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Anti-neutrino Cross Sections for Neon and Deuterium

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Abstract

Recent anti-neutrino experiments have shown a marked difference in the quantity $R_A(x) = (\mathrm{d}\sigma^{\bar{\nu}\mathrm{n}}/\mathrm{d}x)/(\mathrm{d}\sigma^{\bar{\nu}\mathrm{p}}/\mathrm{d}x)$ between deuterium and neon. It has been suggested that this difference may be related to an effect discovered by the European Muon Collaboration (EMC). We test several popular and apparently successful explanations of the EMC effect against the data. We find that none of the models examined can explain the observations.

1. Introduction

Recent bubble chamber measurements (Allesia *et al.* 1981; Asratyan *et al.* 1985, 1986) using $\bar{\nu}$ beams have allowed an investigation of the neutron and proton structure functions in Ne and D. The reactions involved are

$$\bar{\nu}+\mathrm{n} \rightarrow \mu^+ + \mathrm{X}^-, \qquad \bar{\nu}+\mathrm{p} \rightarrow \mu^+ + \mathrm{X}^0. \tag{1,2}$$

It has been suggested that such measurements could be used as complementary tests of the various models which have been proposed to explain the EMC effect (Aubert *et al.* 1983). We have chosen to test the dynamical rescaling model of Close and colleagues (CRRJ) (Close *et al.* 1983, 1985; Jaffe 1983; Jaffe *et al.* 1984) and the 'off-mass-shell' model of Dunne and Thomas (1986*a*, 1986*b*) in this way.

The quantities that have been determined experimentally are the x-dependence of the cross-section ratios for the two nuclei, where the cross sections are determined from the charge in the final hadronic state,

$$R_A(x) = \left(\frac{\mathrm{d}\sigma^{\bar{\nu}\mathrm{n}}}{\mathrm{d}x}\right)_A \bigg/ \left(\frac{\mathrm{d}\sigma^{\bar{\nu}\mathrm{p}}}{\mathrm{d}x}\right)_A, \tag{3}$$

and the ratio of the cross-section ratios for the two nuclei

$$\rho(x) = R_{\mathrm{Ne}}(x)/R_{\mathrm{D}}(x). \tag{4}$$

In the quark–parton model (Leader and Predazzi 1982) we can write the general cross section for reactions (1) at high energies ($E \gg M$) as

$$\frac{\mathrm{d}^2\sigma^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}}{\mathrm{d}x\,\mathrm{d}y} = \frac{G^2 ME}{\pi}\{(1-y+\tfrac{1}{2}y^2)F_2^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}(x)-(y-\tfrac{1}{2}y^2)xF_3^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}(x)\}, \tag{5}$$

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where

$$x = Q^2/2M\nu\,, \qquad y = \nu/E \tag{6, 7}$$

are the usual scaling variables, M is the nucleon mass, and F_2 and F_3 are the structure functions

$$F_2^{\bar{\nu}\mathrm{n}}(x) = 2x\{d(x)+\bar{u}(x)+\bar{s}(x)\}\,, \tag{8}$$

$$F_2^{\bar{\nu}\mathrm{p}}(x) = 2x\{u(x)+\bar{d}(x)+\bar{s}(x)\}\,, \tag{9}$$

$$xF_3^{\bar{\nu}\mathrm{n}}(x) = 2x\{d(x)-\bar{u}(x)-\bar{s}(x)\}\,, \tag{10}$$

$$xF_3^{\bar{\nu}\mathrm{p}}(x) = 2x\{u(x)-\bar{d}(x)-\bar{s}(x)\}\,. \tag{11}$$

Inserting these into equation (5) and integrating out the y-dependence we find that

$$\frac{\mathrm{d}\sigma^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}}{\mathrm{d}x} = \frac{G^2ME}{\pi}\{\tfrac{2}{3}F_2^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}(x)-\tfrac{1}{3}xF_3^{\bar{\nu}\mathrm{n},\,\bar{\nu}\mathrm{p}}(x)\}\,, \tag{12}$$

