quarks in a non-relativistic quark model using an adiabatic approach. All necessary formalisms and derivations can be found in Liu *et al.* (1993) and are not repeated here. It can be expressed in closed but very lengthy form, so we summarise the necessary formulas in the Appendix. In order to give a clearer idea of its behaviour, we provide a plot for

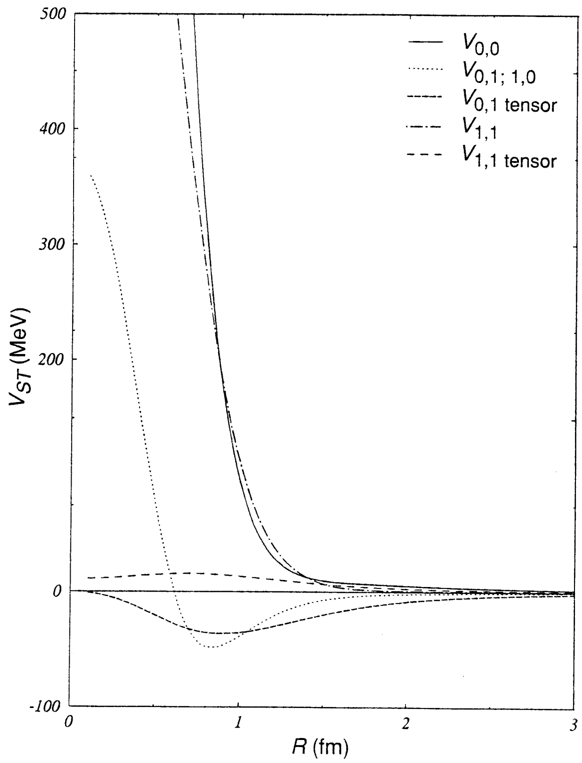

Fig. 1. Quark–pion coupling potential $V_{q\pi}$ for the four spin–isospin channels indicated.

the four spin–isospin channels in Fig. 1. This quark–pion coupling potential has several interesting features:

(a) a repulsive core at short range similar to that of the Paris and Reid soft core potentials for all spin–isospin channels. [Note that the use of some cut-off procedures for the OPEP such as those by form factors or phenomenological means (Kukulin 1991) does not retain this core-like behaviour.]

(b) a reasonably strong medium-range attraction due to the inclusion of the quark structure of the nucleon.

This treatment of one-pion exchange is similar to that of Braüer *et al.* (1985, 1990), except that the pion is treated as a point-like* particle here. It was shown (Liu *et al.* 1993) that the short-range part of this potential term is very sensitive to the contact term of the quark–pion potential. Since the NN potential resulting from quark–pion coupling is finite anyway, we did not put in unknown parameters. By this consistent treatment of OPEP at short distance, the ambiguities caused by its different parametrisations in meson exchange and quark model calculations are reduced. This permits more definitive comparisons

* The idea of treating the pion as a point particle (or a collective degree of freedom) has a long history from chiral symmetry considerations. PCAC and modern theories like the cloudy bag model all lend support to such an approximation (see Weinberg 1990; Thomas 1981, 1983; Théberge *et al.* 1980, 1981; Manohar and Georgi 1984).

between meson exchange and quark model pictures. We found that the attraction in the pion–quark potential is not strong enough to bind the deuteron, mainly because it has too short a range due to the use of harmonic confinement. We added the σ contribution to simulate the two-pion exchange process.

The potential terms for the σ, ρ, ω, η and δ exchange were taken to be the same as in conventional meson exchange calculations. For the sake of completeness, the explicit expressions of these meson exchange terms are given here:
Vector mesons ω and ρ:

$$\begin{aligned}V(\mu,r) = \frac{g^2}{4\pi}\mu\Bigg[&\left(1+\frac{\mu^2}{2m^2}+\frac{\mu^2}{6m^2}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)Y(\mu r)\\ &-\frac{3\mu^2}{2m^2}Z(\mu r)\boldsymbol{L}\cdot\boldsymbol{S}-\frac{\mu^2}{12m^2}T(\mu r)S_{12}\Bigg]\\ &+\frac{gf}{4\pi}\frac{\mu^3}{2m^2}[(1+\tfrac{2}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)Y(\mu r)-4Z(\mu r)\boldsymbol{L}\cdot\boldsymbol{S}-\tfrac{1}{3}T(\mu r)\boldsymbol{S}_{12}]\\ &+\frac{f^2}{4\pi}\frac{\mu^3}{6m^2}[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 Y(\mu r)-\tfrac{1}{2}T(\mu r)\boldsymbol{S}_{12}]\,.\end{aligned} \tag{6}$$

Scalar mesons σ and δ:

$$V(\mu,r) = -\frac{g^2}{4\pi}\mu\left[\left(1-\frac{\mu^2}{4m^2}\right)Y(\mu r)+\frac{\mu^2}{2m^2}Z(\mu r)\boldsymbol{L}\cdot\boldsymbol{S}\right]. \tag{7}$$

where

$$Y(x)=\exp(x)/x,\qquad Z(x)=\left(\frac{1}{x}+\frac{1}{x^2}\right)Y(x),\qquad T(x)=\left(1+\frac{3}{x}+\frac{3}{x^2}\right)Y(x)\,.$$

For ρ and δ, there is an extra isospin operator $\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2$.

