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Photons in Curved Space–Time

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Abstract

Because photons are described by quantum mechanical wavefunctions that have a nonzero spatial extent it follows that they can be influenced by curved space–time. It is generally assumed that this tidal interaction is far too small to have a significant effect. This paper argues that there is a significant effect that results in an interaction between the photon and the material causing the curved space–time in which the photon loses energy to low energy secondary photons. The energy loss is a function of the space–time curvature and is proportional to distance. The only situation fully considered is that of a photon in curved space–time due to a uniform distribution of matter. Because the energy loss rate is very small it is difficult to observe in the laboratory and therefore its major applications are in astronomy. If the intergalactic density of matter is n hydrogen atoms m^{-3}, then the predicted value for the 'Hubble' constant (assuming no universal expansion) is $51\cdot68\, n^{1/2}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The theory can solve the virial mass discrepancy observed in clusters of galaxies and it makes a definite prediction about their relative magnitudes. Other astronomical applications are considered.

1. Introduction

Although the tidal stress on a fundamental particle moving along its geodesic in curved space–time is very small, it is not negligible. This paper argues that this tidal stress can produce an energy loss mechanism that is of considerable astrophysical importance. In particular, this loss can explain the Hubble redshift without requiring universal expansion. This paper will concentrate on the derivation of the energy loss relation and its application to the Hubble redshift. The crucial idea is that as the geometry of null geodesics in curved space–time is that of a three-dimensional surface in four-dimensional hyperspace, then momentum is not confined to three dimensions but has four spatial components. Suppose a photon starts off with all its momentum in the observable, three spatial dimensions, then in curved space–time the photon's trajectory is curved so that some momentum is transferred to the fourth dimension. This momentum transfer corresponds to an effective energy loss by the photon. The basic assumption of this paper is that the momentum transfer is not permanent but is subject to the Heisenberg uncertainty principle, in that it can only stay in the fourth dimension for a distance inversely proportional to the momentum. Then it appears as new low energy secondary photons. In other words, the fourth dimension is only accessible on a temporary basis. The main thrust of the analysis is to obtain the magnitude of this deflected momentum. This is done by examining the behaviour of

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a bundle of photons in curved space–time and extrapolating the results to a single photon.

The result of this analysis of the interaction between a photon and the curved space–time produced by a uniform distribution of matter is that a photon suffers a fractional energy loss proportional to the square root of the density of the medium, and proportional to the distance travelled. It is argued that if there is any other interaction occurring then the energy loss can be inhibited. This means that laboratory tests are very difficult and that the major application of the theory is in astronomy where the requirements of large distances and low densities can be found. As well as the prediction of a Hubble redshift many other astronomical applications are considered. Not only can it resolve the virial mass discrepancy that is observed in clusters of galaxies, but there is a definite prediction of an as yet unobserved relationship between the velocity of a galaxy in a cluster and its redshift.

2. Energy Loss Relationship

The following analysis keeps within the geometric optics approximation to wave propagation in curved space–time and does not consider post geometric effects such as coupling between the particles spin and the space–time curvature. It is therefore implicit that the wavelength is short compared with distances over which the amplitude, wavelength or polarisation varies, and is also short compared with the minimum radius of curvature of space–time. The basic results of the geometric optics approximation (Misner *et al.* 1973) are that rays are null geodesics, the polarisation vector is perpendicular to the rays, and the amplitude is an adiabatic invariant. Consider a bundle of null geodesics that has a cross-sectional area normal to a central reference geodesic, then the product of this area times the square of the wave amplitude is invariant. Physically this means that as the wave propagates its total energy remains constant. Because of the 'focussing theorem' the cross-sectional area will remain constant or decrease and hence the amplitude remains the same or increases. Geometrically, the bundle of rays follow a three-dimensional surface that always has a non-negative curvature. These results are often interpreted in the photon model by saying that photons are point-like particles that travel on null geodesics and that the total number of photons is conserved. The electromagnetic properties of photons are due to the combined effects of a large number of photons. Here it is argued that photons are particles with physical extent and wave-like properties and that individual photons are subject to tidal stress due to the curvature of space–time. In the standard model the curvature of space–time only influences the geometry of the photon bundle and the trajectories of individual photons are completely described by null geodesics.

