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The Clustering and Evolution of QSOs*

B. J. Boyle

Anglo-Australian Observatory, P.O. Box 296,
Epping, N.S.W. 2121, Australia.
Present address: Institute of Astronomy, Madingley Road,
Cambridge, CB3 0HA, U.K.

Abstract

Recent advances in instrumentation, particularly at the Anglo-Australian Telescope, have greatly increased the number of quasi-stellar objects (QSOs) identified at faint magnitudes ($B > 20$ mag) and high redshifts ($z > 2.2$). As a result, significant progress has been made in the study of QSO clustering and evolution in the last two to three years. This paper reviews the results obtained and discusses their relevance to models of galaxy formation and the large-scale structure of the universe.

1. Introduction

QSOs are the most readily observable class of astronomical object at $z > 1$ and so afford a unique opportunity to investigate the nature of the universe at high redshift. Their clustering properties can yield valuable information on large-scale structure in the early universe, discriminating between different models for galaxy formation. Their evolution in number density and/or luminosity with cosmic epoch provides a direct insight into the epoch of QSO formation which in turn constrains the epoch of galaxy formation.

Information on QSO clustering and evolution is drawn primarily from the statistical analysis of large unbiased QSO catalogues. In the optical, such catalogues are normally constructed from spectroscopic observations of QSO candidates selected by their anomalous colours from machine measurements of photographic plates (see Smith 1986 for a review). Until recently the prohibitive amounts of telescope time required to obtain spectroscopic confirmation for large numbers of faint QSO candidates had limited suitable QSO surveys to $B < 20$ mag and $z < 2.2$ (e.g. Schmidt and Green 1983; Marshall *et al.* 1984). At these 'bright' magnitudes little discrimination was afforded between competing models for QSO evolution (see below) and the space density of QSOs was too low to conduct a detailed analysis of their clustering properties at scales less than $50\,h^{-1}$ Mpc (Osmer 1981). At $z > 2.2$ the situation was even worse; small number statistics and possible selection effects plagued most attempts to establish the reality of any decline in the space density of QSOs at high redshifts from the few available samples of $z > 2.2$ QSOs (Osmer 1982).

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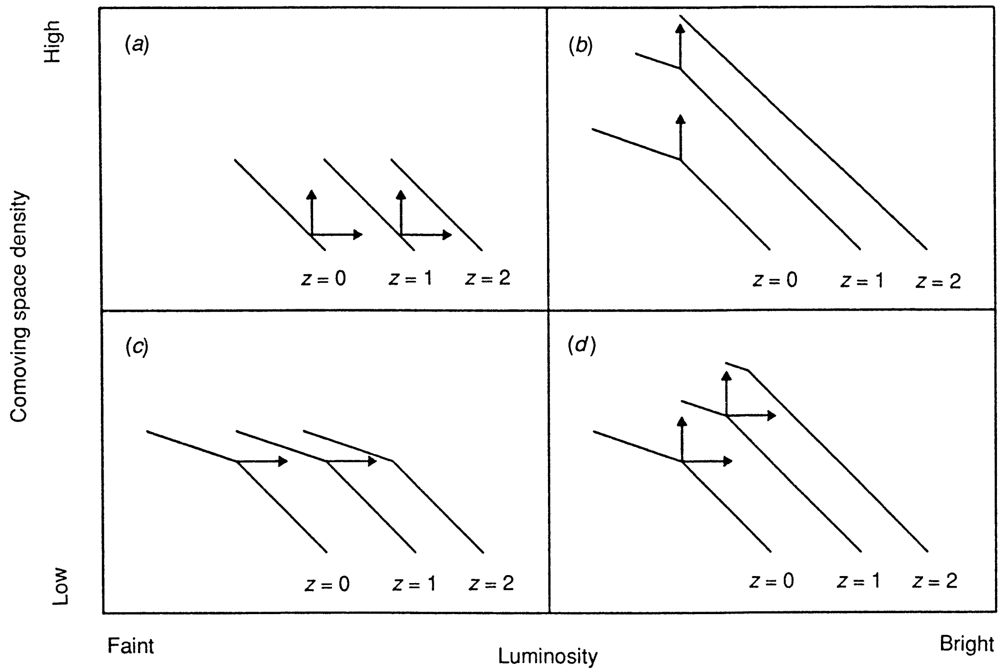

Fig. 1. Schematic representation of the QSO luminosity function. See text for details.