A monopole form factor of the form

$$\frac{\Lambda^2}{\Lambda^2+k^2}$$

is applied at each vertex. The cut-off mass Λ is treated as a free parameter for each meson fitted. The meson–nucleon coupling constants are also treated as parameters but are restricted to physically acceptable ranges in the fitting. This treatment of heavy mesons is basically the same as in the Bonn-r potential (Machleidt *et al.* 1987), but the parameters are refitted.

We mention that we were unable to use any microscopic models for them. There have been attempts to understand the heavier meson exchange in the quark model framework (Yazaki 1990) and to treat the baryon–meson vertex in the quark pair creation model (Yu and Zhang 1984, 1986) but we feel more quantitative development is needed to apply them in fitting the experimental data. This is certainly an important issue in understanding short-range NN dynamics and should be further pursued. In the present calculation, only the local part of the meson exchange potential is retained. This is for consistency with the treatment of the pion force, where non-local effects are averaged out in the adiabatic approach.

For the OGE potential V_{OGE}, we use the one given by Holinde (1984), which has a simple analytic form and is summarised in the Appendix. The main feature of this potential is a central short-range repulsion of about 400 MeV for all spin–isospin channels. The spin–orbit force is not included due to the ambiguities about its role in single-hadron studies, since there are suggestions that the OGE spin–orbit force may be cancelled out by those from other sources such as confinement (Shimizu 1989). Earlier work (Warke and Shanker 1980) showed that the strength of the OGE spin–orbit force is very weak—about two orders of magnitude smaller than its phenomenological counterpart. Therefore its effect is very small anyway. The OGE tensor force is also very weak—just a few MeV—and plays a negligible role. Basically, V_3 has the same ingredients as that of Bráuer *et al.* (1985, 1990), the differences being that the parameters are fewer and constrained here since we do not introduce form factors at quark–pion vertices. Because of this similarity, we feel that it is appropriate to use the parameters of the Tübingen group (Faessler *et al.* 1983; Faessler and Fernandez 1983; Fernandez 1987; Bráuer *et al.* 1985, 1990; Holinde 1984) for the OGE, i.e. $\alpha_s = 0{\cdot}97$, $a = 34{\cdot}5$, $m = 355$. We also tried the parameters used by the Tokyo group, i.e. $\alpha_s = 1{\cdot}39$, $a = 62{\cdot}5$, $m = 300$, and no significant difference was found in the fit.

In model 5, V_π is the conventional OPEP with form factor cut-offs instead of quark model extensions. In fact V_5 is similar to the Bonn-r potential but the momentum-dependent terms are neglected and the parameters are refitted. The fitting parameters for all the potentials are listed in Table 1.

Table 1. Coupling constants and cut-off masses for mesons

The f/g ratios for ρ and ω are kept constant at $6{\cdot}1$ and 0 respectively. The pion–nucleon coupling constant f^2 used in models 1–4 is $0{\cdot}075$, which corresponds to $g_\pi^2 = 13{\cdot}86$ used in model 5. Λ and m are in MeV

	Model 1	Model 2	Model 3	Model 4	Model 5
β	2·06	2·24	1·89	2·13	—
g_ρ^2	1·05	1·25	—	1·20	1·0
Λ_ρ	968·46	992·0	—	954·65	1498·80
g_ω^2	9·04	9·20	—	9·95	23·89
Λ_ω	1100·0	1142·0	—	1153·60	1988·40
$g_{\sigma(T=1)}^2$	9·58	8·20	13·98	8·38	8·62
$m_{\sigma(T=1)}$	551·24	547·16	585·24	505·72	509·38
$g_{\sigma(T=0)}^2$	1·15	1·65	1·38	2·70	22·50
$m_{\sigma(T=0)}$	402·0	401·50	350·0	400·08	660·0
Λ_σ	1552·50	1744·60	1296·70	1798·10	1730·70
g_η^2	—	0·03	—	0·54	5·76
Λ_η	—	846·0	—	890·23	1403·0
g_δ^2	—	4·62	—	1·67	5·59
Λ_δ	—	1414·50	—	1645·80	1580·0
g_π^2	—	—	—	—	13·86
Λ_π	—	—	—	—	1500·0

We feel a few comments are due about the use of the adiabatic method for the OGE and quark–pion potentials. Though it is well known that the short-range NN interaction is highly non-local in RGM calculations (Takeuchi *et al.* 1989; Oka and Yazaki 1984; Suzuki and Hecht 1983; 1984), it is also known (Shimizu 1989; Holinde 1984) that adiabatic approaches give a reasonable qualitative description. In fact the energy-dependent potential derived from an RGM kernel in Suzuki and Hecht (1983, 1984) is very similar to the adiabatic calculation of Holinde (1984). Another reason for using the adiabatic approach is not in the methodology, but rather the fact that the constituent quark model has its own limitations. At the present stage of development, it would be unreasonable to expect the quark model to give a precise description of the NN interaction, or one that is more accurate than the hadron spectroscopy. We therefore focus on qualitative discrepancies between vector meson exchange and gluon exchange mechanisms.

