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Interferometers, Aberration and the Sagnac Effect

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Abstract

In this note we show that the usual expression for diurnal aberration (see e.g. Woolard and Clemence 1966) is not valid for some types of radio interferometers. The difference (usual minus correct) is a constant of the instrument, and is absorbed in the calibration procedure. The analysis of multibaseline VLBI may also be affected.

1. Why Does an Interferometer Need to Correct for Aberration?

Consider the simple geometry of Fig. 1 *a*, where we show an east–west interferometer. As judged by an observer at the pole, the source is seen on the interferometer meridian; that is, the polar observer (one at rest in the inertial frame containing the source) would see the wavefront arriving simultaneously at the two antennas. The observer would also learn that the subsequent passage from the antennas to the central mixer introduced a differential delay: the signal from A would arrive before the signal from B. An observer at the interferometer centre M would observe the same delay, but would interpret it as a positional offset—the aberration angle. In the more general case (Fig. 1 *b*) both observers would see the wavefront arriving obliquely to the interferometer axis. Two corrections are then needed when calculating the delay that would be observed at the centre M: between the instant the wavefront arrives at A and subsequently at B the Earth will have rotated through a small angle, reducing the effective baseline; secondly, as discussed above, the propagation time A to M is less than the time B to M. It is the sum of these two effects which leads to the usual expression for the aberration angle, and it is the second of these two effects which may be absent in certain classes of radio interferometers.

2. Difference in Propagation Times

We could calculate the difference [(A to M)−(B to M)] following the traditional discussion in texts on special relativity. An alternative approach views the results as a manifestation of the Sagnac effect. In 1913 Sagnac showed that the propagation time of a light signal around a closed path on a rotating frame depends on whether the signal travels with or against the sense of rotation. The phenomenon is of relevance in clock synchronisation experiments (Saburi *et al.* 1976); it also has a practical use in modern laser gyroscopes (Chow *et al.* 1985).

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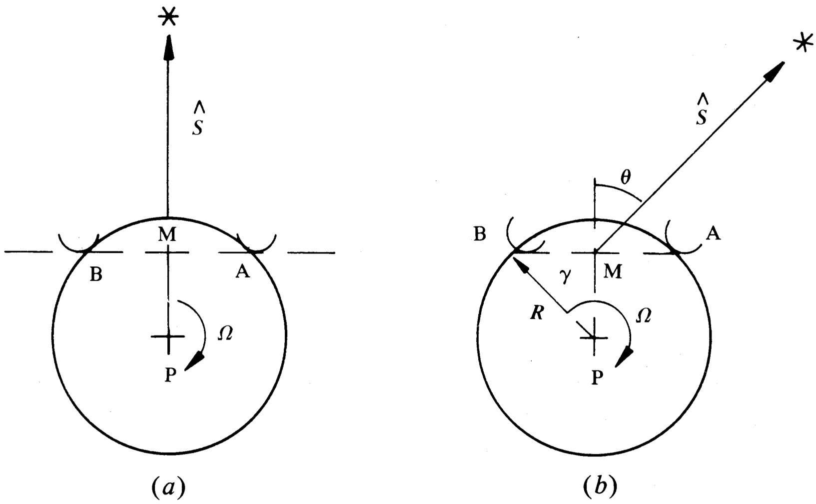

Fig. 1. (*a*) The east–west interferometer (AB): at this instant, as seen by an inertial observer, the source at $\hat{S}$ is on the interferometer meridian. (*b*) The general case: the source is at an angle θ, as seen by the inertial observer.

Around a closed path of area A (as projected on to the equatorial plane) the observed time difference is

$$\Delta\tau = 4A\Omega/c^2 .$$

Consider now the triangle ABP in Fig. 1, where P is the point on the Earth's axis in the plane normal to the axis and containing the line AB. From symmetry, the propagation times (A to P) and (B to P) are equal and, similarly, (P to A) equals (P to B). Thus the effect is due simply to the difference in propagation times [(A to B)−(B to A)]. Since M is midway, we have

$$\Delta[(\text{A to M})-(\text{B to M})] = 2A\Omega/c^2 , \qquad \Delta\tau \approx LR\Omega/c^2 .$$

where L is the interferometer baseline and R the Earth's radius.