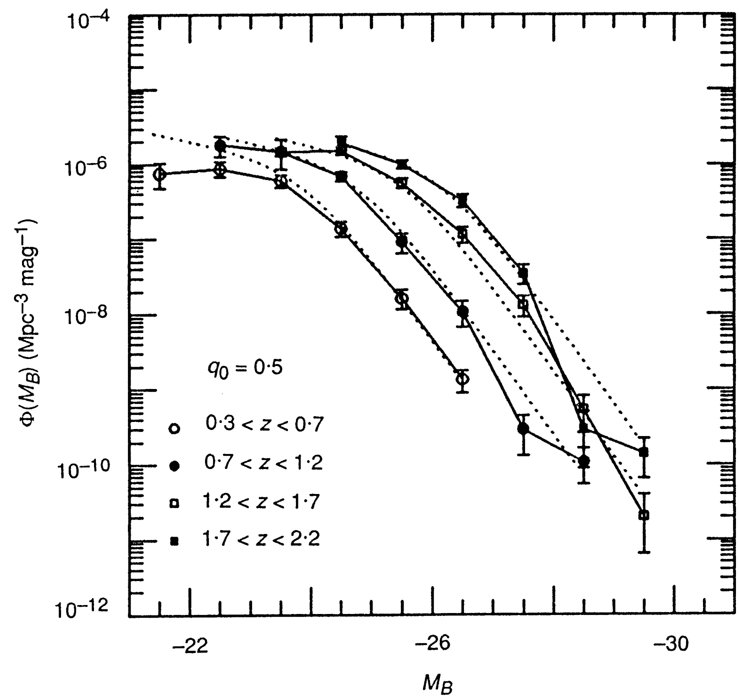

Fig. 2. The QSO luminosity function at $z < 2.2$ derived by Boyle *et al.* (1988). The dotted lines denote the prediction of the model described in the text.

Over the last three years, however, advances in instrumentation (in particular the introduction of multi-object spectroscopy) have resulted in a vast increase in the numbers of spectroscopically identified QSOs at faint magnitudes and high redshifts. This review discusses the new results on QSO clustering and evolution derived from such samples, with specific reference to those obtained with the Anglo-Australian Telescope (AAT).

2. QSO Evolution

The evolution in the number density or luminosity of QSOs with cosmic epoch is most directly established from the redshift dependence of the QSO luminosity function (LF). For any given redshift interval, the QSO LF simply represents the comoving space density of QSOs as a function of absolute magnitude or luminosity. From the schematic diagram in Fig. 1*a*, it is clear that, if the LF exhibits no conspicuous features in the magnitude range surveyed, the indicated redshift dependence of the QSO LF can be interpreted either as a uniform increase in space density (a vertical shift in the LF) or as a uniform increase in luminosity (a horizontal shift in the LF) towards higher redshift.

Unfortunately, whilst conventional spectroscopic techniques limited surveys to $B < 20$ mag, the derived LF exhibited a featureless power law form (Schmidt and Green 1983; Mitchell *et al.* 1984; Marshall *et al.* 1984) as represented by Fig. 1*a*, and consequently little discrimination between evolutionary models was possible. The first indications that a feature may be present in the QSO LF at low luminosities came from Koo (1983) who was able to carry out a limited spectroscopic survey of QSO candidates to $B < 22.5$ mag. The importance of such a feature in the LF is readily demonstrated from Figs 1*b*–1*d*; affording a straightforward determination of whether the LF evolves with redshift by increasing in space density (Fig. 1*b*), luminosity (Fig. 1*c*) or, indeed, a combination of the two (Fig. 1*d*).

With the advent of multi-object spectroscopy, however, the number of QSOs identified with $B > 20$ mag has dramatically increased, thus enabling detailed studies to be made of the LF at low luminosities. Using the FOCAP fibre-optic system at the AAT, Boyle *et al.* (1990) spectroscopically identified 420 faint ($B \lesssim 21$ mag) QSOs selected by their ultra-violet excess (UVX) from machine measurements of UK Schmidt Telescope (UKST) U and J photographic plates. Of these, 397 formed a complete sample with $0.3 < z < 2.2$. The QSO LF derived from this sample clearly shows a 'break' feature at low luminosities (Fig. 2).

