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Inhomogeneously Heated Electron Thermal Bottleneck in Semiconductors

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Abstract

General phenomenological criteria for the origin of the inhomogeneously heated electron thermal bottleneck in semiconductors are obtained. Different physical situations in solid-state physics are discussed in which the electron bottleneck is described in terms of multi-valued characteristics. A microscopic model of the electron bottleneck in layer semiconductors is suggested and investigated in detail. A novel phenomenon—the cold electon bottleneck in layer semiconductors—is predicted. It is shown that, unlike the hot electron bottleneck, the cold electron one manifests itself in the formation of two (one) S-N shaped or locked loop-type sections on the current–voltage characteristics (CVC). Various methods of experimental detection of such bottlenecked CVC are discussed.

1. Introduction

In 1941 Van Vleck introduced the notion of the bottleneck phenomenon for describing the interaction between magnet and phonon subsystems in solids. Later Faughman and Strandberg (1961), Stoneham (1965), Wooldridge (1969), Brya *et al.* (1968) and Korolev *et al.* (1975) investigated in detail this effect in paramagnetic crystals. To be quite precise, the bottleneck phenomenon can be observed not only in the above-mentioned paramagnetic system. Its sphere is much wider. As a matter of fact, this phenomenon may characterise the finite value of the velocities of solid-state subsystem interactions of different physical nature. Conwell (1967), Bass and Gurevich (1975) and also Bass *et al.* (1984) showed that the electron–phonon energy channel is characterised by the limited relaxation time. Owing to the latter, hot electrons occur in such media. There are more complicated physical systems with more than two species of interacting quasiparticles. For example, Bass *et al.* (1984) considered semiconductors consisting of three subsystems: electron, short-wave phonon and long-wave phonon. In magnetic semiconductors a magnon subsystem will be added to these (Bass and Oleynik 1976).

It is necessary to emphasise the fact that under certain conditions any channel of interaction which has a limited relaxation time may lead to the bottleneck phenomenon. Up to the present homogeneous nonequilibrium electron states have been investigated in different physical bottleneck systems. Let us trace their basic features for the example of hot electrons.

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2. Bottleneck of Uniformly Heated Electrons and Their Multi-valued Characteristics

With the volume thermostat available (usually it is a crystal lattice), the amount of homogeneous electron heating is determined by the following equation:

$$P(T_e)(T_e - T_p) = W(T_e, \lambda), \tag{1}$$

where T_e and T_p are the temperatures of electrons and phonons respectively. The quantity $P(T_e)$ characterises the intensity of the electron–phonon energy interaction, $W(T_e, \lambda)$ is the heat evolution rate and λ is the external parameter depending on the amplitude of external influences (current, electric field, super frequencies, light or sound waves etc.). It should be stressed that in practice $W(\lambda)$ is usually a monotonically increasing function [see e.g. the works of Conwell (1967), Bass and Gurevich (1975), Bass *et al.* (1984), Haken (1978), Thompson (1982), Gurevich and Mashkevich (1989)].

Special properties of the heating process are determined by the dependence of $T_e(\lambda)$. In line with (1) the form of the function $T_e(\lambda)$ is defined by concrete temperature dependences of the 'source' $[W(T_e)]$ and the 'sink' $[\sim P(T_e)]$ of the system nonequilibrium.

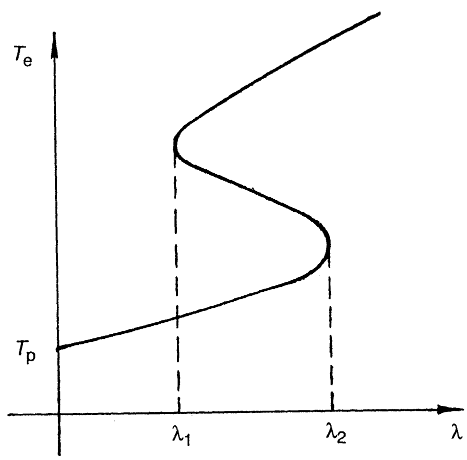

Fig. 1. Dependence of electron temperature T_e on the amplitude of the imposed external force λ. Here (λ_1, λ_2) is the interval where the function $T_e(\lambda)$ is multi-valued.