and thus for $R(x)$ we have

$$R_A(x) = (F_{2A}^{\bar{\nu}\mathrm{n}}-\tfrac{1}{2}xF_{3A}^{\bar{\nu}\mathrm{n}})/(F_{2A}^{\bar{\nu}\mathrm{p}}-\tfrac{1}{2}xF_{3A}^{\bar{\nu}\mathrm{p}}) \tag{13}$$

$$= \frac{\tfrac{1}{3}xd_A(x)+x\bar{u}_A(x)+x\bar{s}_A(x)}{\tfrac{1}{3}xu_A(x)+x\bar{d}_A(x)+x\bar{s}_A(x)}\,. \tag{14}$$

In this work we shall concentrate on the region $x \geqslant 0\cdot3$ where possible enhancements of the nuclear sea should be negligible (Llewellyn-Smith 1983; Ericson and Thomas 1983). Then there is a negligible difference between F_2 and F_3 and in fact in comparing the two models we shall only compute the ratio

$$R_A'(x) = F_{2A}^{\bar{\nu}\mathrm{n}}(x)/F_{2A}^{\bar{\nu}\mathrm{p}}(x)\,. \tag{15}$$

2. The Models

For the parton distributions in the free nucleon we have used the parametrisation of Buras and Gaemers (1978) (modified for three flavours). We now turn to the models for the nuclear corrections to these distributions.

(a) Dynamical Rescaling

The key observation behind so-called dynamical rescaling is that the EMC effect can be reproduced by shifting the momentum scale at which the nucleon structure function is evaluated (Close *et al.* 1983):

$$F_2^A(x, Q^2) = F_2^N(x, \xi Q^2)\,, \tag{16}$$

where ξ is a function of A and Q^2. It was further suggested that a change in confinement scale of the nucleon from R to R_A could lead to (16) with

$$\xi(A, Q^2) = (R_A/R)^{2\alpha_s(\mu_A^2)/\alpha_s(Q^2)}. \tag{17}$$

The essential physics is then supposed to be an increase in the distances quarks move as A increases; see, however, Llewellyn-Smith (1985).

For our calculations we have used the parameters obtained by Close *et al.* (1985) with a Reid soft-core potential for the correlation function between nucleons. These parameters may be slightly optimistic, but nevertheless reasonable. We have also had to use the (presumably good) approximation that the proton and neutron radii are equal.

These parameters are

(a) Nucleon r.m.s. radius: Close *et al.* (1985) used $R_{\rm rms} = 0\cdot 9$ fm and obtained $R_{\rm Ne}/R = 1\cdot 104$ and $R_{\rm D}/R = 1\cdot 015$.

(b) Scale μ_A^2: Using the value $\mu_{A=56}^2 = 0\cdot 50 \pm 0\cdot 11\ {\rm GeV}^2$ obtained by Close *et al.* (1985) we find that $\mu_{A=20}^2 = 0\cdot 55 \pm 0\cdot 12\ {\rm GeV}^2$ and $\mu_{A=2}^2 = 0\cdot 65 \pm 0\cdot 14\ {\rm GeV}^2$.

(c) QCD scale parameter Λ: The parametrisation of Buras and Gaemers (1978) gives a best fit with $\Lambda = 0\cdot 3$ GeV. However, Close *et al.* (1985) used a value of $\Lambda = 0\cdot 25 \pm 0\cdot 10$ GeV. The two values give similar results.

Using these parameters we calculated the rescaling parameter $\xi(A, Q^2)$, and we could then evaluate the neutron and proton structure functions for each nucleus

$$F_{2A}^{\bar{\nu}{\rm n},\bar{\nu}{\rm p}}(x, Q^2) = F_{2N}^{\bar{\nu}{\rm n},\bar{\nu}{\rm p}}(x, \xi(A, Q^2)Q^2), \tag{18}$$

and subsequently their ratio, and finally the ratio of these ratios for Ne and D. We did this for values of Q^2 from 2 to 20 GeV^2.

