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Diquarks and the Bosonisation of QCD

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Abstract

Previously the functional integral formulation of quantum chromodynamics (QCD) has been transformed into one involving colour singlet and colour octet bilocal fields describing $\overline{q}q$ states. While useful in determining the effective action for the observable colour singlet mesons, this formulation is of no use in determining the effective action for the baryon states. Here we show that there exists an alternative bosonisation of QCD in which the colour singlet meson fields and the colour triplet diquark fields form a complete set of functional integration variables. These diquark fields play an essential role in the colour singlet baryon states.

1. Introduction

In quantum chromodynamics (QCD) the hadrons are understood as quark bound states bound by gluon exchange. Thus while the fundamental quantum field theory (QFT) is defined in terms of quark and gluon fields, the observables (i.e. the hadrons) correspond to fields in another QFT which, for a long time, has been known to describe low energy hadronic physics very well. Hence one of the most fundamental problems in QCD is to discover if and how the quark-gluon QFT may be transformed into a meson-baryon QFT. The meson-baryon action in that QFT must specify meson and baryon masses and their various coupling constants, and all of these parameters must be determined by the fundamental quark-gluon dynamics. There has in fact been considerable progress in the study of this problem, particularly as far as the meson sector of the hadronic QFT is concerned. Early studies of this were by Kleinert (1976) and Schrauner (1977), but these ignored colour and the baryon sector. In Roberts and Cahill (1987), an extension of Cahill and Roberts (1985), it was shown how QCD could be formally transformed, using functional integral methods, into a QFT involving bilocal fields describing colour singlet ($\mathbf{1}_C$) and colour octet ($\mathbf{8}_C$) $\overline{q}q$ states. This formulation is ideal for studying the realisations of chiral symmetry in QCD. Praschifka *et al.* (1987*a*, 1987*b*) and Roberts *et al.* (1988, 1989) have shown that the bilocal $\mathbf{1}_C$ $\overline{q}q$ fields may be expanded in terms of local fields which correspond to the meson fields. This was demonstrated by deriving the effective action for these meson fields. The meson masses occurring in this effective action were shown, in Cahill *et al.*

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(1987), to be same as those that arise in a Bethe-Salpeter description of $\overline{q}q$ bound states. This illustrates the general rule that the powerful functional methods lead very efficiently to the same results as would follow from older QFT methods.

However the above bosonisation of QCD in terms of $\mathbf{1}_c$ and $\mathbf{8}_c$ bilocal bose fields suffers from one major deficiency, and that is that the $\mathbf{8}_c$ fields correspond to unbound $\overline{q}q$ states. This is because gluon exchange in such states is repulsive (this is most easily seen in Cahill *et al.* 1987). Hence the $\mathbf{8}_c$ boson fields do not permit any sensible expansion into local fields. Thus it has been completely unclear up to now as to what should be done with these unphysical bose fields. The purpose of this paper is to show that there is an alternative bosonisation of QCD. In this bosonisation the bilocal fields that arise are only the $\mathbf{1}_c$ $\overline{q}q$ fields that arose in the first bosonisation and $\overline{\mathbf{3}}_c$ qq diquark fields (and their $\mathbf{3}_c$ $\overline{q}\overline{q}$ antimatter partners). This result is important for two reasons. First, because gluon exchange between q and q in $\overline{\mathbf{3}}_c$ states is attractive (see Cahill *et al.* 1987), their exists, as we will discuss here, an expansion of the bilocal diquark fields into local diquark fields, with each such local diquark field describing a particular diquark bound state. The masses of these states will be seen to be identical to those derived in Cahill *et al.* (1987) using Bethe-Salpeter techniques. Second, the $\overline{\mathbf{3}}_c$ qq states play a fundamental role in baryon structure because baryons in QCD are three quark colour singlet states and hence [see Cahill *et al.* (1987), (1989, present issue p. 129)] any two of the quarks are necessarily in $\overline{\mathbf{3}}_c$ states. Hence the diquark boson fields that arise in the new bosonisation of QCD are the components of the baryons, and we are clearly on the path to a meson-baryon effective action description of QCD. Interestingly the diquark $\mathbf{6}_c$ states, for which gluon exchange is repulsive, do not arise in the new bosonisation. Hence it could be said that we have replaced the unphysical $\mathbf{8}_c$ $\overline{q}q$ sector by the physical $\overline{\mathbf{3}}_c$ and $\mathbf{3}_c$ diquark sector.