As mentioned above, in the classical interpretation the bottleneck in systems like (1) is revealed in electron heating: $T_e > T_p$. Here the function $T_e(\lambda)$ may possess some specific features connected with the properties of the 'source' and 'sink'. In fact it follows from (1) that

$$\frac{dT_e}{d\lambda} = \frac{W'_\lambda}{[P(T_e)(T_e - T_p)]'_{T_e} - W'_{T_e}}, \tag{2}$$

where the prime defines the corresponding first derivative. From (2) one can see that for $W'_{\lambda} > 0$ and $\lambda > 0$ in the temperature region where

$$W'_{T_e} > [P(T_e)(T_e - T_p)]'_{T_e} \tag{3}$$

a decreasing section on the dependence $T_e(\lambda)$ will take place (Fig. 1). With an increase of temperatue T_e, inequality (3) changes into its opposite. This results in the appearance of the second rising branch of the function $T_e(\lambda)$ (Fig. 1). Consequently, the bottleneck at the realisation of the inequality (3) is described by an S-shaped function $T_e(\lambda)$ in any nonequilibrium system of type (1). This case occurs, for example, in semiconductors undergoing electron heating by electric (Conwell 1967; Bass and Gurevich 1975; Bass *et al.* 1984), thermal (Vakser 1981) or sound (Bugaev and Vakser 1984) fields. These are different modifications of the overheating instability in semiconductors.

3. Thermal Bottleneck of Inhomogeneously Heated Electrons: Phenomenological Criteria

Let us consider the manifestation of the bottleneck in semiconductors with $\nabla T_e \neq 0$. We investigate this phenomenon for the example of a one-temperature ($T_e = T_p = T$) medium in which the problem under consideration reveals itself with great clarity. In this problem, which is one-dimensional with respect to the z coordinate, the corresponding thermal balance equation is

$$\frac{\mathrm{d}Q}{\mathrm{d}z} = W(T, \lambda), \tag{4}$$

where Q is the density of the heat flux in the z direction and $W(T, \lambda)$ is a nonlinear non-equilibrium source as above (Vakser 1988, 1989*a*, 1989*b*). It is noteworthy that the quantity Q may consist of two terms. The first is the density of the heat stream, proportional to ∇T, and the second is proportional to λ, the so-called 'extraneous heat force'. A steady state in such systems is achieved through a balance between heat release and heat removal outside the specimen via its boundaries due to the heat stream Q. It is obvious that in the systems considered the boundary conditions are of paramount importance in the formation of the non-equilibrium state.

To make sure let us consider some types of boundary conditions. Due to the fact that later on only the nonlinearity of boundary conditions, but not their concrete form will be of importance, their traditional Newtonian form will be used (Carslaw and Jaeger 1959).

Symmetrical boundary conditions are considered first:

$$Q\,|_{z=0,a} = \mp\eta(T)(T - T_0)\,|_{z=0,a}\,, \tag{5}$$

where $\eta(T)$ is the surface heat transfer coefficient, a is the length of the sample and T_0 is the temperature of the external thermostat. The integration of (4) over the volume with regard to expression (5) gives the relationship (1), where the following substitutions are made:

$$P \to \eta/a, \quad T_{\mathrm{p}} \to T_0, \quad T_{\mathrm{e}} \to \overline{T} = \frac{1}{a}\int_0^a T(z)\,\mathrm{d}z,$$

and where the bar denotes averaging over the coordinate. And now it is easy to see that the thermal bottleneck for $\nabla T \neq 0$ and symmetrical boundary conditions has no characteristic features as compared with the case $\nabla T = 0$.