The redshift dependence of the LF is one of increasing luminosity, rather than increasing number density, towards higher redshifts. Maximum likelihood techniques applied to the data-set (Boyle *et al.* 1988) reveal that, in a $q_0 = 0.5$ universe, the LF $[\Phi(L,z)]$ for QSOs with $M_B < -23$ can be represented by a two power law function:

$$\Phi \propto L^{-3.8\pm0.15} \qquad L > L^*(z)$$

$$\Phi \propto L^{-1.4\pm0.20} \qquad L < L^*(z),$$

where $L^*(z)$ is the luminosity of the break. The redshift dependence of this feature is given by

$$L^*(z) = (1+z)^{3.15\pm0.1} L(0).$$

The 'flattening' of the QSO number-magnitude relation at $B > 20$ mag (Crampton *et al.* 1987; Marano *et al.* 1988; Koo and Kron 1988) provides further evidence for a feature in the LF whose evolution with redshift is predominantly one of increasing luminosity towards higher redshift. Extrapolating the LF to $z = 0$ gives good agreement with estimates of the present day Seyfert LF (Cheng *et al.* 1985).

The simplest physical interpretation of this evolution is that it results from a single generation of long-lived (10^{10} yr) QSOs. However, under the conventional supermassive black hole model for QSO energy generation (Rees 1984), this model predicts larger remnant black hole masses ($10^9 - 10^{10} M_\odot$) and Eddington ratios (~ 0.01) in low redshift QSOs and Seyferts than are inferred from emission line/continuum studies (Wandel and Mushotsky 1986; Padovani 1989). Smaller black hole masses ($10^7 - 10^8 M_\odot$) are predicted in models which invoke successive generations of short lived (10^8 yr) QSOs to explain the observed evolution (Koo 1986) but such models require a great deal of 'fine tuning' to account for the constant comoving number density of QSOs at $z < 2.2$ (Cavaliere and Padovani 1988). A third explanation is the 'recurrent' model proposed by Cavaliere and Padovani (1988) in which QSO activity occurs recurrently in a significant fraction of the galaxy population; possibly fuelled by declining episodes of interactions with companion galaxies (Smith *et al.* 1986). In this case, the resulting black hole masses would be $10^8 - 10^9 M_\odot$. Since all models make different predictions for the remnant black hole mass, it is clear that detailed measurements of the masses of the central regions of nearby galaxies and active galactic nuclei will play a fundamental role in discriminating between them. At present the evidence for supermassive black holes in nearby galaxies is inconclusive (Dressler 1989), but observations with the Hubble Space Telescope should help to resolve the issue.

Just as multi-object spectroscopy has increased the number of $z < 2.2$ QSOs, the development of low resolution, high efficiency CCD spectrographs has recently led to a dramatic increase in the number of QSOs identified with $z > 2.2$. At these redshifts the UVX technique for selecting QSOs breaks down and alternative methods must be found. Using a sophisticated search in 4-dimensional multi-colour space to select candidates from UKST photographic plates, Warren *et al.* (1988) have identified 25 QSOs in the redshift range $2.9 < z < 4.5$ with the Faint Object Red Spectrograph at the AAT. The LF derived from this sample is shown in Fig. 3 together with the $z \sim 2$ LF derived by Boyle *et al.* (1988).

For $q_0 = 0.5$, Warren *et al.* (1988) find that, relative to $z = 2$, the comoving space densities of QSOs with $M_B = -26$ are down by a factor of 2 at $z = 3$ and by a factor of 6 at $z = 4$. Note, however, that this is only true for faint ($M_B \gtrsim -27$) QSOs; the flattening of the QSO LF at $z \geq 3$ as seen by Warren *et al.* (1988) will result in a much less dramatic decrease in the number density with redshift for the most luminous QSOs at this redshift. For example, Miller *et al.* (1990) find no evidence for any decrease in the comoving number density of