3. Results and Concluding Remarks

The resulting fits to NN phase parameters for some lower partial waves are plotted in Fig. 2. In our calculation, the S-waves are fitted with a weight factor of 50, i.e. the errors of the S-waves are reduced by a factor of 50, so that they are reproduced more accurately and consistently. This makes the role of the short-range force more prominent, and the S-waves serve as a more stringent constraint on the parameters. This procedure pushes the discrepancies to higher partial waves, i.e. the P and D waves. Of course, without the weight factor in the S-wave the higher partial wave results will be better, but in our view it is more suitable to judge the short-distance dynamics with a good and consistent S-wave fit. The quality of the fits is similar as long as the value of the weight factor is greater than 20. The P and higher partial waves are fitted using the data given by Arndt *et al.* (1987), Bugg (1990) and Bugg and Bryan (1992), without any weighting factors. The NN scattering parameters are roughly described, though there are some discrepancies for the high-energy region, especially for D-waves. Our hope is that these fits will enable us to learn the salient features of various models and make some qualitative statements about meson and gluon exchange mechanisms. We now discuss the results and dynamics of various models. We find two comparisons helpful in understanding the results, i.e. quark dynamics versus phenomenological parametrisation, and vector meson versus gluon exchange.

Firstly, the conventional meson exchange (i.e. model 5) gives the best fit, except for $^3D_3$. But this is achieved with a hard form factor of $1\cdot5$ GeV for the one-pion exchange, and with large coupling constants (over 20) for the ω and σ mesons. On the other hand, quark model calculations do not find such hard form factors (Liu *et al.* 1993), and the coupling constants are more reasonable. In fact the ω coupling in models 1–4 is in line with its SU(3) strength (at $\boldsymbol{k}^2 = 0$). This reduction comes from the short-range behaviour of the quark–pion potential $V_{q\pi}$, which is repulsive in all channels. However, for model 5, the one-pion exchange becomes attractive at short range in odd-parity states (Liu *et al.* 1993). To compensate this attraction, a large ω coupling is needed. Therefore, the overly large ω coupling is related to the short-range quark dynamics.

Secondly, we compare the vector meson exchange and OGE results, i.e. models 1, 2 and 4 versus model 3. The most significant difference between vector meson