Quite a different physical situation occurs under nonsymmetrical boundary conditions. As noted elsewhere (Vakser 1988, 1989*a*, 1989*b*, 1990), this nonsymmetry may be connected both with the inequality of surface heat transfer coefficients ($\eta\,|_{z=0} \neq \eta\,|_{z=a}$) and with the inequality of surface temperatures ($T\,|_{z=0} \neq T\,|_{z=a}$). To demonstrate this, an abruptly nonsymmetrical problem is considered:

$$T\,|_{z=0} = T_0, \qquad Q(T,\lambda)\,|_{z=a} = \eta(T)(T - T_0)\,|_{z=a}\,. \tag{6}$$

Carrying out the standard procedure of averaging (4) over the volume and taking into account the conditions (6), one can obtain for the average temperature $\overline{T}$

$$Q(\overline{T},\lambda)\,|_{z=0} + W(\overline{T},\lambda)\,a = \eta(T)(T - T_0). \tag{7}$$

Equation (7) is derived proceeding from the assumption that $T\,|_{z=0,a} \approx \overline{T}$ and $\overline{W(T,\lambda)} \approx W(\overline{T},\lambda)$. From (7) it follows that

$$\frac{\mathrm{d}\overline{T}}{\mathrm{d}\lambda} = \left(a\frac{\partial W}{\partial \lambda} + \frac{\partial Q\,|_{z=0}}{\partial \lambda}\right) \Bigg/ \left(\frac{\mathrm{d}}{\mathrm{d}\overline{T}}[\eta(\overline{T})(\overline{T} - T_0)] - \frac{\partial W}{\partial \overline{T}}\,a - \frac{\partial Q\,|_{z=0}}{\partial \overline{T}}\right). \tag{8}$$

Comparison of (2) and (8) testifies to the fact that the bottleneck at $\nabla T \neq 0$ and $Q\,|_{z=0} \neq 0$ may have considerable effect on the properties of the $\overline{T}(\lambda)$ function. It is clear from (8), that unlike (2), not only the denominator of (8) at

$$a\frac{\partial W}{\partial T} > \frac{\mathrm{d}}{\mathrm{d}T}[\eta(T)\,(T - T_0)] - \frac{\partial Q\,|_{z=0}}{\partial T}, \tag{9}$$

but also the numerator at

$$-\frac{\partial Q\,|_{z=0}}{\partial \lambda} > a\frac{\partial W}{\partial \lambda} > 0 \tag{10}$$

can change their signs. Consequently, the function $\overline{T}(\lambda)$ can possess both S-type and N-type sections. If requirements (9) and (10) are satisfied simultaneously within the same λ interval, the characteristic $T(\lambda)$ will have S-N shape branches. Under special conditions a locked loop-type curve will appear.

It is worth mentioning that unlike criteria (3) for $\nabla T = 0$, which hold for hot $T_{\mathrm{e}} > T_0, T_{\mathrm{p}}$ electrons, the relationships (9) and (10) at $\nabla T \neq 0$ can be carried out for cold $\overline{T} < T_0, T_{\mathrm{p}}$ electrons as well.

Different cases of this situation have been considered for inhomogeneously heated electrons in uniform semiconductors (Vakser 1988, 1989*a*, 1989*b*, 1990). In these papers the specific properties of the bottleneck for hot and cold electrons have been investigated within the framework of the phenomenological approach to boundary conditions. The primary criteria (9) and (10) were satisfied due to certain relations between surface and bulk parameters of semiconductors. Now let us consider another solid-state model possessing the microscopic mechanism of this phenomenon.

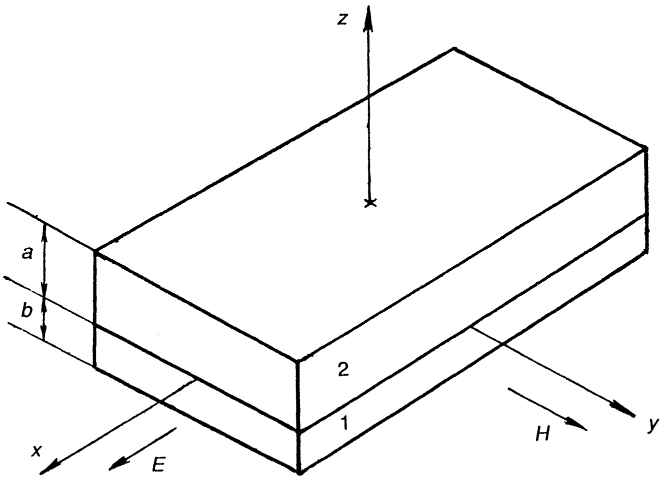

Fig. 2. Layer semiconductor where media 1 and 2 are homogeneous isotropic semiconductors, while ***E*** and ***H*** are external electric and magnetic fields.