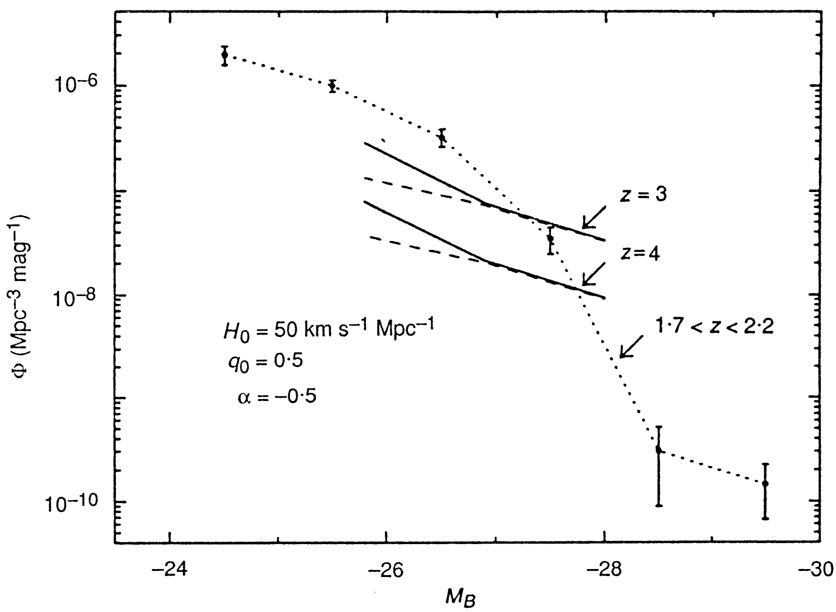

Fig. 3. QSO luminosity functions at $z = 3$ and $z = 4$ from Warren *et al.* (1988) compared with that by Boyle *et al.* (1988) at $1.7 < z < 2.2$ (dotted line). The solid and dashed lines represent the calculation of the LF with and without corrections for incompleteness.

$M_B = -28$ QSOs between $z = 2.2$ and $z = 4$. Moreover, some observations even support a continuing increase in the space density of the highest luminosity QSOs at $z > 3$ (Hazard *et al.* 1986).

A decline in the comoving number density of faint QSOs at high redshifts is also seen in the CCD grism survey of Schmidt *et al.* (1988). They observe a seven-fold decrease in the space density of QSOs with $M_B \lesssim -25.9$ between $z = 2$ and $z = 3.3$. In contrast, Koo and Kron (1988) find no significant evidence *at any magnitude* for a decrease in the comoving space density of faint ($M_B \sim -26$) QSOs over the entire redshift range $1.1 < z < 3.5$. Moreover, recent FOCAP observations of faint QSOs in the range $2.2 < z < 2.9$ by Boyle, Jones and Shanks (in preparation) reveal no significant decrease in the number density of QSOs over this redshift range (even before correction for possible incompleteness); although the statistics are sufficiently poor to also be consistent with the Warren *et al.* (1988) result at $z = 3$.

It is not clear the extent to which the above differences can be attributed to varying degrees of incompleteness in the original catalogues (Smith 1989). Certainly, even sophisticated multi-colour searches are subject to redshift and magnitude-dependent biases (Miller and Mitchell 1989) although most practitioners of this craft would claim that the biases are well known and can be corrected for. Nevertheless, differences do remain between the space densities of $z > 3$ QSOs derived from different surveys. Although the greatest body of evidence is perhaps in favour of a turn-around in the number density of QSOs between $2 < z < 3$, the data are still not inconsistent with a uniform space density of QSOs out to $z > 3$. Clearly, however, all these surveys do agree that the rapid evolution in luminosity witnessed at $z < 2.2$ cannot continue on much beyond $z = 2.5$; pointing to a fundamental change in the QSO population at this epoch.

The redshift at the which the comoving surface density of QSOs begins to decline—the 'redshift cut-off'—has a fundamental bearing on galaxy formation models. The Cold Dark Matter (CDM) model, with its 'late' epoch of galaxy formation, is already significantly constrained by the number of high redshift QSOs identified. Efstathiou and Rees (1988) have demonstrated that a constant comoving number density of QSOs over the range $2 < z < 4$ is only consistent with CDM if QSOs are short-lived and radiate at about the Eddington limit, contrary to some physical models of QSO evolution at $z < 2$ (see above). Efstathiou and Rees (1988) also conclude that the discovery of a significant number of QSOs with $z > 5$ would begin to pose serious problems for the CDM hypothesis.