The problem of computing the tidal stress on a photon is attacked by considering the geometry of a bundle of null geodesics. At any point on the reference geodesic we can erect Riemann normal coordinates, which provide a local inertial reference frame. Then within the assumptions of geometric optics this frame is Minkowskian. Consider a cylindrical coordinate system with all the photons parallel to the z-axis at the origin. Then as the bundle of photons propagate along null geodesics the cross-sectional area decreases or at least remains the same. There is no differential change in the longitudinal direction. In general there will be a change in the average direction of the bundle due to gravitational deflection. However, this is of no interest here and will be ignored. It is the second order term that depends on the local curvature that is important. Since the change in area is scale invariant, and neglecting possible higher order terms, the cross-sectional area A satisfies the differential equation

$$\frac{1}{A}\frac{\mathrm{d}^2A}{\mathrm{d}z^2} = -\epsilon^2, \tag{1}$$

where ϵ^2 is proportional to the local gaussian curvature. Consistent with the geometric optics approximation, we assume that ϵ^2 has negligible variation over the distances of interest. The geometry is that of a three-dimensional surface of a four-dimensional hypersphere of radius $1/\epsilon$.

Consider a plane wave with finite lateral extent and let the wave travel a distance z, which is large compared with a wavelength but small compared with $1/\epsilon$. In four spatial dimensions the wave has been deflected by an angle ϵz and therefore if p_0 is the original momentum, the momentum deflected into the fourth spatial dimension is $p_0 \sin\epsilon z$, and the energy associated with this deflected momentum is $E_0 \sin\epsilon z$, where E_0 is the original energy. Basically we are assuming that the change in momentum of a body travelling along a three-dimensional curved surface in four dimensions is the same as that for a body travelling along a two-dimensional surface in three dimensions, or a one-dimensional surface in two dimensions. In all cases the trajectory can be reduced to a circular path in two dimensions. The only difference with the four-dimensional space is that we cannot directly perceive the curvature. This change in momentum is quite general in that it only depends on the properties of wave propagation in the approximation of geometric optics. It must therefore apply to the propagation of a bundle of photons. In this case we identify the momentum density with a photon density, and find the same result for the total deflected energy and momentum.

How can these ideas be applied to a single photon? The area change is a proportional one so that arguments about the photon being too small to be affected are not valid. More importantly, the wave equation used to describe the photon obeys the laws of geometric optics and its propagation is modified by the curvature. Havas (1966) has pointed out that the concept of a single photon is rather tenuous. There is no way we can tell the difference between a single photon and a bundle of photons which combine to have the same energy, momentum and spin. This is because photons obey Bose–Einstein statistics which is the quantum mechanical equivalent of the principle of superposition. However, the wave equation that describes the photon in quantum mechanics only applies to three spatial dimensions. The effective energy is given by an operator whose expectation is taken over three dimensions; it does not cover the energy deflected into the fourth dimension. This energy is 'lost' from the photon. In the limit where ϵz is very small, which usually applies, we find this energy loss to be

$$E = \epsilon z E_0, \tag{2}$$

where E_0 is the energy of the primary photon. Since the energy E is lost to the primary photon we interpret this as an energy loss equation which can be written as

$$\mathrm{d}E_0/\mathrm{d}z = -\epsilon E_0. \tag{3}$$

If the curvature is constant this can be integrated to get

$$E(z) = E_0 \exp(-\epsilon z). \tag{4}$$

What happens to the energy? The only reasonable explanation is that it gets emitted as low energy photons. The energy can only remain 'lost' in the fourth dimension for a time consistent with the Heisenberg uncertainty principle. The primary starts off at $z = 0$ in an initial state, with all its energy and momentum confined to three dimensions, then as it travels the momentum imbalance builds up, until eventually at z, it emits two (or possibly more) secondary photons and regains its initial state, but with a slightly reduced energy. At least two secondaries are required in order to preserve spin and momentum. The primary then repeats this procedure. An estimate of the interaction length between emission of secondaries can be obtained from the uncertainty principle applied to the primary photon. This is equivalent to observing that radiation efficiency requires that the pathlength between emissions is comparable with the wavelength of the secondaries. If there is any energy loss ΔE then the uncertainty principle applied to the primary photon relates it to a distance Δz, where