4. Electron Thermal Bottleneck in Layer Inhomogeneous Semiconductors: Microscopic Model

We consider a two-layer semiconductor consisting of a wide-gap medium 1 ($-b < z < 0$) and a narrow-gap medium 2 ($0 < z < a$) (see Fig. 2). An electric current of density $j = j_x$ flows under the influence of the static electric field $E = E_x$. The static magnetic field $\boldsymbol{H}$ lies in the structure plane and is transverse to the direction of $\boldsymbol{j}$: $H = H_y$ (Fig. 2). It is assumed that the thicknesses of the layers satisfy the inequalities

$$b \lesssim L \ll a, \tag{11}$$

where L is the electron cooling length (see e.g. Bass and Gurevich 1975). We take an ideal thermal contact between media 1 and 2 to be realised on the plane $z = 0$, i.e. the conditions of continuity of electron temperatures and heat fluxes, the adiabatic thermal boundary conditions for electrons on the outer plane $z = +a$ and condition (5) on the plane $z = -b$:

$$Q_1 \,|_{z=-b} = -\eta(T_{e_1} - T_0), \qquad T_{e_1} \,|_{z=0} = T_{e_2} \,|_{z=0},$$

$$Q_1 \,|_{z=0} = Q_2 \,|_{z=0}, \qquad Q_2 \,|_{z=a} = 0. \tag{12}$$

The indices 1 and 2 refer here and below to the two layers.

In the approximations of quasi-neutrality and equilibrium phonons, characterised by the temperature T_p, the heat balance equations (in the one dimensional problem with respect to z) consist of (4) for layer 1 and (1), with the term dQ_2/dz to be added to the left-hand part, for layer 2, where $W_{1,2}=j_{1,2}E$ is the rate of Joule heat. The densities of thermal fluxes are

$$Q_{1,2} = -\kappa_{1,2}(T_e)\frac{dT_{e_{1,2}}}{dz} + \phi_{1,2}(T_{e_{1,2}}, H)\,E, \tag{13}$$

and those of currents are

$$j_{1,2} = \sigma_{1,2}(T_e)\,E + \frac{\phi_{1,2}(T_{e_{1,2}}, H)}{T_{e_{1,2}}}\frac{dT_{e_{1,2}}}{dz}. \tag{14}$$

Here $\kappa_{1,2}$ and $\sigma_{1,2}$ are the thermal and electric conductivities, while $\phi_{1,2}$ are the Nernst–Ettingshausen coefficients (Bass and Gurevich 1975).

Our target is to plot the CVC of the whole structure

$$\bar{j}\,(E) = \frac{1}{b+a}\left(\int_{-b}^{0} j_1\,dz + \int_{0}^{a} j_2\,dz\right), \tag{15}$$

subject to the electron thermal bottleneck. It is clear from (15) and (14) that for solving this problem it is necessary to find partial voltage–temperature characteristics (VTC) $\overline{T}_{1,2}(E)$. [It will be shown below that certain combinations of E and H fields behave as the parameter λ in Section 3; see equation (18).] The VTC are found by solving simultaneously the above-mentioned equations of thermal balance and boundary conditions (12).

In general, for an arbitrary degree of nonequilibrium ($|T_e - T_{0,p}| \approx T_{0,p}$), due to the nonlinearity of differential equations solving this problem requires some numerical calculation. However, we shall use another way which allows for analytical calculation. We make some simplifying assumptions, which do not influence our final conclusions. These are as follows: (1) the dominant relaxation momentum mechanism of electrons at layer 1 is scattering from the dislocations ($d\ln\sigma_1/d\ln T_1=1$); (2) the magnetic field is classically high; (3) the conductivity of layer 1 is considerably greater than layer 2, i.e. $\sigma_1 \gg \sigma_2$ and $\phi_1 \gg \phi_2$; and (4) media 1 and 2 are nondegenerate homogeneous isotropic semiconductors.