In Section 2 we derive the new bilocal meson-diquark bosonisation of QCD, and in Section 3 we extract the effective action for, as an example, the $J^P = 0^+$ diquark states, which naturally introduces the form factors for these bound states and which ensure the absence of any divergences in the integrations which determine the parameters in the action. Section 4 summarises the results. The proof of a determinant identity is given in the Appendix.

2. Bilocal Fields

In this section we show how the generating functional of QCD, which involves functional integration over quark and gluon fields may be transformed to one involving bilocal meson and diquark fields. The transformation essentially amounts to a change of variables in the functional integrations, but importantly to a set of variables which is far more useful in analysing the low energy states of QCD—the hadrons.

The generating functional for QCD in Euclidean metric is

$$Z[J^a_\mu,\overline{\eta},\eta] =$$

$$\int D\overline{q}Dq\prod_{a\mu}DA^a_\mu\Delta_f[A^a_\mu]\exp\Big(-S[A^a_\mu,\overline{q},q]+\int d^4x(\overline{\eta}q+\overline{q}\eta+J^a_\mu A^a_\mu)\Big), \tag{1}$$

where

$$S[A_\mu^a,\overline{q},q]=\int d^4x\Big(\tfrac{1}{4}F_{\mu\nu}^aF_{\mu\nu}^a+\frac{1}{2\xi}(\partial_\mu A_\mu^a)^2+\overline{q}\{\gamma_\mu(\partial_\mu-ig\tfrac{\lambda^a}{2}A_\mu^a)\}q\Big),$$

$$F_{\mu\nu}^a=\partial_\mu A_\nu^a-\partial_\nu A_\mu^a+gf^{abc}A_\mu^bA_\nu^c, a=1,2...8,$$

and where $J_\mu^a,\overline{\eta},\eta$ are external sources and $\Delta_f[A]$ is the Faddeev-Popov determinant. When the sources are put equal to zero then

$$Z(T)=Z(0)\sum_n \exp(-E_nT),$$

where T is the Euclidean time interval and $\{E_n\}$ is the energy spectrum of QCD. Here we consider massless quarks and as well as the local colour symmetry the action then has global chiral symmetry $G=U_L(N_f)\otimes U_R(N_f)$. As shown in Roberts and Cahill (1987) (1) may be written as (we may now put $J=0$)

$$Z[0,\overline{\eta},\eta]=\exp\left(W_1\left(ig\frac{\delta}{\delta\eta(x)}\frac{\lambda^a}{2}\gamma_\mu\frac{\delta}{\delta\overline{\eta}(x)}\right)\right)\int D\overline{q}Dq\exp\left(-S[\overline{q},q]+\int(\overline{\eta}q+\overline{q}\eta)\right), \quad (2)$$

where

$$S[\overline{q},q]=\int d^4xd^4y\Big(\overline{q}(x)\gamma.\partial_x\delta^4(x-y)q(y)+$$

$$+\tfrac{1}{2}g^2\overline{q}(x)\frac{\lambda^a}{2}\gamma_\mu q(x)D_{\mu\nu}^{ab}(x-y)\overline{q}(y)\frac{\lambda^b}{2}\gamma_\nu q(y)\Big), \quad (3)$$

$$W_1[J_\mu^a]=\sum_{n=3}^{\infty}\int d^4x_1...d^4x_n\frac{1}{n!}D_{\mu_1...\mu_n}^{a_1...a_n}(x_1...x_n;\xi)\prod_{i=1}^{n}J_{\mu_i}^{a_i}(x_i).$$

Here W_1 involves the $n\geq 3$ point connected gluon Green functions. To simplify the presentation of the following we shall use the gauge in Cahill and Roberts (1985) in which the $n=2$ point function in (3) is expressed as

$$g^2D_{\mu\nu}^{ab}(x)=\delta^{ab}\delta_{\mu\nu}\int\frac{d^4q}{(2\pi)^4}\frac{4\pi\alpha(q^2)}{q^2}e^{iq.x}=\delta^{ab}\delta_{\mu\nu}D(x),$$

and the case of more general gauges may be easily determined by the application of the results of Roberts and Cahill (1987).