3. QSO Clustering

In recent years there has been rapid progress in the measurement of QSO clustering. This has primarily arisen as a result of the availability of the large, faint ($B < 20$ mag) QSO catalogues whose surface densities and sizes are sufficiently high to permit a detailed study of QSO clustering at scales $< 50\,h^{-1}$ Mpc ($H_0 = 100\,h$ km/s/Mpc). Previously, studies of QSO clustering were restricted to larger scales with no significant detection of clustering (Osmer 1981; Chu and Zhu 1983).

The first tentative detection of QSO clustering was made by Shaver (1984) from an analysis of the heterogeneous Veron-Cetty and Veron (1984) catalogue. This clustering was confirmed at the 4.2σ level by Shanks *et al.* (1987, 1988) using the homogeneous Boyle *et al.* (1990) sample of 392 QSOs with $z < 2.2$. Shanks *et al.* (1988) identified 25 QSO pairs with comoving separations $<10\,h^{-1}$,* whereas on a random null hypothesis only 11.1 pairs would have been expected. At small scales, Shanks *et al.* (1988) also found that the 2-point QSO correlation function, $\xi_{QQ}(r)$, was consistent with the -1.8 power law form derived for galaxies (see Fig. 4):

$$\xi_{QQ}(r) = B_{QQ}\ r^{-1.8}$$

with an observed amplitude of $B_{QQ} = 33 \pm 12$ at $z = 1.4$ [the mean QSO redshift in the Boyle *et al.* (1990) sample]; consistent with the amplitude derived by Shaver (1984). Similar levels of clustering at these scales have also been identified in the samples of Barbieri *et al.* (1987) and Crampton *et al.* (1987) by Iovino and Shaver (1988), although at a lower significance level.

In order to compare the amplitude of the QSO correlation function at $z = 1.4$ with that of the galaxy correlation function (B_{GG}) at $z = 0$, Shanks *et al.* (1988) adopted two models for the evolution of the galaxy correlation function. First,

* Unless otherwise stated, $q_0 = 0.5$ will be assumed throughout this section.

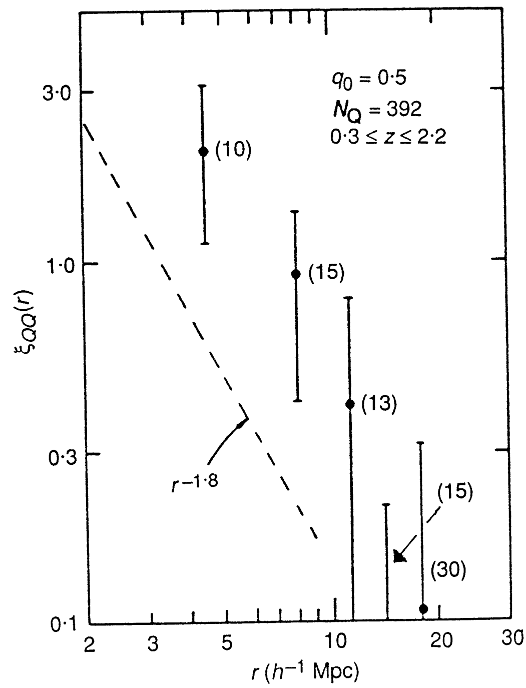

Fig. 4. The QSO 2-point correlation function at small scales (Shanks *et al.* 1988). The numbers in parentheses refer to the QSO pair count at that separation. A −1.8 power law is indicated for comparison.