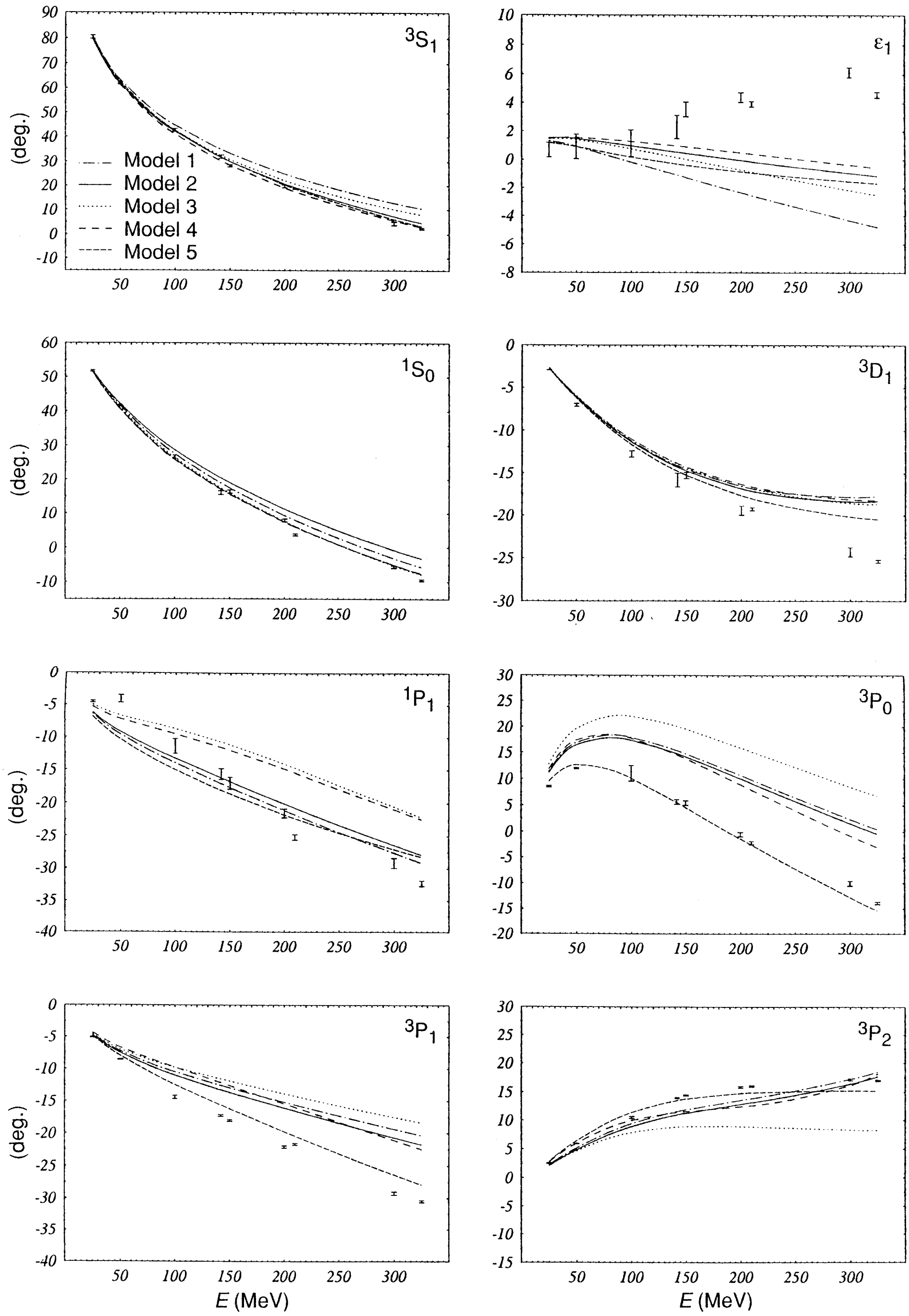

Fig. 2. Plots of phase parameters from various models. Experimental data are from recent analyses by Arndt *et al.* (1987) and Bugg (1990). Keys are in the $^3S_1$ plot.