3. Implications

A VLB interferometer records the signals at A and B on tape, eliminating the transmissions A to M and B to M. The term $\Delta\tau$ will therefore be missing. This means that:

> if the clocks at A and B (which control the recording) are both synchronised by a common technique to the polar clock, and
>
> if the full aberration correction is applied, then the observer will make one of three equivalent deductions: that all source coordinates are in error by $\Delta\tau$; or that there is a further clock error of $\Delta\tau$; or that the VLB coordinate frame is misaligned with respect to the frame of the connected element interferometers.

If the observer derives the 'clock error' by aligning the interferometer on a standard source, then the effect is absorbed by the calibration process.

The critical question is the synchronisation of the clocks: if each observatory clock is 'synchronised' with a clock situated on the polar axis, then it is simply the diurnal aberration which is affected. If the reference clock were at the solar system barycentre, then the annual aberration would be affected.

We note that this 'error' arises from the way we chose to attack the problem, treating the interferometer as a single telescope. It will not be seen in the more appropriate formation for VLBI interferometers (Cohen and Shaffer 1971; Shapiro 1976), where the delay at each antenna is calculated explicitly.

Several other types of radio interferometer will be similarly affected. The Australia Telescope (Frater 1984) is a connected-element interferometer in which the signals from each antenna are sent to the central site as a stream of digitised data. Since the subsequent processing takes into account the time associated with each sample, the propagation time from antenna to processor is irrelevant. The clocks which drive the digitisers are synchronised in a symmetrical manner: the round-trip delay (M to A to M) is measured; half the delay is adopted as the transmission time; a clock pulse, advanced by this transmission time, is then sent to antenna A for synchronisation purposes.

Equally, a connected-element interferometer would not show the constant $\Delta\tau$ term if the mixer M were placed at the pole.

4. Other Consequences for VLBI: Closure Phase

Interferometers with independent clocks have a fundamental difficulty: absolute phase information is lost. This means that one cannot produce a map simply by Fourier inverting the observed visibilities. Techniques have been devised to overcome this problem (Readhead and Wilkinson 1978) which rely on 'closure phase'. As was first shown by Jennison (1958), reliable phase information can still be derived under these circumstances, provided that there are more than two antennas. One forms the algebraic sum of the observed phases measured on a closed triangle of baselines, and in this way the unknown contribution from each clock is removed. The algebraic sum is called the closure phase. We now examine the consequences of measuring closure phase on a stationary and a rotating frame.

Stationary Frame

Given three antennas at $\boldsymbol{r}_i$ and a source at $\hat{\boldsymbol{S}}$ we measure for baseline ij, the phase

$$\phi_{ij} = (2\pi/\lambda)(\boldsymbol{r}_j - \boldsymbol{r}_i)\,.\,\hat{\boldsymbol{S}}\,.$$

Thus $\sum \phi_{ij} = 0$, if measured cyclically over three baselines. Adding an arbitrary phase offset to any antenna will make no difference to the cyclic sum.

Rotating Frame

Clearly, the triangle defined by the positions occupied by each antenna as it receives the wavefront will differ slightly from the triangle defined by the positions occupied by the antennas on the Earth, because of the rotation of the Earth in the interval between the instant of the wavefront reaching the first antenna and reaching the last. This of itself would not alter the phase closure property, as one can ensure that at each instant all three phase calculations refer to the same wavefront (Rogers *et al.* 1974), but care must be taken to allow correctly for the Sagnac effect.