For both D and Ne there was little change to the ratio of cross sections (see Fig. 1), probably because the changes in each structure function due to rescaling cancelled each other. Thus the Ne–D ratio $\rho(x)$ did not deviate much from unity: about 1% at large x (see Fig. 3).

(b) Off-shell Effects

Dunne and Thomas (DT) (1986*a*, 1986*b*) considered the meaning of the renormalisation scale μ^2. In perturbative QCD the moments $M_{\rm N}(Q^2)$ of a non-singlet structure function depend on μ^2 as

$$M_{\rm N}(Q^2) = M_{\rm N}(\mu^2)\{\alpha_s(Q^2)/\alpha_s(\mu^2)\}^{d_{\rm N}}. \tag{19}$$

However, as $M_{\rm N}(Q^2)$ is an observable, it should not depend on the choice of renormalisation point. To avoid this the moments must depend on a mass scale in such a way that the unphysical dependence on μ^2 is eliminated. DT chose the invariant mass of the target as the mass scale. Treating the nucleus as a collection of off-mass-shell nucleons we again arrive at (16), but with ξ now given by

$$\xi(A, Q^2) = (M_{\rm N}^2/P_A^2)^{\alpha_s(P_A^2)/\alpha_s(Q^2)}, \tag{20}$$

where P_A^2 is the invariant mass of the bound nucleon. The effect of this change of scale is smaller than that in the rescaling model.

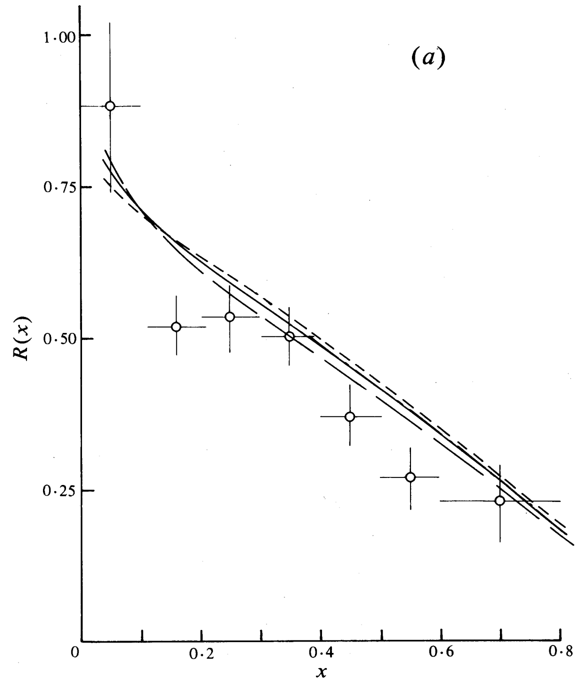

Fig. 1. Ratio of neutron to proton cross sections $R(x)$ for (*a*) deuterium and (*b*) neon. The data are for $W > 2$ GeV and $Q^2 > 2\ \text{GeV}^2$ (where W is the final hadronic invariant mass). The curves are for the dynamical rescaling CRRJ model: solid curve, no rescaling; short-dashed curve, rescaled at $Q^2 = 2\ \text{GeV}^2$; long-dashed curve, rescaled at $Q^2 = 20\ \text{GeV}^2$.

Along with this rescaling, we are also led to consider the Fermi motion of the nucleons, modified from the procedure of Llewellyn-Smith (1985) by allowing for the single particle binding of a nucleon in the shell-model state i. After some manipulation we arrive at an expression for the structure functions having used the harmonic oscillator model for the shell-model states:

$$F_2^A(x, Q^2) = \sum_i \int_x^A \mathrm{d}y\, f_i(y)\, F_2^N(x/y,\ \xi_i(Q^2)Q^2), \tag{21}$$

where we have defined y as $(p_i^0 + k_z)/M_{\mathrm{N}}$ and

$$f_i(y) = N_i \int \mathrm{d}^2 k_{\mathrm{T}}\, \rho_i\{k_{\mathrm{T}}^2 + (M_{\mathrm{N}}\, y - p_i^0)^2\}, \tag{22}$$

with N_i a normalisation constant. [Analytic expressions for $f_i(y)$ are given by DT.]