It is at this stage that the present analysis differs markedly from the earlier bosonisation, for we now perform a Fierz rearrangement of the quartic part of (3) which introduces the Grassmannian structures which ultimately lead to the meson and diquark states of QCD.

Consider the following Fierz spin identity:

$$\gamma^{\mu}_{rs}\gamma^{\mu}_{tu} = K^{a}_{ru}K^{a}_{ts}; \{K^{a}\} = \{\mathbf{1}, i\gamma_5, \tfrac{i}{\sqrt{2}}\gamma^{\mu}, \tfrac{i}{\sqrt{2}}\gamma^{\mu}\gamma_5\}. \tag{4}$$

Using the following properties of the charge conjugation matrix $C = \gamma^2\gamma^4$;

$$C^2 = -\mathbf{1}, C^{-1}\gamma^{\mu}C = -\gamma^{\mu T},$$

where T denotes the transpose, we also obtain,

$$\gamma^{\mu}_{rs}\gamma^{\mu}_{tu} = (K^{a}C^{T})_{rt}(C^{T}K^{a})_{us}. \tag{5}$$

For the generators of the ($N_C = 3$) colour group we have (Cvitanovic 1976)

$$\lambda^{a}_{\alpha\beta}\lambda^{a}_{\gamma\delta} = 2(\delta_{\alpha\delta}\delta_{\beta\gamma} - \tfrac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta}),$$

which on using

$$\sum_{\rho=1}^{3} \epsilon_{\rho\alpha\gamma}\epsilon_{\rho\beta\delta} = \delta_{\alpha\beta}\delta_{\gamma\delta} - \delta_{\alpha\delta}\delta_{\gamma\beta},$$

we obtain the Fierz colour identity

$$\lambda^{a}_{\alpha\beta}\lambda^{a}_{\gamma\delta} = \tfrac{4}{3}\delta_{\alpha\delta}\delta_{\beta\gamma} + \tfrac{2}{3}\sum_{\rho}\epsilon_{\rho\alpha\gamma}\epsilon_{\rho\delta\beta}. \tag{6}$$

We also have the Fierz flavour identity

$$\delta_{ij}\delta_{kl} = F^{c}_{il}F^{c}_{kj}, \{F^{c}\} = \{\sqrt{\tfrac{1}{3}}\mathbf{1}, \sqrt{2}T^{1}, ...\sqrt{2}T^{8}\}, \tag{7}$$

for $N_F = 3$ flavours where $T^{a} = \frac{1}{2}\lambda^{a}$ are the generators of the $SU(3)$ flavour group, and

$$\delta_{ij}\delta_{kl} = H^{f}_{ik}H^{f}_{lj}; \{H^{f}_{ik}\} = \{\frac{i}{\sqrt{2}}\epsilon_{nik}, n = 1,2,3; S^{m}_{ik}, m = 1, ..., 6\}, \tag{8}$$

where $\{S^{m}, m = 1, ...6\}$ are the 6 symmetric 3×3 matrices given in (A3).