$$\Delta E \, \Delta z = hc/4\pi, \tag{5}$$

and using equation (3) to get ΔE, the average distance between the emission of secondaries is

$$\Delta z = (hc/4\pi\epsilon E_0)^{\frac{1}{2}}. \tag{6}$$

There is an important difference between this tidal interaction and the majority of particle interactions. Normal interactions occur in a small volume and there is no interference between different interactions of the same or different types. However, this tidal interaction requires a slow build up of an energy discrepancy and the secondaries are only emitted when the discrepancy can no longer be maintained. Any other interaction that occurs during the process can interfere with the tidal interaction. Although the energy discrepancy built up prior to the interfering interaction could be emitted as supplementary secondary photons, the very short pathlength compared with their wavelength suggests that the radiation efficiency would be very low. The result is that if there is another interaction that has an interaction length much shorter than Δz, then the tidal energy loss is inhibited. Then, as the interaction length becomes comparable with Δz, we expect a transition zone as the energy loss builds and reaches its full value when the interaction length is much greater than Δz. This inhibition is similar to the influence of a plasma on the emission of synchrotron radiation (Pacholczyk 1970) in which the intensity of the low frequency radiation in a plasma is less than that in a vacuum. The difference is because the low frequency radiation has a different phase velocity in a plasma and eventually loses phase coherence with its source. For the tidal interaction it is the velocity of the primary photon that is important. For efficient radiation of low energy photons it must maintain phase coherence with the gravitational field over a distance commensurate with the secondary wavelength. This inhibitory effect is considered later in discussing possible laboratory experiments.

3. Calculation of Curvature

The curvature parameter ϵ can be obtained from the Raychaudhuri (1955) equation which describes the evolution of the volume of a bundle of geodesics. If we assume that the geodesic bundle has zero shear and vorticity it states that

$$\frac{1}{V}\frac{\mathrm{d}^2 V}{\mathrm{d}s^2} = -R_{\alpha\beta}\, U^\alpha U^\beta\,, \tag{7}$$

where s is a suitable affine parameter, $R_{\alpha\beta}$ is the Ricci tensor, U^α is the 4-velocity on the reference geodesic and V is the volume of the bundle of geodesics. This equation can be obtained directly from the equation of geodesic deviation by restricting the acceleration of the displacements to be parallel to the displacements, which is equivalent to having zero shear and vorticity. Because of symmetry the shear and the vorticity of a geodesic bundle are zero for homogeneous and isotropic media. The applicability of equation (7) can be extended by using the Goldberg–Sachs theorem (Chandrasekhar 1979) which states that if the Riemann tensor is algebraically special then the shear and vorticity are zero. The Kerr solution for the external field of a rotating mass, and the special case of the Schwarzschild solution for the external field of a non-rotating mass, satisfy this theorem. Since the Ricci tensor is zero for the exterior solutions the right-hand side of (7) is zero for both these cases. The result is that there is change of volume and, therefore, there is no predicted photon interaction with the external gravitational field of a mass such as the Sun or the Earth. There is still the possibility that there is an interaction that depends on factors other than a simple volume change.

For null geodesics the pathlength z is a suitable affine parameter. Note that the focussing theorem states that the right-hand side of (7) is always non-positive. Then, since there is no change in the longitudinal direction, the change in volume is entirely due to a change in area. Therefore, combining equation (1) with (7) we get

$$\epsilon = (R_{\alpha\beta}\, U^\alpha U^\beta)^{\frac{1}{2}}\,. \tag{8}$$

This relationship is only a function of the Riemann geometry of space–time and does not depend on any particular gravitational theory. However, Einstein's general theory of gravitation gives a particularly elegant evaluation. Direct application of the field equations with the stress energy–momentum tensor $T_{\alpha\beta}$ and metric tensor $g_{\alpha\beta}$ gives