Under these assumptions equation (4) for medium 1 becomes linear (Vakser 1989*a*, 1989*b*). The thermal equation for medium 2, after its integration over the coordinate z subject to conditions (12) and neglecting electron heating will set the boundary (at $z=0$) value of the heat flux

$$Q_1\,|_{z=0} = Q_2\,|_{z=0} = \int_0^a P_2(T_{e_2})(T_{e_2} - T_p)\,dz. \tag{16}$$

Bearing in mind the fact that characteristic spatial temperature variations are localised at the cooling length L, formula (16) may be rewritten in the form

$$Q_2\,|_{z=0} = P_2(T_{e_2})(T_{e_2} - T_p)\,L\,|_{z=0}\,. \tag{17}$$

The latter condition together with (12) and (4) make the problem analytically soluble.

We will not dwell on the procedure for solving the equations because it is analogous to one carried out elsewhere (Vakser 1989*a*, 1989*b*). We will instead discuss the final results and their physical meaning.

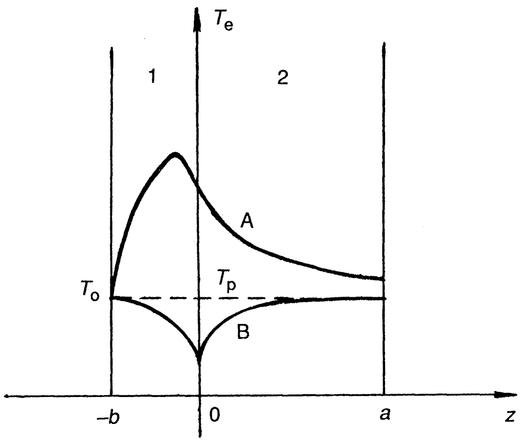

Fig. 3. Spatial distribution of electron temperature $T_e(z)$ for $T_0 = T_p$. Curve A corresponds to the case $H > 0$ and $E > 0$ or $H < 0$ and $E < 0$. Curve B corresponds to the case $H < 0$ and $E > 0$ or $H > 0$ and $E < 0$.

The temperature distribution at $\lambda b < \pi$ and $\eta = \infty$ is given by

$$T_{e_1}(z) = \frac{T_e(0)\sin\lambda(z+b) - T_0 \sin\lambda z}{\sin\lambda b}, \qquad \lambda = \left(\frac{\sigma_1}{\kappa_1 T_0}\right)^{\frac{1}{2}} HE. \tag{18}$$

The contact temperature $T_e(0)$ is found from (17). Graphical analysis of the equation obtained (which is similar to 7) shows that for positive directions of the external fields ($E > 0$ and $H > 0$) there are hot electrons [$T_e(z) > T_0$, see Fig. 3 curve A] in the system. Conditions (9) and (10) of multi-valued hot electrons VTC and CVC in the case of two-layer semiconductor structure reduce to the form

$$s < s_{cr}(\lambda b) \quad \text{and} \quad P_2^0 > P_{cr}^0, \tag{19}$$

where s is the 'rate' of the energy relaxation process in medium 2: $P_2(T_e) = P_2^0 T_{e_2}^{s-1}$, $P_2^0 = n_2 \nu_e$, where n_2 is the electron concentration, ν_e the equilibrium value of energy frequency (Conwell 1967; Bass *et al.* 1984); s_{cr} and P_{cr}^0 are the critical values of s and P_2^0. It is easy to obtain $s_{cr} \approx -0{\cdot}4$ to $-0{\cdot}5$ for $T_e \approx 3T_p$ to $5T_p$ and $P_{cr}^0 \approx (\kappa_1/b) f(s, \kappa_1, \kappa_2, L, b, \lambda b)$, where f is a dimensionless function of the order of one. Inequalities (19) are the necessary and sufficient conditions of multi-valued bottleneck hot electrons VTC and CVC.