Anticommuting the Grassmann elements in the quartic part of (3) and using the Fierz identities (4)-(8), we obtain

$$\begin{aligned} S[\overline{q}, q] = \int d^4x d^4y \Big[& \overline{q}(x)\gamma.\partial\,\delta^4(x-y)q(y) + \\ & -\tfrac{1}{6}\overline{q}(x)K^{a}F^{c}q(y)D(x-y)\overline{q}(y)K^{a}F^{c}q(x) + \\ & -\tfrac{1}{12}\overline{q}(x)K^{a}\epsilon^{\rho}H^{f}\overline{q}(y)^{cT}D(x-y)q(y)^{cT}K^{a}\epsilon^{\rho}H^{f}q(x)\Big], \end{aligned} \tag{9}$$

where $q^c = Cq$, $\overline{q}^c = \overline{q}C$, ϵ^ρ is a colour matrix with elements $\epsilon_{\rho\alpha\beta}$, and where Dirac, colour and flavour indices are suppressed. It is straightforward to check that in the quartic terms in (9) $\overline{q}(y)K^aF^cq(x)$ are $\mathbf{1}_c$ $\overline{q}q$ fields with the flavour ($\mathbf{1}_F$ or $\mathbf{8}_F$) representations determined by F^c, while $q(y)^{cT}K^a\epsilon^\rho H^f q(x)$ are $\overline{\mathbf{3}}_c$ qq fields with the flavour ($\overline{\mathbf{3}}_F$ or $\mathbf{6}_F$) representations determined by the H^f. These results follow from the colour and flavour representations of the quark fields. The (integral) spin of these boson fields is determined by the K^a. It is very significant to the physics of QCD that it is only these $\overline{q}q$ and qq (and their $\overline{qq}$ conjugate) colour representations which arise from the above Fierz rearrangements.

It is convenient to introduce the notation $\{M_m^\theta\} = \{\sqrt{\tfrac{4}{3}}K^aF^c\}$ and $\{M_d^\phi\} = \{i\sqrt{\tfrac{2}{3}}K^a\epsilon^\rho H^f\}$ for the various tensor products in (9). We now extend the usual procedure of producing the quartic terms in (2), in which $S[\overline{q},q]$ has the form of (9), by means of functional integrals for bilocal fields, and (2) can be written (up to unimportant factors)

$$Z[0,\overline{\eta},\eta] = \exp(W_1)\int D\overline{q}DqD\mathcal{B}D\mathcal{D}^\star D\mathcal{D}\exp\Big(\int\int\Big[-\overline{q}(x)\gamma.\partial\delta^4(x-y)q(y)$$

$$-\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2D(x-y)}-\frac{\mathcal{D}^\phi(x,y)\mathcal{D}^\phi(x,y)^\star}{2D(x-y)}$$

$$-\overline{q}(x)\frac{M_m^\theta}{2}q(y)\mathcal{B}^\theta(x,y)-\tfrac{1}{2}\overline{q}(x)\frac{M_d^\phi}{2}\overline{q}(y)^{cT}\mathcal{D}^\phi(x,y)^\star$$

$$-\tfrac{1}{2}\mathcal{D}^\phi(x,y)q(y)^{cT}\frac{M_d^\phi}{2}q(x)\Big]+\int(\overline{\eta}q+\overline{q}\eta)\Big), \tag{10}$$

where $\mathcal{B}^\theta(x,y) = \mathcal{B}^\theta(y,x)^\star$ are 'hermitean' bilocal fields (* denotes complex conjugation). The functional integrations over $\mathcal{D}$ and $\mathcal{D}^\star$ denote integrations over the real and imaginary parts of $\mathcal{D}$. To confirm the results of the above bilocal functional integrations it is necessary to expand the exponential in powers of the Grassmann elements before doing the integrations, and then resumming the resulting series, which gives the exponentiated quartic terms in (9).

The integrations over the Grassmann elements may now be performed (see Berezin (1966) for the general theory of such integrations) and we obtain

$$Z[0,\overline{\eta},\eta] = \exp(W_1)\int D\mathcal{B}D\mathcal{D}D\mathcal{D}^\star(\mathrm{Det}\mathcal{F}^{-1}[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}])^{\frac{1}{2}}$$

$$\times\exp\Big(-\int\int\frac{\mathcal{B}^\theta(x,y)\mathcal{B}^\theta(y,x)}{2D(x-y)}-\int\int\frac{\mathcal{D}^\phi(x,y)\mathcal{D}^\phi(x,y)^\star}{2D(x-y)}+\tfrac{1}{2}\int\Theta\mathcal{F}\Theta^T\Big), \tag{11}$$