In using the harmonic oscillator model we have had to fit the well depth and $\hbar\omega$ to the separation energies of the major shells in the Ne and D nuclei. There is one further effect to consider, namely that neutrons are more tightly bound than protons in the Ne nucleus. The available data give separation energies for protons, so we have had to estimate the separation energies for neutrons based on the proton data and the fact that neutrons do not experience Coulomb repulsion. In the results this does not

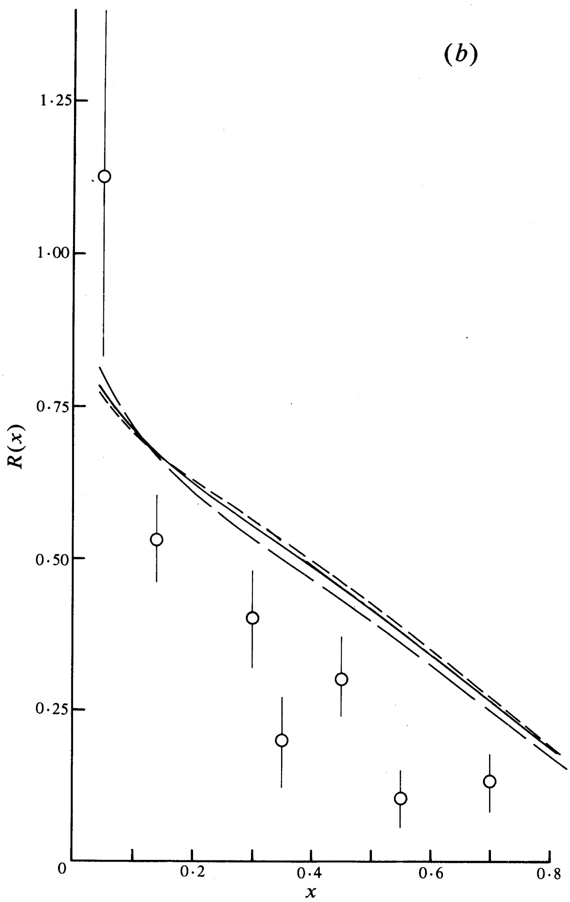

Fig. 1. *(Continued)*

appear to make much difference to the ratio of structure functions, the major effects being the rescaling and Fermi motion.

The parameters used were

(a) QCD scale parameter Λ: we used the best fit value found by Buras and Gaemers (1978) for their parametrisation, $\Lambda = 0\cdot3$ GeV.

(b) Invariant masses of bound nucleons and well depths (Negele 1970; Jacob and Maris 1973; Barrett and Jackson 1977):

(i) Deuterium ($A = 2,\ Z = 1$):
The energies of the proton and neutron are slightly split, owing to the proton–neutron mass difference; proton energy $M_0 = 930\cdot7$ MeV, neutron energy $= 932\cdot0$ MeV; $\hbar\omega = 7$ MeV.

(ii) Neon ($A = 20,\ Z = 10$):
0s 2 protons $M_0 = 897$ MeV, 2 neutrons $M_0 = 889$ MeV;
0p 6 protons $M_1 = 912$ MeV, 6 neutrons $M_1 = 905$ MeV;
0d 2 protons $M_2 = 927$ MeV, 2 neutrons $M_2 = 922$ MeV;
$\hbar\omega = 14$ MeV for protons, $\hbar\omega = 16$ MeV for neutrons.