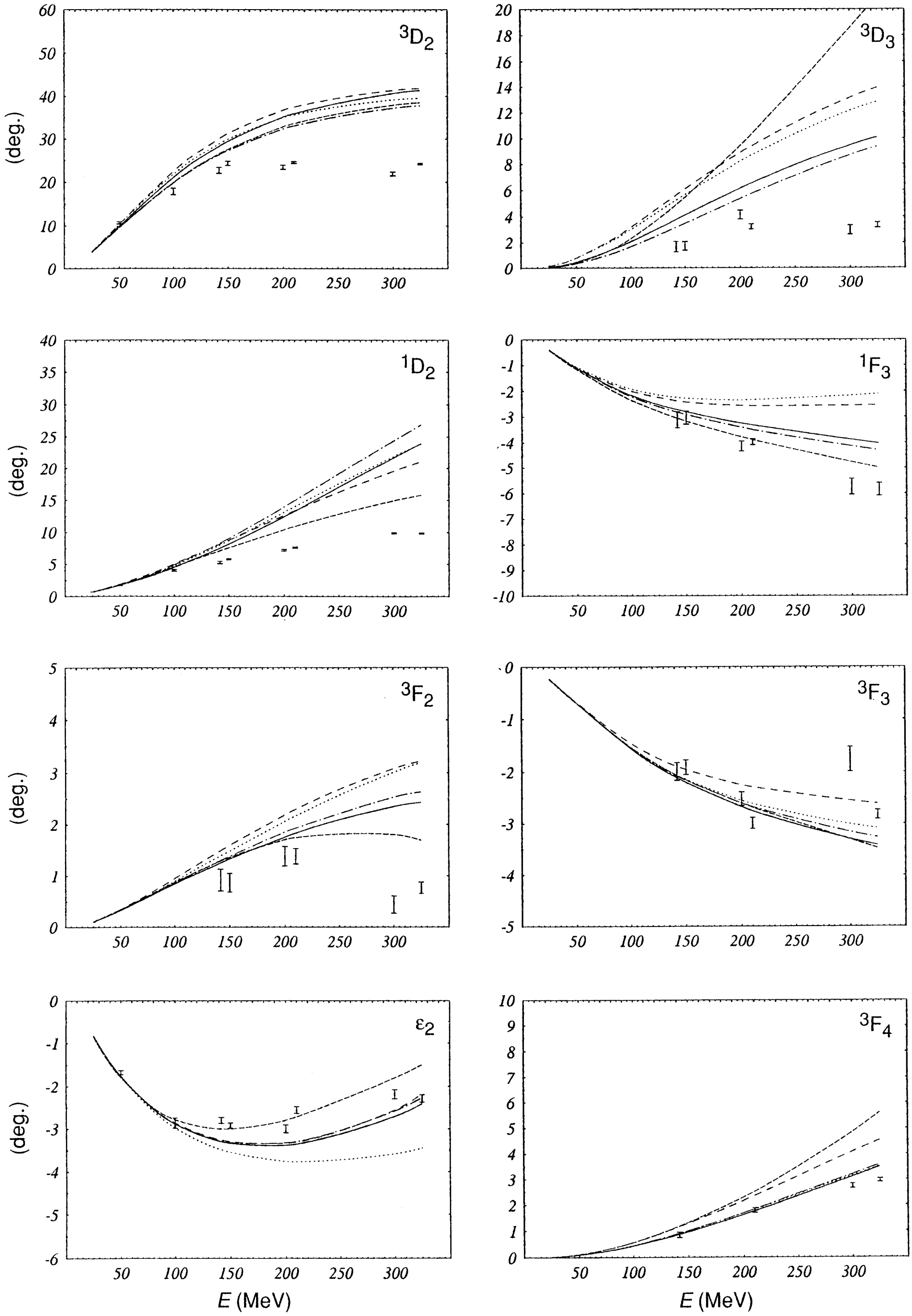

Fig. 2. (*Continued*)

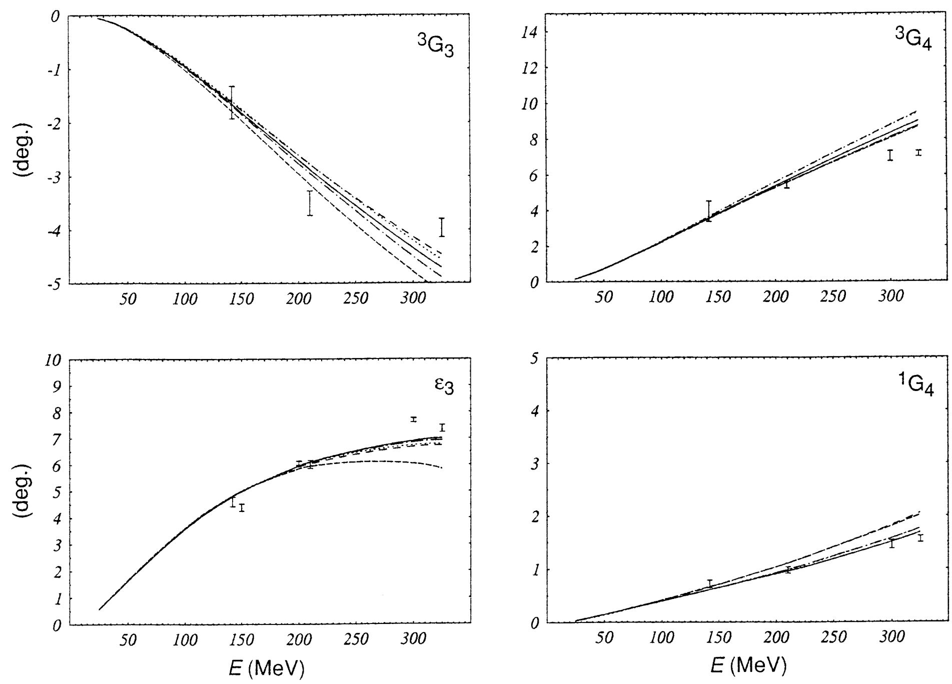

Fig. 2. (*Continued*)

and OGE fits occurs in the P waves. For D waves, vector meson exchange and OGE give similar results, both having some problems at the high energy end. For higher partial waves, the quality of fit becomes much better as the OPEP tail becomes dominant and the various models considered here are in good agreement. It is seen that the vector meson exchange potentials V_1 and V_2 give better results than the OGE potential V_3. The biggest difference is found in the triplet P waves. The main problem for the OGE is to get the P-wave splitting, as has been stressed elsewhere (Bráuer *et al.* 1985, 1990; Morimatsu *et al.* 1984). In vector meson exchange models, the splitting is roughly achieved, with the ρ meson exchange being the main contributor pushing the $^3P_2$ (and other P waves) up. There are two reasons for the failure of model 3 to give the splitting, namely its weak spin–orbit force and the restriction in the form of the short-range part of the one-pion exchange force. It seems that without some dramatic operation, it is very hard to produce the ^3P wave splittings with OGE. The correlation between $^3P_2$ and $^3P_0$ states is very difficult to reconcile for the OGE, even with very flexible parameters for the σ meson.

It is noteworthy that the result of the fourth model (i.e. vector mesons plus OGE) shows an improvement over those of models 1–3. In many states, the V_4 fit has a mutual compensation between vector meson and OGE. This suggests the possibility that with a more sophisticated quantitative treatment, a combination of vector meson exchange and OGE might provide a better explanation of the short-range NN interaction.

In summary, by comparing short-range mechanisms of vector meson exchange and one-gluon exchange, it appears that the vector mesons incorporate extra dynamics not accounted for by OGE. There could be corresponding non-perturbative effects not yet included (or not thoroughly investigated) in hadron spectroscopy studies. The point of view that one-gluon exchange gives sufficient short-range dynamics for the NN interaction is not confirmed in this study, even with the σ exchange treated phenomenologically. It seems difficult to reconcile the quark model with the parameters used in the conventional meson exchange calculations. We feel that a full explanation of the NN interaction also needs a better understanding of the vector meson exchange at a more microscopic level.