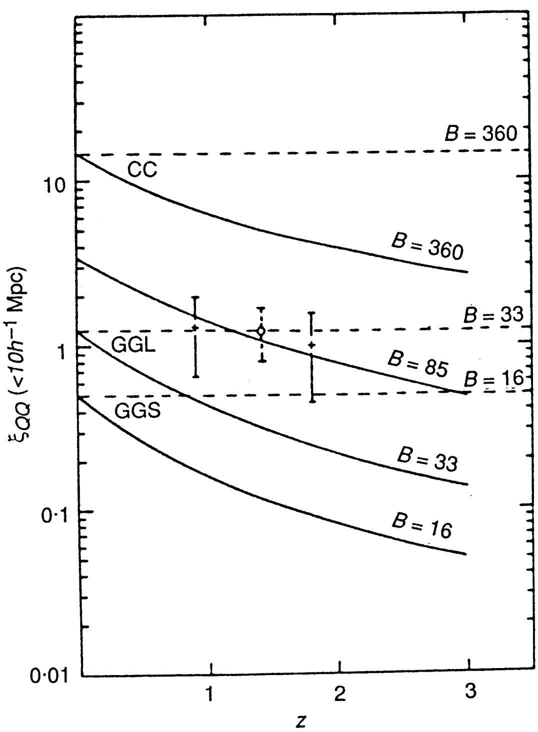

Fig. 5. The amplitude of ξ_{QQ} as a function of redshift. The solid and dashed lines represent the 'stable' and 'comoving' models respectively. The lines marked GGL and GGS represent models based on the range of amplitudes derived for ξ_{GG}. The CC model is based on the rich cluster correlation function (Bahcall and Soneira 1983).

a model in which the galaxy clustering is stable and the B_{GG} is large. In such a 'stable' model the density contrast (i.e. strength) of the clustering decreases at higher redshifts where the proper QSO background density is higher by $(1+z)^3$. However, in models of biased galaxy formation in which the galaxies are more strongly clustered than the mass (Davis *et al.* 1985), the galaxy clusters may still be expanding almost as fast as the Hubble flow. In such a 'comoving' model the amplitude of the galaxy correlation function will remain constant with redshift.

Shanks *et al.* (1988) found that the amplitude of the QSO correlation function was greater than that of galaxies in the 'stable' model but less than that for clusters of galaxies in the same model; this suggests a stable group environment for QSOs (consistent with direct CCD observations of QSOs; Yee and Green 1987). However, in the 'comoving' model the amplitude of the QSO correlation function at $z = 1.4$ is consistent with that of the galaxy correlation function (see Fig. 5). Although Shanks *et al.* (1988) found no evidence for any significant evolution with redshift in the amplitude of the QSO correlation function, the errors on the estimates of B_{QQ} in separate redshift bins were sufficiently large to prevent any discrimination between the 'stable' and 'comoving' models for galaxy formation.

In contrast, analyses of several smaller homogeneous samples by Kruszewski (1988) and Shaver (1988) have yielded apparent support for a strong decrease in the amplitude of the QSO correlation function with redshift. However, both these claims rest predominantly on the high values of the B_{QQ} derived at $z < 0.5$, where the samples used may be contaminated by radio galaxies which are known to exhibit strong clustering (Peacock *et al.* 1988). By combining all the results from the various surveys together there may be some evidence for a mild decrease in the strength of QSO clustering with redshift (Iovino and Shaver 1988), although the present data are insufficient to rule out any model of QSO clustering.

Even with the availability of much larger samples of faint QSOs, the QSO correlation function still exhibits no significant signal at scales $>10\,h^{-1}$ Mpc (Shanks *et al.* 1988; Osmer and Hewett 1988). Some isolated groups with comoving sizes in the range $10\,h^{-1}$ Mpc $< r < 100\,h^{-1}$ Mpc have been identified (Crampton *et al.* 1988) but their statistical significance is hard to assess as they do not yet contribute significantly towards the amplitude of the correlation function at these scales. However, the appearance of any large-scale structure in the QSO correlation function may have significant cosmological importance. In gravitational instability models, any small features in ξ_{QQ} at large scales will grow as $(1+z)^{-2}$ until $z = 0$ in an $\Omega_0 = 1$ universe, whereas little growth is expected in a low Ω_0 universe (Peebles 1980). Any observed increase in the large-scale amplitude of ξ_{QQ} towards low redshift would therefore imply $\Omega_0 = 1$. In contrast, the position of any large-scale feature is not expected to evolve significantly with redshift. The redshift dependence of such a feature in models with different values of q_0 would therefore provide a sensitive q_0 test, since differences in calculated comoving separations amount to 40% between $q_0 = 0.1$ and $q_0 = 0.5$ models, even over the limited redshift range $0 < z < 1.4$. Although Shanks *et al.* (1988) found no significant evidence for the QSO correlation function to develop systematically positive correlations at large scales ($r > 20\ h^{-1}$ Mpc) towards lower redshift, an increased scatter about $\xi_{QQ} = 0$ at these scales was observed at $z < 1.4$. However, much larger surveys will be required before any large-scale features in ξ_{QQ} can be identified reliably.