where $\Theta = (\overline{\eta}, -\eta^T)$, and

$$\mathcal{F}^{-1}[\mathcal{B},\mathcal{D},\mathcal{D}^\star] = \begin{pmatrix} -\mathcal{D} & G^{-1T} \\ -G^{-1} & -\overline{\mathcal{D}} \end{pmatrix},$$

$$G^{-1}(x,y,[\mathcal{B}]) = \gamma.\partial\delta^4(x-y) + \mathcal{B}(x,y),$$

$$\mathcal{B}(x,y) = \mathcal{B}^\theta(x,y)\frac{M_m^\theta}{2},$$

$$\overline{\mathcal{D}}(x,y) = D^\phi(x,y)^\star \frac{M_d^\phi}{2} C^T,$$

$$\mathcal{D}(x,y) = \mathcal{D}^\phi(y,x)C^T\frac{M_d^\phi}{2}.$$

The anticommutation of the Grassmann elements in (10) causes the matrix valued bilocal fields $\mathcal{D}(x,y)$ and $\overline{\mathcal{D}}(x,y)$ to be completely antisymmetric (including the two space-time variables). This ensures that, when the local diquark fields are introduced in Section 3, the diquark states satisfy the Pauli exclusion principle. Then, for example, for S-wave states the $\overline{\mathbf{3}}_f$ diquarks must have spin 0 whilst the S-wave $\mathbf{6}_f$ diquarks are spin 1.

Using the identity (A1) from the Appendix we may rewrite the determinant in (11) as

$$\mathrm{Det}\mathcal{F}^{-1} = (\mathrm{Det}(G[\mathcal{B}]^{-1}))^2\mathrm{Det}(\mathbf{1} + G[\mathcal{B}]\overline{\mathcal{D}}G[\mathcal{B}]^T\mathcal{D}),$$

and (11) may be written

$$Z(T) = \int D\mathcal{B}D\mathcal{D}D\mathcal{D}^\star \exp\Big(\mathrm{TrLn}(G[\mathcal{B}]^{-1}) + \tfrac{1}{2}\mathrm{TrLn}(\mathbf{1} + G[\mathcal{B}]\overline{\mathcal{D}}G[\mathcal{B}]^T\mathcal{D})$$
$$-\int\int \frac{\mathcal{B}^\theta\mathcal{B}^\theta}{2D} - \int\int \frac{\mathcal{D}^\phi\mathcal{D}^{\phi\dagger}}{2D} - R[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}]\Big), \qquad (12)$$

where

$$\exp(-R[\mathcal{B},\mathcal{D},\overline{\mathcal{D}}]) = \Big(\exp(W_1)\exp(\tfrac{1}{2}\int \Theta\mathcal{F}\Theta^T)\Big)\Big|_{\overline{\eta},\eta=0}.$$

Equation (12) is the main result of this work, and shows that (1), which described QCD in terms of the fundamental quark and gluon variables, may be reformulated as a functional integral over colour singlet $\overline{q}q$ and colour triplet qq bilocal fields. That is, once the various n-point gluon propagators are known the quark degrees of freedom of QCD may be completely replaced by the above bilocal fields. The effective action defined by (12) is similar to that in Cahill and Roberts (1985) and subsequent papers, but differs in the important point that G only involves $\mathbf{1}_c$ fields and not $\mathbf{8}_c$, and in the presence of extra terms containing $\overline{\mathbf{3}}_c$ and $\mathbf{3}_c$ diquark fields. We show later how local fields emerge from the bilocal fields, but the very important point is that diquarks are now shown to play a fundamental role in the rigorous reformulation of QCD. Of course they will ultimately emerge as constituents of the $\mathbf{1}_c$ baryons. We note that if we neglect the $\mathcal{R}$ contribution to the effective action then we obtain the global colour symmetry model (GCM) of the meson-diquark bosonisation of QCD. The essence of this model is to compensate the neglect of $\mathcal{R}$ by using an effective gluon 2-point function. The features of the GCM are that it is Lorentz and chirally invariant, includes the colour algebra and models quark confinement.