$$\epsilon = \{8\pi G(T_{\alpha\beta}\, U^\alpha U^\beta - \tfrac{1}{2} T g_{\alpha\beta}\, U^\alpha U^\beta) + \Lambda g_{\alpha\beta}\, U^\alpha U^\beta\}^{\frac{1}{2}}\,, \tag{9}$$

where T is the contracted form of $T_{\alpha\beta}$, and Λ is the cosmological constant. For photons with null geodesics we have $g_{\alpha\beta}\, U^\alpha U^\beta = 0$, and the two last terms in (9) are zero. Equation (9) has a particularly simple form for a perfect fluid where the pressure is negligible compared with the density, which is true for most astrophysical gases and plasmas. For photons we can combine equation (3) with (9) for a perfect fluid at rest to get

$$\mathrm{d}E_0/\mathrm{d}z = -(8\pi G\rho/c^2)^{\frac{1}{2}} E_0\,, \tag{10}$$

where ρ is the density of the medium. This equation was derived by a simpler heuristic argument in an earlier paper (Crawford 1979). This is the essential result of this paper. It states that a photon travelling in a medium, which is rare enough for there to be effectively no other interaction, loses energy at a rate proportional to the square root of the local density.

4. Hubble Redshift

The obvious application of equation (10) is to see if it can explain the Hubble redshift without the necessity of universal expansion. The simple model is that the redshift is due to the energy loss by the photons in passing through the intergalactic medium. It is a 'tired light' model for explaining the observed redshift distance relationship. In fact for a constant density intergalactic medium it predicts an exponential redshift distance function (cf. equation 4). For small distances a linear relationship is a good approximation. It can be shown (Crawford 1987; present issue p. 459) that the diffuse X-ray background is consistent with free–free emission from a very hot intergalactic plasma. Let this plasma have a density of n hydrogen atoms m^{-3}, then the observed Hubble redshift H is given by (10) as

$$H = 51\cdot 68\, n^{\frac{1}{2}}\,\mathrm{km\,s^{-1}\,Mpc^{-1}}\,. \tag{11}$$

For the best-fit density to the X-ray observations (Crawford 1987) of $n \leqslant 2\cdot 05\ \mathrm{m}^{-3}$, this gives $H \leqslant 74\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, a value in good agreement with current observations. The inequality arises because of the unknown clumpiness of the intergalactic medium. This clumpiness could easily give a reduction by a factor of about 2 in the predicted value for H. Although this result was derived for a specific cosmological model, we note that Marshall *et al.* (1980) using a standard model found a density of the intergalactic gas of 36% of the closure density. The closure density is defined as $3H^2/8\pi G$, and comparison with equation (10) shows that this tidal energy loss theory predicts a density of one-third of the closure density in order to give the same redshift relationship, which is to be compared with their value of 36%. The significance of this result is that the density of the intergalactic medium and hence the predicted Hubble constant is not critically dependent on cosmological models.

If there is a high degree of clumping of the intergalactic gas the measurement of the Hubble redshift is likely to have systematic errors, especially for nearby galaxies. For instance, a gas cloud similar in density and size to those inside clusters of galaxies would produce an additional effective velocity of about 2000 $\mathrm{km\,s^{-1}}$ for objects seen through it. Indeed, we expect to see 'holes' in the redshift distribution due to the effects of high density clouds between us and the object. This may well be the explanation of the observations by Karoji *et al.* (1976), who found that galaxies seen through clusters have a larger redshift than those selected away from clusters. They used apparent magnitudes to make a relative normalisation for the distribution in distance, and found that the redshift velocity of radio galaxies seen through a cluster was about 2500 $\mathrm{km\,s^{-1}}$ greater than that for galaxies away from clusters.