The form of the CVC, taking into account the above assumptions, is as follows:

$$\bar{j}(E) = \frac{b}{a}\left(\sigma_1 E \frac{\tan\frac{1}{2}\lambda b}{\frac{1}{2}\lambda b} \frac{T(0)+T_0}{2T_0} + \frac{\phi_1(H)}{T_0} \frac{T(0)-T_0}{b}\right). \tag{20}$$

The inversion of the direction of one of the two fields (i.e. $E > 0$ and $H < 0$ or $E < 0$ and $H > 0$) changes the situation radically. These variations are connected with the formation of cold electrons in the system owing to the influence of the Ettingshausen effect (Bass *et al.* 1984). Let us consider in detail the physical sense of this phenomenon. In weak enough electric fields the transverse magnetic field accomplished spatial sorting of electrons according to their energies. This is an exhibition of the action of the 'extraneous thermal force'. Because of the two different field directions the hot ($T_{e_1} > T_0 = T_p$) electrons of layer 1 are localised at the plane $z = -b$, where they are thermalised by external thermostat [see conditions (12) at $\eta = \infty$]. Here the cold ($T_{e_1} < T_0 = T_p$) electrons are localised on the contact $z = 0$. It should be noted though that a similar energy separation process takes place in layer 2. Because $n_2 \ll n_1$ and $\phi_2 \ll \phi_1$, this process in medium 2 in comparison with medium 1 may be neglected (see Fig. 3, curve B). In this case the cold electrons of layer 2 serve as an external 'heater' for the cold electrons of medium 1. For a steady regime and carrying out requirements (9) [or (19) where the left-hand inequality has to be changed into the opposite one with $s_{cr} \geq 2$], the VTC and CVC (20) will become multi-valued. The physical sense of these mathematical conditions is the regime of the cold electron bottleneck described through multi-valued characteristics. Physically speaking this situation takes place in the case when the rate of thermal removal from electrons of layer 2, through the contact $z = 0$, to electrons of

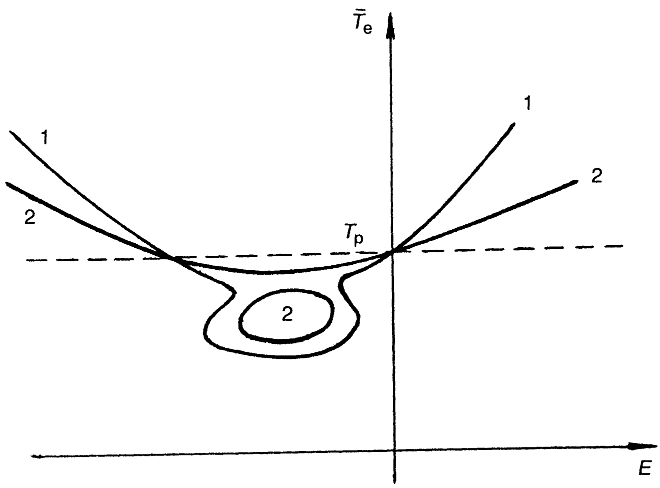

Fig. 4. Voltage–temperature characteristics of semiconductor structure corresponding to the case $T_p = T_0$, $s > s_{cr}$, $P_2^0 > P_{cr}$ and $H > 0$. Curves 1 and 2 conform to different values of P_2^0 : $1 - P_2^{0(1)} > P_{cr}$ and $2 - P_2^{0(2)} > P_2^{0(1)} > P_{cr}$.