3. Local Diquark Fields

The fundamental result (12) is that QCD may be written in terms of colour singlet and colour triplet boson bilocal fields. As shown before for the mesons each bilocal field corresponds to an infinite set of local fields, each with its own mass and each corresponding to one observable or physical meson. We shall illustrate the physical content of (12) for the diquarks by extracting the action for the scalar diquarks and hence finding the mass functional for these states. Consider the local field expansions of the bilocal diquark fields

$$\mathcal{D}^{\phi}(x,y) = \mathcal{D}^{\phi}_{V}(x-y) + \sum_{k} d^{\phi}_{k}(\frac{x+y}{2})\Gamma^{\phi}_{k}(x-y), \tag{13}$$

where $\{\Gamma^{\phi}_{k}(z)\}$ is a complete set of real functions (arbitrary at this stage) and the local diquark fields $d^{\phi}_{k}(w)$ are considered the expansion coefficients. The $\mathcal{D}^{\phi}_{V}(x-y)$ are the translation invariant minima of the action for the diquark bilocal fields in (12), which would be diquark condensate fields if they are non zero. We shall not consider them here.

Let us now consider one specific diquark state—the $J^P = 0^+$ scalar, which is important for the nucleons. This state corresponds to the M^{ϕ}_{d} for which $K^a = i\gamma_5$ and $H^f = \sqrt{\frac{1}{2}}i\epsilon^f$. To second order the action for this field is, from (12) (and neglecting the $\mathcal{R}$ contribution) on expanding the functional Ln,

$$S_0[d^\star,d] = -\tfrac{1}{2}\mathrm{Tr}\Big(G[\mathcal{B}]d^\star\Gamma\frac{M_d}{2}C^T G[\mathcal{B}]^T C^T\frac{M_d}{2}d\Gamma\Big) + \int\int \frac{d\Gamma d^\star\Gamma}{2D} + .., \tag{14}$$

in which we suppress the flavour and colour superscripts M_d and on d and $d^\star$ (the d fields $\in \mathbf{3}_f$ and $\mathbf{3}_c$ representations). Extracting the long wavelength parts of this action (see the Appendix of Cahill and Roberts (1985) for the method) we obtain

$$S_0[d^\star,d] = \frac{f^2[\Gamma]}{2}\int \mathrm{d}^4x\Big(\partial_\mu d^\star(x)\partial_\mu d(x) + M_0^2[\Gamma]d^\star(x)d(x)\Big) + ... \tag{15}$$

In Cahill *et al.* (1987), using the equivalent Bethe-Salpeter integral equation formulation, it was also shown there that the diquark form factor is determined by the minimisation $\delta M[\Gamma]/\delta\Gamma = 0$. In this way, in general, the complete set of Γ's in (13) is to be determined. Because the mass spectrum of these various states is rapidly increasing, the expansion in (13) may be truncated after only a few terms if we are interested only in low energy hadronic phenomena. By modelling the gluon 2-point function it has been possible to study the scalar diquark state (Praschifka *et al.* 1988*a*) and from that study emerged the discovery of the cause of the quark constituent mass effect. In fact the ratio of the constituent quark mass to the diquark mass was found to be $\approx \frac{1}{2}$ suggestive of weak binding, whereas the quark propagators were confining. A comprehensive study of the constituent quark mass effect in diquark states, as well as the meson states, was reported in Praschifka *et al.* (1988*b*).

4. Summary

We have shown here that the generating functional for QCD, initially defined in (1) in terms of quark and gluon fields, may be rewritten, in (12), in terms of colour singlet and triplet bilocal boson fields. These bilocal fields may be expanded in terms of local fields and, as shown here and elsewhere, describe the physical colour singlet mesons and the diquark components of the colour singlet baryons. As before once the various gluon *n*-point functions are known this new formulation allows a divergence free and very practical means of determining low energy hadronic physics from QCD. Again it is very significant that it is only those states needed for colour singlet mesons and colour singlet baryons that arise in this formulation. The other colour states, which from a group theory point of view could have been present, i.e. those corresponding to unbound and hence unphysical $\mathbf{8}_c$ $\bar{q}q$ and $\mathbf{6}_c$ qq states, do not occur.