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References

Arndt, R. A., *et al.* (1987). *Phys. Rev.* D **35**, 128.
Bräuer, K., *et al.* (1985). *Z. Phys.* A **320**, 609.
Bräuer, K., *et al.* (1990). *Nucl. Phys.* A **507**, 599.
Bugg, D. V. (1990). *Phys. Rev.* C **41**, 2708.
Bugg, D. V., and Bryan, R. A. (1992). *Nucl. Phys.* A **540**, 449.
Diakonov, D. I., Petrov, V. Yu, and Pobyltsa, P. Y. (1988). *Nucl. Phys.* B **36**, 809.
Faessler, A., *et al.* (1983). *Nucl. Phys.* A **402**, 555.
Faessler, A., and Fernandez, F. (1983). *Phys. Lett.* B **124**, 145.
Fernandez, F. (1987). *Phys. Lett.* B **188**, 314.
Holinde, K. (1984). *Nucl. Phys.* A **415**, 477.
Kukulin, V. I. (1991). Conf. on Pauli Principle in Few-body Systems, Tokyo, October 1991.
Liu, G. Q., *et al.* (1993). *Nucl. Phys.* A **556**, 331.
Machleidt, R., Holinde, K., and Elster, Ch. (1987). *Phys. Rep.* **149**, 1.
Manohar, A., and Georgi, H. (1984). *Nucl. Phys.* B **234**, 189.
Morimatsu, O., Yazaki, K., and Oka, M. (1984). *Nucl. Phys.* A **424**, 412.
Myhrer, F., and Wroldsen, J. (1988). *Rev. Mod. Phys.* **60**, 629.
Neudatchin, V. G., *et al.* (1991). *Phys. Rev.* C **43**, 2499.
Obukhovsky, I. T., and Kusainov, A. M. (1990). *Phys. Lett.* B **238**, 142.
Oka, M., and Yazaki, K. (1984). *Int. Rev. Nucl. Phys.* **1**, 489.
Reid, R. V. Jr (1968). *Ann. Phys. (New York)* **50**, 411.
Shimizu, K. (1989). *Rep. Prog. Phys.* **52**, 1.
Suzuki, Y., and Hecht, K. T. (1983). *Phys. Rev.* C **27**, 299.
Suzuki, Y., and Hecht, K. T. (1984). *Nucl. Phys.* A **420**, 525.
Takeuchi, S., *et al.* (1989). *Nucl. Phys.* A **504**, 777.
Théberge, S., *et al.* (1980). *Phys. Rev.* D **22**, 2838.
Théberge, S., *et al.* (1981). *Phys. Rev.* D **23**, 2106(E).
Thomas, A. W. (1981). *J. Phys.* G **7**, L283.
Thomas, A. W. (1983). *Adv. Nucl. Phys.* **13**, 1.
Warke, C. S., and Shanker, R. (1980). *Phys. Rev.* C **21**, 2643.
Weinberg, S. (1979). *Physica* A **96**, 327.
Weinberg, S. (1990). *Phys. Rev. Lett.* **65**, 1181.
Yazaki, K. (1990). *Prog. Part. Nucl. Phys.* **24**, 353.
Yu, Yuo-wen and Zhang, Zong-ye (1984). *Nucl. Phys.* A **426**, 557.
Yu, Yuo-wen and Zhang, Zong-ye (1986). *Nucl. Phys.* A **455**, 737.

Appendix: Explicit Expressions for the Potential Terms

The Quark–Pion Coupling Potential $V_{q\pi}$ of Liu et al. (1993):

$$V_{q\pi} = 9V_{36}^{(\mathrm{nqe})} + V_{36}^{(\mathrm{qe})} + 4V_{14}^{(\mathrm{qe})} + 4V_{34}^{(\mathrm{qe})} + 2V_{12}^{\mathrm{qe}} + 4V_{23}^{\mathrm{qe}} + 6(V_{12}^{(\mathrm{nqe})} - V_{12}^{(\mathrm{s})}), \quad \text{(A1)}$$

where the various components are

$$V_{36}^{(\mathrm{nqe})} = \frac{1}{N(R)} \tfrac{25}{81} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left\{ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \left[R_{36}^{S} + (-1)^{S+T} R_{36}^{\mathrm{e}S} \right] + \boldsymbol{S}_{12} R_{36}^{T} \right\}, \quad \text{(A2)}$$