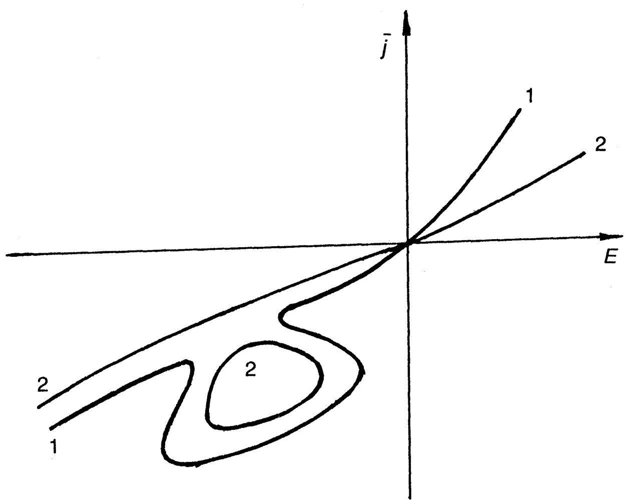

Fig. 5. Current–voltage characteristics of a two-layer semiconductor in the regime of the cold electron thermal bottleneck for $H > 0$. Designations of curves 1 and 2 are the same as in Fig. 4.

layer 1 exceeds the rate of heat removal via the electons of layers 1 into the external thermostat, through the boundary $z = -b$.

It is easy to demonstrate that, depending on the relations between the parameters of layers, the numbers of multi-valued sections on VTC and CVC in question varies from one to two (Figs 4 and 5). Under certain conditions a situation is possible where the lower branches of VTC and CVC split off forming a locked loop-curve (Figs 4 and 5, curve 2).

5. Conclusions

We sum up the theoretical results and discuss the possibilities of experimental investigation of the phenomenon at issue. It is clear from our discussion that the inhomogeneously heated electron bottleneck qualitatively differs from the uniformly heated electron one. First, it has greater variety of multi-valued CVC. Second, it has the possibility of the locked loop-type curve formation on the CVC. Third, here is the further prospect of creation of multi-functional devices of both active and passive type. It should be mentioned that reconstructing one type of device into another may be accomplished in a contactless way by means of the magnetic field [compare different sections of curves 1 and 2 in Figs 4 and 5 corresponding to various values of $\lambda(H)$].

The phenomenon may be investigated experimentally in planar geometry structures with an abrupt n–n$^+$ junction, consisting of two layers with close values of electron affinity. The principal criteria (19) are carried out in those sandwiches in which the low conductivity layer is made of a narrow-gap semiconductor with Kane dispersion law and the high conductivity layer is a wide-gap semiconductor with a quadratic electron energy dispersion law. In Table 2·1 of Sharma and Purohit (1974) one can see such pairs of semiconductors: InP–InSb, CdTe–InSb, CdS–PbTe, CdTe–Te, CdS–PbS, CdTe–PbTe etc.

The main requirement for the formation of a multi-valued cold electron CVC ($s > s_{cr} \gtrsim 2$) will be satisfied under dominant energy relaxation in layer 2 via either piezoelectric or deformation acoustic phonons, depending on the value of the nonquadratic parameter of the Kane dispersion law: $\alpha = d \ln p / d \ln \epsilon$, where p and ϵ are the electron quasi-momentum and electron energy.

Other convenient objects for observation of the phenomenon are graded-gap semiconductors $Cd_xHg_{1-x}Te$ or $Pb_{1-x}Sn_xTe$, in which x depends on coordinates (for details see Krotkus and Dobrovolskis 1988). In the case of $b \approx 10^{-2}$–10^{-3} cm, $H \approx 10^3$ Oe, $a \approx 0 \cdot 1$–$0 \cdot 5$ cm, $n_1 \approx 10^{14}$–10^{15} cm^{-3}, $n_2 \approx 10^{12}$–10^{13} cm^{-3}, $T_0 = T_p = 4 \cdot 2$ K, it follows from the calculations that the multi-valued sections on VTC and CVC in the region of cold electrons will arise in the interval of electric fields with an upper limit not exceeding $0 \cdot 1$ to $0 \cdot 01$ V cm^{-1}.

The problem under study is connected with the exhibition of the electron bottleneck at the longitudinal (along $\boldsymbol{j}$) characteristic $j = j_x(E)$. At the same time, naturally, transverse (in the z direction) characteristics of the bottleneck specimen will have their own pecularities as well (the thermo emf for example). This problem, however, in addition to the calculation of the corresponding device based on the phenomenon predicted, requires further investigation which will consitute the subject of a separate paper.

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