5. Virial Theorem for Clusters of Galaxies

A long standing problem in understanding the dynamics of clusters of galaxies is that the mass predicted from the virial theorem is much larger than the observed mass of the galaxies (Dressler 1978). The virial theorem states that for a number of masses in gravitational equilibrium the average potential energy is twice the average kinetic energy. The kinetic energy in clusters of galaxies is computed from differential redshifts of the individual galaxies interpreted as line of sight velocities. The kinetic energy is computed from these velocities on the assumption that the transverse velocities have the same distribution. The potential energy is computed from distances, and masses derived from luminosities and mass-to-light ratios. X-ray observations have shown that there is a considerable quantity of gas in the centres of clusters. Suppose that the differential redshifts are not due to different velocities, but are essentially due to energy loss by the photons in the gas. The more distant galaxies in the cluster would have a larger redshift than the nearer ones. This is in addition to the overall redshift caused by the intergalactic gas between us and the cluster. From equation (18) we note that the extra redshift is proportional to the square root of the cluster gas density. Therefore, the root mean square velocity, the velocity dispersion, is also proportional to the square root of the gas density. The X-ray intensity from the cluster gas is due to free–free interactions (bremsstrahlung) and is proportional to the square of the gas density. The result of this extremely simple model is that the X-ray intensity should be proportional to the fourth power of the velocity dispersion, which is in agreement with the analysis by Quintana and Melnick (1982) who found an exponent of $4 \cdot 0 \pm 0 \cdot 7$. It will be shown in a later paper that a better model that includes observed gas distributions makes predictions that are in excellent agreement with observations.

An important consequence of this model is that galaxies with larger redshifts are deeper into the cluster gas and therefore further away, and therefore fainter. The model makes the definite prediction that in a cluster there is a correlation between the redshift of each galaxy and its apparent magnitude (or any other independent distance measure). This is one result that is a crucial test of the theory. The major difficulty in making the test is that the relative distance range is small, and hence the predicted spread in magnitudes is small, and because the intrinsic spread in magnitude is much larger, a large number of measurements are needed to get a statistically significant result.

6. Laboratory Tests

One of the major disadvantages of this theory is that it does not readily predict a result that can be tested in the laboratory. This arises because the interaction length of other interactions is small enough to inhibit the energy loss process. It is difficult to have sufficient density to get a measurable energy loss without having competing interactions. As an example of a possible experiment we consider the gravitational redshift experiment by Pound and Snider (1965). They measured the gravitational redshift of $14 \cdot 4$ keV γ rays passing through a vertical, $22 \cdot 5$ m path in helium. The predicted fractional energy change from equation (10) is $-1 \cdot 25 \times 10^{-12}$. The typical pathlength between emission of secondaries (equation 6) is 11 m, and the typical frequency for the secondaries is about 1 MHz. Even for a very strong source it would be very difficult to detect the secondaries because of their very low power and broad spread in frequency. Pound and Snider observed a fractional energy change that was in agreement with the gravitational redshift of $2 \cdot 5 \times 10^{-15}$, and not the much larger value predicted here. Their experiment measured the difference between the frequency shift of upward moving photons to that of downward moving photons and was deliberately designed to cancel one-way effects. Nevertheless, they would have seen the large shift predicted here. However, it has been argued above that if the photons have an interaction length from some other interaction that is much smaller than that predicted by equation (6), then the effect would be greatly reduced. In this case there is such an interaction; the coherent forward scattering that produces the macroscopic effect of refractive index. An estimate of the interaction length is provided by the extinction length given by the Ewald and Oseen extinction theorem (Jackson 1975). In the high frequency limit the electrons are essentially free and the refractive index is the same as that for a plasma. For a plasma with a density of n electrons m^{-3}, and for X-ray photons with wavelength λ, the extinction distance has the value $(\lambda n r_0)^{-1}$, where r_0 is the classical electron radius. A simple calculation shows that the extinction distance in a plasma is just that distance over which an electromagnetic wave has a phase change of one radian compared with propagation in a vacuum.

For the Pound and Snider experiment the extinction length is 0·15 m, a value much smaller than the typical decay distance of 11 m, which shows that these results do not invalidate the photon decay theory. It is of interest to note that in an earlier experiment Pound and Rebka (1960) did observe an unexplained one-way energy loss, but for less than the value predicted above. This analysis also shows that the laboratory confirmation of the theory would be very difficult, because it requires the observation of very high energy photons over long distances through a low density medium in order to eliminate the interference from other interactions.