$$\begin{aligned} V_{36}^{(\mathrm{qe})} &= \frac{1}{N(R)} \left[9 - \tfrac{1}{3}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \tfrac{25}{81} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right] \left[Q_{36}^{S} + (-1)^{S+T} Q_{36}^{\mathrm{e}S} \right] \\ &\quad + \frac{1}{N(R)} \left(\tfrac{2}{9} - \tfrac{50}{243} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \boldsymbol{S}_{12} (-1)^{S+T} Q_{36}^{\mathrm{e}T}, \end{aligned} \quad \text{(A3)}$$

$$\begin{aligned} V_{14}^{(\mathrm{qe})} &= \frac{1}{N(R)} \left[\tfrac{25}{9} + \tfrac{1}{27}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \tfrac{61}{81} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right] \left[Q_{14}^{S} + (-1)^{S+T} Q_{14}^{\mathrm{e}S} \right] \\ &\quad + \frac{1}{N(R)} \left(\tfrac{1}{81} + \tfrac{7}{243} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \boldsymbol{S}_{12} Q_{14}^{T}, \end{aligned} \quad \text{(A4)}$$

$$\begin{aligned} V_{34}^{(\mathrm{qe})} &= \frac{1}{N(R)} \left[5 + \tfrac{1}{9}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \tfrac{85}{81} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right] \left[Q_{34}^{S} + (-1)^{S+T} Q_{34}^{\mathrm{e}S} \right] \\ &\quad + \frac{1}{N(R)} \left(\tfrac{1}{27} - \tfrac{5}{243} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \right) \boldsymbol{S}_{12} \left[Q_{34}^{T} + (-1)^{S+T} Q_{34}^{\mathrm{e}T} \right], \end{aligned} \quad \text{(A5)}$$

$$V_{12}^{(\mathrm{nqe})} = \frac{5}{N(R)} \left[R_{12}^{S} + (-1)^{S+T} R_{12}^{\mathrm{e}S} \right], \quad \text{(A6)}$$

$$\begin{aligned} V_{12}^{(\mathrm{qe})} &= \frac{1}{N(R)} \left[5 + \tfrac{13}{9}(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \tfrac{205}{81} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \right] \\ &\quad \times \left[Q_{12}^{S} + (-1)^{S+T} Q_{12}^{\mathrm{e}S} \right], \end{aligned} \quad \text{(A7)}$$

$$V_{23}^{(\mathrm{qe})} = V_{34}^{(\mathrm{qe})}, \quad \text{(A8)}$$

$$V_{12}^{(s)} = 5 R_{12}^{S}. \quad \text{(A9)}$$

In the above formulas, the superscript '(nqe)' denotes a non-quark-exchange term, '(qe)' denotes a quark exchange term, and '(s)' represents the single cluster term. The subscripts (ij) means a pion is exchanged between the quark pairs i

and j. The radial integrals R, Q in the above equations are given by the following expressions:

$$R^S_{36} = G_0 F_S(R)\,, \qquad R^{eS}_{36} = -\,G_0 F_0 \exp(-\tfrac{3}{2}\beta^2 R^2)\,,$$

$$R^S_{12} = G_0 F_0\,, \qquad R^{eS}_{12} = -\,G_0 F_0 \exp(-\tfrac{3}{2}\beta^2 R^2)\,,$$

$$Q^S_{36} = Q^S_{12} = -\tfrac{3}{4} G_0 \exp(-\tfrac{1}{2}\beta^2 R^2) F_0\,, \qquad Q^{eS}_{36} = \tfrac{3}{4} G_0 \exp(-\beta^2 R^2) F_S(R)\,,$$

$$Q^S_{34} = -\,\tfrac{3}{4} G_0 \exp(-\tfrac{1}{2}\beta^2 R^2) F_S(\tfrac{1}{2}R)\,, \qquad Q^{eS}_{34} = \tfrac{3}{4} G_0 \exp(-\beta^2 R^2) F_S(\tfrac{1}{2}R)\,,$$

$$Q^S_{14} = -\,\tfrac{3}{4} G_0 \exp(-\tfrac{1}{2}\beta^2 R^2) F_S(R)\,, \qquad Q^{eS}_{14} = Q^{eS}_{12} = \tfrac{3}{4} G_0 \exp(-\beta^2 R^2) F_0\,,$$

$$R^T_{36} = G_0 F_T(R)\,,$$

$$Q^T_{34} = -\,\tfrac{9}{4} G_0 \exp(-\tfrac{1}{2}\beta^2 R^2) F_T(\tfrac{1}{2}R)\,, \qquad Q^{eT}_{34} = \tfrac{9}{4} G_0 \exp(-\beta^2 R^2) F_T(\tfrac{1}{2}R)\,,$$

$$Q^T_{14} = -\,\tfrac{9}{4} G_0 \exp(-\tfrac{1}{2}\beta^2 R^2) F_T(R)\,, \qquad Q^{eT}_{36} = \tfrac{9}{4} G_0 \exp(-\beta^2 R^2) F_T(R)\,.$$