It will be shown elsewhere that this bosonisation gives rise to a functional integral formulation of the baryon states in QCD, leading to approximations for baryon structure, mass spectrum and couplings to mesons, i.e. the basics of nuclear physics.

It is also possible to repeat the argument (Cahill *et al.* 1988) against the Skyrmion model in which baryons are modelled as topological solitons of the Euler-Lagrange equations of the purely mesonic sector of the action in (12). We see that to derive the Skyrmion model we must arbitrarily discard the diquark part of the effective action and, as well, judiciously select only certain of the long wavelength terms of the meson sector so as to ensure stability of these solitons. However it is becoming much more definite that baryons are three quark states and then any two of the quarks are necessarily in a $\bar{\mathbf{3}}_c$ state, and that these quark pairs are most easily understood in terms of their bound states—the diquark states. We refer readers to Fredriksson and Jändel (1982) and Anselmino *et al.* (1987) for discussions of various aspects of diquarks and to Skytt and Fredriksson (1988) for a compilation of the diquark literature.

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Appendix

(a) We first show that for $n \times n$ matrices A, B, C and D we have the identity

$$\mathrm{Det}\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \mathrm{Det}(CB)\mathrm{Det}(C^{-1}DB^{-1}A - \mathbf{1}). \tag{A1}$$

We have the well known identities, for $n \times n$ matrices $\mathbf{1}, a, a'$ and b,

$$\mathrm{Det}\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ a' & b \end{pmatrix} = \mathrm{Det}\begin{pmatrix} \mathbf{1} & a \\ \mathbf{0} & b \end{pmatrix} = \mathrm{Det}(b).$$

Now

$$\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ a' & b \end{pmatrix}\begin{pmatrix} \mathbf{1} & a \\ \mathbf{0} & b \end{pmatrix} = \begin{pmatrix} \mathbf{1} & a \\ a' & d \end{pmatrix},$$

where $d = a'a + b^2$. Thus, taking the determinants of both sides, we get

$$\mathrm{Det}(b^2) = \mathrm{Det}(d - a'a) = \mathrm{Det}\begin{pmatrix} \mathbf{1} & a \\ a' & d \end{pmatrix}. \tag{A2}$$

The decomposition

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}\begin{pmatrix} \mathbf{1} & A^{-1}B \\ C & D \end{pmatrix},$$

and (A2) then allow us to write

$$\begin{aligned} \mathrm{Det}\begin{pmatrix} A & B \\ C & D \end{pmatrix} &= \mathrm{Det}(A)\mathrm{Det}(D - CA^{-1}B) \\ &= \mathrm{Det}(A)\mathrm{Det}(BDB^{-1} - BCA^{-1}) \\ &= \mathrm{Det}(A)\mathrm{Det}(B)\mathrm{Det}(C)\mathrm{Det}(C^{-1}DB^{-1} - A^{-1}), \end{aligned}$$

which gives (A1). In Section 2 identity (A1) is used for functional determinants which arise from the Grassmannian integrations of the quark fields.

(b) The elements of the matrix set $\{S^m, m = 1,\ldots 6\}$ are encoded in the relation

$$\sum_{m=1}^{6} a_m S^m = \frac{1}{\sqrt{2}}\begin{pmatrix} a_1\sqrt{2} & a_4 & a_5 \\ a_4 & a_2\sqrt{2} & a_6 \\ a_5 & a_6 & a_3\sqrt{2} \end{pmatrix}. \tag{A3}$$

Alternatively from (8) we may obtain (C. D. Roberts, personal communication 1988)

$$\delta_{ij}\delta_{kl} = H^f_{ik}H^f_{lj}; \qquad \{H^f, f = 1,\ldots,9\} = \{F^c, c = 7,5,2,0,1,3,4,6,8\}.$$

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