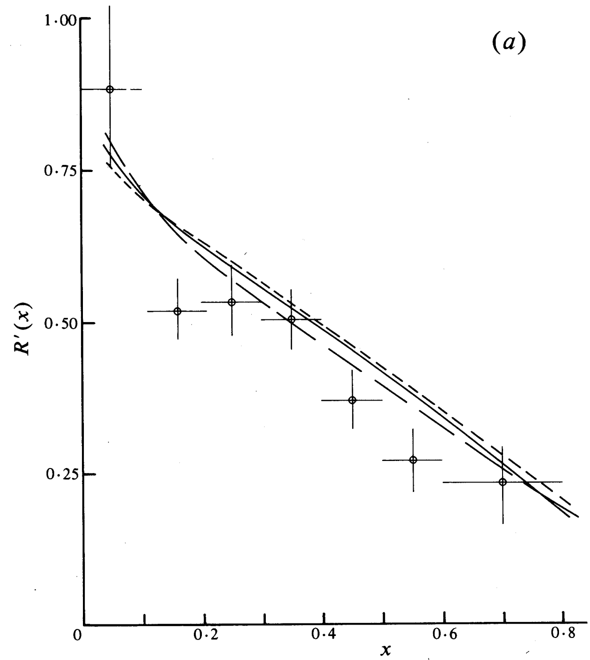

Fig. 2. Ratio of neutron to proton structure functions $R'(x)$ for (*a*) deuterium and (*b*) neon. The data are for $W > 2$ GeV and $Q^2 > 2\ \mathrm{GeV}^2$. The curves are for the off-mass-shell DT model: solid curve, no rescaling; short-dashed curve, rescaled at $Q^2 = 2\ \mathrm{GeV}^2$; long-dashed curve, rescaled at $Q^2 = 20\ \mathrm{GeV}^2$.

Using these parameters we performed the integrations in equations (21) and (22) to find our structure functions and the appropriate ratios.

For D we again find little change in the ratio $R'(x)$ compared with the case of no rescaling (see Fig. 2*a*), but for Ne we note that for medium x, $R'(x)$ is well below the value obtained without rescaling. For larger x, $R'(x)$ rises above the non-rescaled value, and this occurs mainly because of the deficiencies of the harmonic oscillator model at large x (see Fig. 2*b*).

In Fig. 3 we have plotted the predicted values of $\rho(x) = R'_{\mathrm{Ne}}(x)/R'_{\mathrm{D}}(x)$ and the experimental data for $\rho(x)$. Both models predict very little change from unity in the value for $\rho(x)$, the dynamic rescaling CRRJ model deviating about 1%, and the off-mass-shell DT model deviating by about 5%. In the DT model $\rho(x)$ rises above unity at large x, again because the harmonic oscillator model breaks down. However, we believe that the qualitative result of a small deviation from unity would still occur for a more sophisticated nuclear model.

3. Conclusions

With the failure of the CRRJ and DT models to satisfactorily predict the experimental results we must look again at the experiments. The D data of Allesia *et al.* (1981) were obtained from the BEBC bubble chamber at CERN, but the Ne data