The functions $F_S(x)$ and $F_T(x)$ are given by

$$F_S(x) = \tfrac{1}{6} \exp(-\tfrac{1}{2}\beta^2 x^2) \left[\mu^2 F(x) - 2\beta^3 \sqrt{\frac{2}{\pi}} \right]\,, \tag{A10}$$

$$F_T(x) = \tfrac{1}{6} \exp(-\tfrac{1}{2}\beta^2 x^2) \left[\left(\mu^2 - \frac{3\mu}{x} + \frac{3}{x^2} \right) F(x) + \frac{6\mu}{x} g(x) - \sqrt{\frac{2}{\pi}} \left(2\beta^3 + \frac{6\mu}{x^2} \right) \right]\,, \tag{A11}$$

where

$$g(x) = \frac{1}{x} \exp\left(\frac{(\mu - \beta^2 x)^2}{2\beta^2} \right) \operatorname{erfc}\left(\frac{\mu - \beta^2 x}{\sqrt{2}\beta} \right)\,, \tag{A12}$$

$$F(x) = g(x) + g(-x)\,. \tag{A13}$$

Here μ is the pion mass, β the nucleon size parameter, and N the normalisation function given by

$$N(R) = 1 - 3C_{ST} \exp(-\tfrac{1}{2}\beta^2 R^2) + (-1)^{S+T} 3C_{ST} \exp(-\beta^2 R^2) - (-1)^{S+T} \exp(-\tfrac{3}{2}\beta^2 R^2)\,, \tag{A14}$$

where the spin–isospin-dependent constant C_{ST} has the values

$$C_{00} = \tfrac{7}{9}\,, \qquad C_{11} = \tfrac{31}{81}\,, \qquad C_{01} = C_{10} = -\tfrac{1}{27}\,. \tag{A15}$$

In equation (A14), β is the confinement strength constant. Readers interested in the details can consult Liu *et al.* (1993).

One-Gluon Exchange Potential V_{OGE} of Holinde (1984):

$$V_{\mathrm{OGE}} = \frac{4\alpha_S}{N(R)}\left\{C_{ST}\,M_{ST}(R)\left[\frac{8}{R}\mathrm{erf}\left(\frac{\beta R}{2\sqrt{2}}\right) - \frac{1}{R}\mathrm{erf}\left(\frac{\beta R}{\sqrt{2}}\right) - 3\beta\sqrt{\frac{2}{\pi}}\right]\right.$$

$$+\frac{2\pi}{3m^2}\left(\frac{\beta^2}{2\pi}\right)^{\frac{3}{2}}\left[\exp(-\tfrac{1}{2}\beta^2R^2)\left\{d_{ST}^{36} + d_{ST}^{14}\exp(-\tfrac{1}{2}\beta^2R^2) - (-1)^{S+T}\exp(-\tfrac{1}{2}\beta^2R^2)\right.\right.$$

$$\left.[d_{ST}^{36}\exp(-\tfrac{1}{2}\beta^2R^2) + d_{ST}^{14}]\right\} - M_{ST}(R)\left\{4d_{ST}^{34}\exp(-\tfrac{1}{8}\beta^2R^2) + 3C_{ST} + d_{ST}^{12}\right\}\Big]$$

$$+\frac{\boldsymbol{S}_{12}}{4m^2}[F_T(R)\exp(-\tfrac{1}{2}\beta^2R^2)\{e_{ST}^{14} - (-1)^{S+T}e_{ST}^{36}\exp(-\tfrac{1}{2}\beta^2R^2)\}$$

$$\left.-4e_{ST}^{34}\,M_{ST}(R)F_T(\tfrac{1}{2}R)]\right\}, \tag{A16}$$

where

$$M_{ST}(R) = \left[1 - (-1)^{S+T}\exp(-\tfrac{1}{2}\beta^2R^2)\right]\exp(-\tfrac{1}{2}\beta^2R^2)\,.$$

The d and e are spin and isospin matrix elements of the following operators:

$$\hat{d}_{ST}^{36} = \tfrac{3}{4}\left[1 + \tfrac{1}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \tfrac{1}{27}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \tfrac{25}{243}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right],$$

$$\hat{d}_{ST}^{14} = \tfrac{1}{12}\left[1 + \tfrac{1}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \tfrac{1}{3}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \tfrac{1}{27}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right],$$

$$\hat{d}_{ST}^{34} = -\tfrac{1}{4}\left[1 - \tfrac{1}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \tfrac{1}{9}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \tfrac{5}{81}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right],$$

$$\hat{d}_{ST}^{12} = -\tfrac{1}{4}\left[1 + \tfrac{5}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 + \tfrac{5}{9}\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + \tfrac{65}{81}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right],$$

$$\hat{e}_{ST}^{36} = \tfrac{1}{18}\left[1 + \tfrac{25}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right], \quad \hat{e}_{ST}^{14} = \tfrac{1}{36}\left[1 + \tfrac{1}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right], \quad \hat{e}_{ST}^{34} = \tfrac{1}{36}\left[1 - \tfrac{5}{9}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right].$$

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