4. Future Prospects

The study of QSO clustering and evolution has clearly progressed dramatically in the last two to three years. As demonstrated above, it has brought with it the opportunity to study the structure of the universe at early epochs and provide important constraints on models of galaxy formation. However, a number of fundamental questions remain to be fully answered. As far as QSO evolution is concerned, perhaps the most important problem is the precise form of QSO evolution beyond $z = 2.2$ and the position of the 'redshift cut-off'. While considerably more data are needed, particularly in the range $2.2 < z < 3.0$, some thought must be given to the method used to select QSOs at these redshifts. Traditional multi-colour techniques, whilst successful in the range $3.0 < z < 4.5$, are subject to significant selection effects at $2.2 < z < 3$ (Miller and Mitchell 1989) and cannot, at present, identify QSOs with $z \gtrsim 4.7$ (Warren 1988). As discussed in Section 2, these are precisely the redshift ranges which may provide the greatest constraints on the CDM model of galaxy formation.

The evolution with redshift in the strength of QSO clustering will also provide a powerful method of discriminating between galaxy formation models. A more accurate determination of the QSO correlation function at different epochs may also provide a unique opportunity to derive constraints on Ω_0 and q_0 from the evolution of any large-scale features in ξ_{QQ}.

All these projects will, however, probably require a least an order of magnitude more data at faint magnitudes than the ~ 500 QSOs currently available. Although a catalogue of 5000 QSOs with $B < 22.5$ may not seem feasible at present, continuing developments in instrumentation may allow such a catalogue to be realised in the not too distant future. For example, the AAT is currently planning a wide-field capability (Taylor and Gray 1989) which would enable spectra to be obtained for up to 400 objects simultaneously over a 2-degree field. In parallel, large CCDs can also now be used to extend QSO candidate selection to fainter magnitudes (Anderson and Schechter 1988), possibly using variability as the least biased method of selecting faint ($B < 24$ mag) QSOs out to $z = 5$ and beyond. Typical field sizes now available (e.g. the f/1 focal reducer + Thomson 1024×1024 chip at the AAT has a $17.3' \times 17.3'$ field of view) mean that several square degrees can be surveyed each night. With such instrumentation available, the field of QSO clustering and evolution looks set for another significant advance, providing further fundamental insights into the structure and evolution of the universe at high redshift.

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References

Anderson, S.F., and Schechter, P.L. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 125 (Brigham Young: Provo).

Bahcall, N.A., and Soneira, R.M. (1983). *Astrophys. J.* **270**, 20.

Barbieri, C., Cristiani, S., Iovino, A., and Nota, A. (1987). Unpublished.

Boyle, B.J., Shanks, T., and Peterson, B.A. (1988). *Mon. Not. R. astr. Soc.* **235**, 935.

Boyle, B.J., Fong, R., Shanks, T., and Peterson, B.A. (1990). *Mon. Not. R. astr. Soc.* **243**, 1.

Cavalieri, A., and Padovani, P. (1988). *Astrophys. J.* **333**, L33.

Cheng, F.Z., Danese, J., De Zotti, G., and Franceschini, A. (1985). *Mon. Not. R. astr. Soc.* **212**, 857.

Chu, Y.-Q., and Zhu, X.-F. (1983). *Astrophys. J.* **267**, 4.

Crampton, D., Cowley, A.P., and Hartwick, F.D.A. (1987). *Astrophys. J.* **314**, 129.

Crampton, D., Cowley, A.P., and Hartwick, F.D.A. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 254 (Brigham Young: Provo).

Davis, M., Efstathiou, G., Frenk, C.S., and White, S.D.M. (1985). *Astrophys. J.* **221**, 371.

Dressler, A. (1989). *In* 'Active Galactic Nuclei', IAU Symposium No. 134 (Eds D.E. Osterbrock and J.S. Miller), p. 217 (Reidel: Dordrecht).

Efstathiou, G., and Rees, M.J. (1988). *Mon. Not. R. astr. Soc.* **235**, 5P.