7. Other Astronomical Effects

The astronomical effects of this theory are pervasive. The theory must be considered in nearly all cases where redshifts are interpreted as velocities. Particular examples are in the dynamics of the Galaxy, galactic rotation curves and the dynamics of stellar clusters and gas clouds. Another example is the velocity distribution of nearby stars where there has been an unexplained redshift, the K term, left after other galactic rotation velocities have been removed. The verification that the K term is due to tidal interactions is rendered difficult because of the large and unknown density distribution of the interstellar gas.

There is the possibility of explaining some discrepancies in the redshifts of galaxies and quasars (Burbidge 1979) as being due to different line of sight density distributions. It is easy with reasonable estimates of densities and sizes of intergalactic gas clouds to get differential redshifts that correspond to effective velocity differences of up to $10^5\ \mathrm{km\,s^{-1}}$, but to go much higher is difficult. Nevertheless, this theory can explain many of the smaller discrepancies. One effect of these clouds is that the redshift of nearby galaxies may not be a good indicator of their distance. However, the statistics of large numbers should ensure that the redshift is a good measure of distance (with an exponential equation) for large distances.

A more subtle effect of this theory is the broadening of spectral lines seen in absorption. Each spectral line will be broadened by the redshift due to the density of the cloud, as the radiation passes through the cloud. The width of the spectral line will be proportional to the product of the depth of the cloud and the mean value of the square root of its density.

8. Summary

A simple model for the interaction of a photon with matter producing curved space–time has been described. The essential result is that in a dilute gas (or plasma) the photon loses energy at a rate given by equation (10). If the gas density is too large the tidal interaction is inhibited due to interference from other interactions, in particular the coherent forward scattering that is equivalent to refractive index. This tidal interaction can explain the Hubble redshift with an intergalactic density of about 2 hydrogen atoms m^{-3}. As a result, the Universe must either be static or have a much lower rate of expansion than currently believed. A first order analysis suggests that it can resolve the virial mass discrepancy in clusters of galaxies by attributing most of the differential redshifts to energy losses in the plasma that produces the cluster X-ray emission. The theory makes definite predictions that the galaxies in a cluster with higher redshifts are further away than those with smaller redshifts. Under the standard description the correlation is zero due to the time symmetry of the gravitational dynamics. The possibility of laboratory tests of the theory is remote because of the strong inhibition of the tidal interaction by other interactions.

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References

Burbidge, G. (1979). *Nature* **282**, 451–5.
Chandrasekhar, S. (1979). *In* 'General Relativity' (Eds S. W. Hawking and W. Israil) (Cambridge Univ. Press).
Crawford, D. F. (1979). *Nature* **277**, 633–5.
Crawford, D. F. (1987). *Aust. J. Phys.* **40**, 459.
Dressler, A. (1978). *Astrophys. J.* **226**, 55–69.
Havas, P. (1966). *Am. J. Phys.* **34**, 753.
Jackson, J. D. (1975). 'Classical Electrodynamics' (Wiley: New York).
Karoji, H., Nottale, L., and Vigier, J.-P. (1976). *Astrophys. Space Sci.* **44**, 229–34.
Marshall, F. E., Boldt, E. A., Holt, S. S., Miller, R. B., Mushotzky, R. F., Rose, L. A., Rothschild, R. E., and Serlemitsos, P. J. (1980). *Astrophys. J.* **235**, 4–10.
Misner, C. W., Thorne, K. S., and Wheeler, J. A. (1973). 'Gravitation' (Freeman: San Francisco).
Pacholczyk, A. G. (1970). 'Radio Astrophysics' (Freeman: San Francisco).
Pound, R. V., and Rebka, G. A. (1960). *Phys. Rev. Lett.* **4**, 337–41.
Pound, R. V., and Snider, J. L. (1965). *Phys. Rev.* B **140**, 788–803.
Quintanna, H., and Melnick, J. (1982). *Astron. J.* **87**, 972.
Raychaudhuri, A. K. (1955). *Phys. Rev.* **98**, 1123.

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