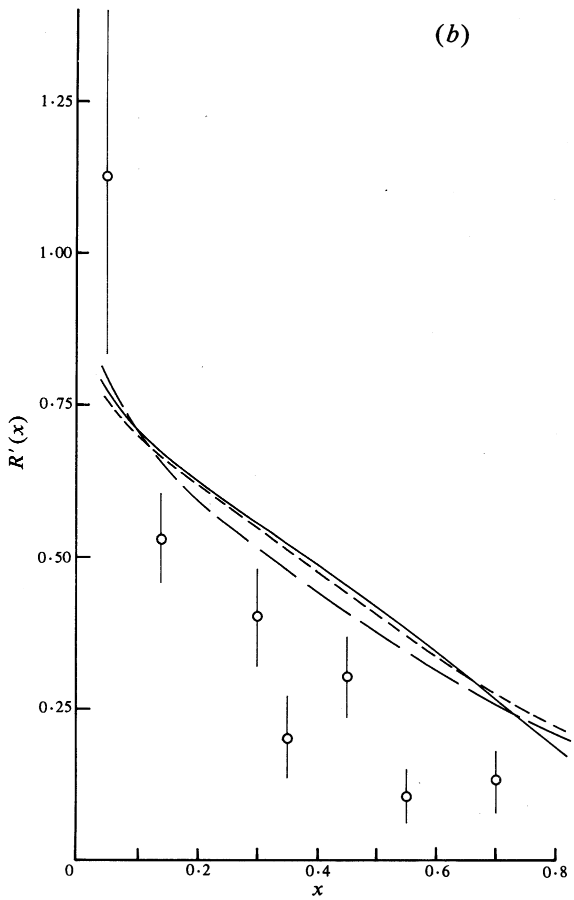

Fig. 2. *(Continued)*

of Asratyan *et al.* (1985, 1986) were obtained from the 15 ft bubble chamber at Fermilab, and there could well be systematic errors between the two. Indeed, whereas our predictions for R generally agree with the data for D, for Ne the data are systematically lower ($x > 0{\cdot}1$). A major difficulty in extracting $\bar{\nu}$n and $\bar{\nu}$p cross sections is that there may well be rescattering corrections in the nuclear medium which alter the charge of the observed final state. Asratyan *et al.* (1986) did analyse the possibility of asymmetry in the $\bar{\nu}$n/$\bar{\nu}$p reactions with Ne nuclei. We can think of no physical explanation for their observation that any such asymmetry would be strongly x-dependent, but independent of y and E.

On the whole we would not expect the EMC effect to be responsible for a value of $\rho \approx 0{\cdot}5$ at large x, because the effects on neutrons and protons tend to cancel each other out, leaving the cross-section ratios R and R' essentially unchanged and giving a ratio ρ very close to unity. If new experiments with carefully controlled systematic errors confirmed the data used here, we would be forced to re-examine our ideas about nuclear corrections for deep-inelastic structure functions (Rith 1986).

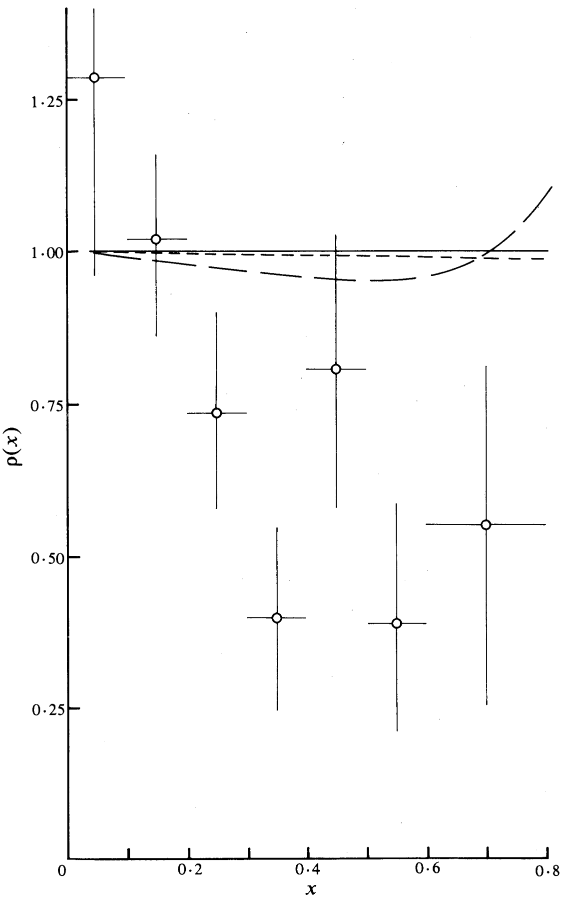

Fig. 3. Comparison of experimental data and theoretical predictions for the ratio $\rho(x) = R'_{\mathrm{Ne}}(x)/R'_{\mathrm{D}}(x)$. The data are for $W > 2$ GeV and $Q^2 > 2\ \mathrm{GeV}^2$. Theoretical curves: solid curve, no rescaling; short-dashed curve, rescaled at $Q^2 = 20\ \mathrm{GeV}^2$ for the dynamical rescaling CRRJ model; long-dashed curve, rescaled at $Q^2 = 20$ GeV for the off-mass-shell DT model.

Acknowledgments

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