Hazard, C., McMahon, R.G., and Sargent, W. (1986). *Nature* **322**, 38.

Iovino, A., and Shaver, P. (1988). *Astrophys. J.* **330**, L13.

Koo, D.C. (1983). *In* 'Quasars and Gravitational Lenses', 24th Liege Astrophysical Colloquium, p. 240 (University of Liege).

Koo, D.C. (1986). *In* 'Structure and Evolution of Active Galactic Nuclei' (Eds G. Giuricin *et al.*), p. 317 (Reidel: Dordrecht).

Koo, D.C., and Kron, R.G. (1988). *Astrophys. J.* **325**, 92.

Kruszewski, A. (1988). *Acta Astronomica* **38**, 155.

Marano, B., Zamorani, G., and Zitelli, V. (1988). *Mon. Not. R. astr. Soc.* **232**, 111.

Marshall, H.L., Avni, Y., Braccesi, A., Huchra, J.P., Tanenbaum, H., Zamorani, G., and Zitelli, V. (1984). *Astrophys. J.* **283**, 50.

Miller, L., and Mitchell, P.S. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 114 (Brigham Young: Provo).

Miller, L., Mitchell, P.S., and Boyle, B.J. (1990). *Mon. Not. R. astr. Soc.* in press.

Mitchell, K.J., Warnock, A., and Usher, P.D. (1984). *Astrophys. J.* **287**, L3.

Osmer, P.S. (1981). *Astrophys. J.* **247**, 762.

Osmer, P.S. (1982). *Astrophys. J.* **253**, 28.

Osmer, P.S., and Hewett, P.C. (1989). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 273 (Brigham Young: Provo).

Padovani, P. (1989). *Astron. Astrophys.* **209**, 27.

Peacock, J.A., Miller, L., Collins, C.A., Nicholson, D., and Lilly, S.J. (1988). *In* 'Large Scale Structures in the Universe', IAU Symposium No. 130 (Eds J. Audouze *et al.*), p. 579 (Reidel: Dordrecht).

Peebles, P.J.E. (1980). 'The Large Scale Structure of the Universe' (Wiley: Princeton).

Rees, M.J. (1984). *Ann. Rev. Astr. Astrophys.* **22**, 471.

Schmidt, M., and Green, R.F. (1983). *Astrophys. J.* **269**, 352.

Schmidt, M., Schneider, D.P., and Green, R.P. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 87 (Brigham Young: Provo).

Shaver, P.A. (1984). *Astron. Astrophys.* **136**, L9.

Shaver, P.A. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 265 (Brigham Young: Provo).

Shanks, T., Fong, R., Boyle, B.J., and Peterson, B.A. (1987). *Mon. Not. R. astr. Soc.* **227**, 739.

Shanks, T., Boyle, B.J., and Peterson, B.A. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 244 (Brigham Young: Provo).

Smith, E.P., Heckman, T.M., Bothun, G.D., Romanshin, W., and Balik, B. (1986). *Astrophys. J.* **306**, 64.

Smith, M.G. (1986). *In* 'Quasars', IAU Symposium No. 119 (Eds G. Swarup and V. Kapahi), p. 17 (Reidel: Dordrecht).

Smith, M.G. (1989). *In* 'Active Galactic Nuclei', IAU Symposium No. 134 (Eds D.E. Osterbrock and J.S. Miller), p. 1 (Reidel: Dordrecht).

Taylor, K.T., and Gray, P.M. (1989). A Wide-Field Multi-Fibre Prime Focus for the AAT. AAO Design Study Report,

Veron-Cetty, M.P., and Veron, P. (1984). A Catalogue of Quasars and Active Nuclei. ESO Scientific Report No. 1.

Wandel, A., and Mushotsky, R.F. (1986). *Astrophys. J.* **306**, L61.

Warren, S.J. (1988). Ph.D. Thesis, University of Cambridge.

Warren, S.J., Hewett, P.S., and Osmer, P.S. (1988). *In* Proceedings of a Workshop on Optical Surveys for Quasars, ASP Conference Series No. 2 (Eds P.S. Osmer *et al.*), p. 98 (Brigham Young: Provo).

Yee, H.K., and Green, R.F. (1987). *Astrophys. J.* **319**, 28.

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