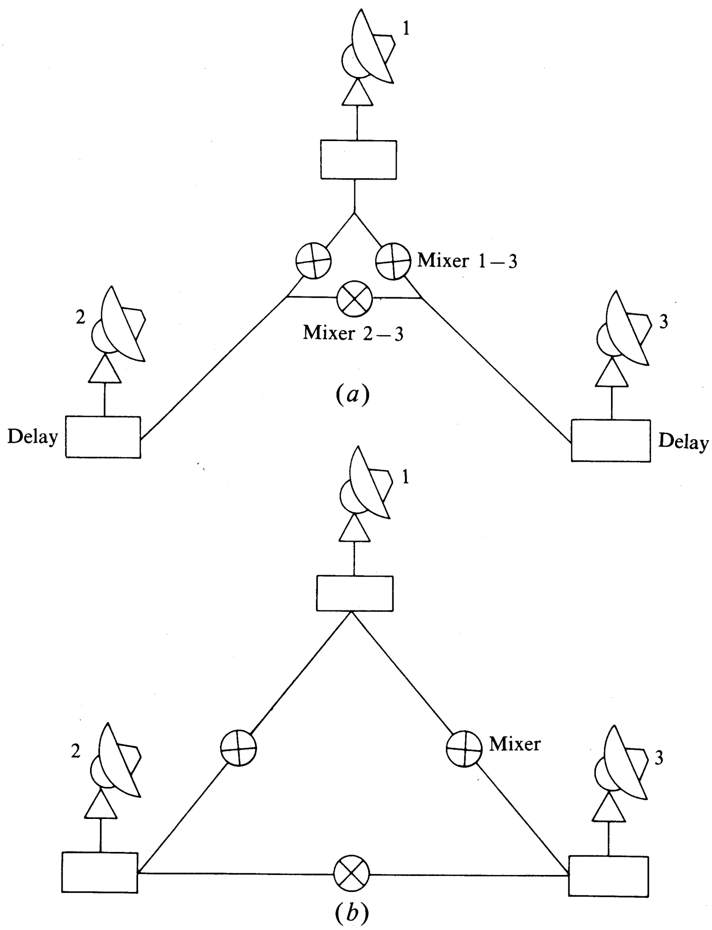

Fig. 2. A three-antenna array in which (*a*) the processing occurs at a central point and (*b*) the processing is distributed to the mid-points of the baselines. The array (*b*) exhibits an error in the closure phase.

In Fig. 2 we show two ways of organising a three-antenna array. In both cases we will provide delays at each antenna to steer the array, to satisfy the common wavefront criterion. The two schemes differ in the location of the mixers—at a common point (Fig. 2*a*) or at the mid-point of each baseline (Fig. 2*b*). We find in case (*b*) an error in the closure phase.

Let ϕ_{ij} be the astronomical phase of baseline ij, and let θ_{ij} be the observed phase. Then, we have

$$\theta_{12} = \phi_{12}+2\pi f\tau_{12},$$

$$\theta_{23} = \phi_{23}+2\pi f\tau_{23},$$

$$\theta_{31} = \phi_{31}+2\pi f\tau_{31},$$

where τ_{ij} is the Sagnac effect described above. Summing to obtain the closure phase, we find that

$$\sum \theta_{ij} = \sum \phi_{ij} +2\pi f \sum \tau_{ij} = \sum \phi_{ij} +2\pi f \frac{4A\Omega}{c^2},$$

where A is the area of the triangle projected onto the equatorial plane. The same phase closure error would surface in VLBI if each baseline were processed (and calibrated) separately.

5. Conclusions

In this note we have argued that there is a fundamental difference between conventional connected-element interferometers and tape-recorder interferometers (VLBI). The difference is in the way the signals are transported from the antenna to the processing correlator. This operational difference translates to an apparent 'clock error', a constant of the instrument. This error is, in general, of little consequence, as it is absorbed in the calibration process. We have identified one set of unlikely circumstances where the effect could be noticed: in a measurement of VLBI closure phase, when each baseline is processed and calibrated separately. Since multi-baseline processors are now routinely available, this problem should not arise.

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