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An Analysis of the $^{12}C(^{16}O,\alpha)^{24}Mg$ Reaction

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Abstract

The angular distribution of α particles from the reaction $^{12}C(^{16}O,\alpha)^{24}Mg$ was calculated, with the help of an α particle model, for the ground state of ^{24}Mg at bombarding energies with ^{16}O of 15·25, 16·48, 17·10, 17·85, 19·30, 20·70 and 21·80 MeV. Calculated results were compared with experimental data and satisfactory agreement was obtained.

1. Introduction

The idea of clustering of nucleons in nuclei was suggested many years ago and it is well known that for light nuclei the shell-model wavefunction can be rewritten in a cluster form (Phillips and Tombrello 1960). With the availability of heavy ion beams of sufficient intensity there has been renewed interest in α cluster models because heavy ion reactions are expected to provide new information on the extent of clustering of nucleons.

Let us consider the $(^{16}O,\alpha)$ reaction in ^{12}C leading to the ground state of ^{24}Mg. The reaction may be assumed to proceed by ejection of an α particle from ^{16}O, the remainder of the projectile then being coupled to the ^{12}C target nucleus to form ^{24}Mg in the ground state.

Groce and Lawrence (1965) have made measurements in the centre-of-mass energy range 6·43–9·64 MeV for the ground state and first six excited states of ^{24}Mg. A more extensive investigation of this reaction has been made at higher energies by Halbert *et al.* (1967). In other experimental work (Patterson *et al.* 1971; Greenwood *et al.* 1972; Shapira *et al.* 1975) the main interest was in the measurement of differential excitation functions.

In an earlier theoretical investigation of the $^{12}C(^{16}O,\alpha)^{24}Mg$ reaction (Nagorcka and Newton 1972), the angular distribution of α particles emerging from this reaction was calculated. We have undertaken the present work in an endeavour to establish to what extent an α cluster-model calculation can describe the angular distribution and differential cross section of this reaction. For simplicity, the problem has been treated in terms of the plane-wave Born approximation (PWBA).

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2. Brief Formulation of the Problem

The ^{12}C and ^{16}O nuclei may be considered to be composed of three and four structureless α particles respectively. Their wavefunctions are taken to be

$$\Psi_C = N_C \exp\left(-\tfrac{1}{2}\alpha_C \sum_{i=1}^{3} \rho_{C_i}^2\right), \qquad \boldsymbol{\rho}_{C_i} = \boldsymbol{r}_i - \boldsymbol{R}_C; \tag{1}$$

$$\Psi_O = N_O \exp\left(-\tfrac{1}{2}\alpha_O \sum_{i=4}^{7} \rho_{O_i}^2\right), \qquad \boldsymbol{\rho}_{O_i} = \boldsymbol{r}_i - \boldsymbol{R}_O. \tag{2}$$

Here N_C and N_O are the normalisation constants, $\boldsymbol{R}_C$ and $\boldsymbol{R}_O$ are the position vectors of the centres of mass of ^{12}C and ^{16}O respectively and are given by

$$\boldsymbol{R}_C = \tfrac{1}{3}\sum_{j=1}^{3} \boldsymbol{r}_j, \qquad \boldsymbol{R}_O = \tfrac{1}{4}\sum_{j=4}^{7} \boldsymbol{r}_j. \tag{3}$$

These types of wavefunctions have been used elsewhere (Thompson and Tang 1968; Tang 1969; LeMere et al. 1976). The width parameters α_C and α_O are chosen to yield the experimentally determined values of $2\cdot453$ and $2\cdot730$ fm for the r.m.s. radii (Barrett 1974) of the nucleon distributions in ^{12}C and ^{16}O, respectively. In this way we obtained $\alpha_C = 0\cdot16619\ \text{fm}^{-2}$ and $\alpha_O = 0\cdot13736\ \text{fm}^{-2}$.

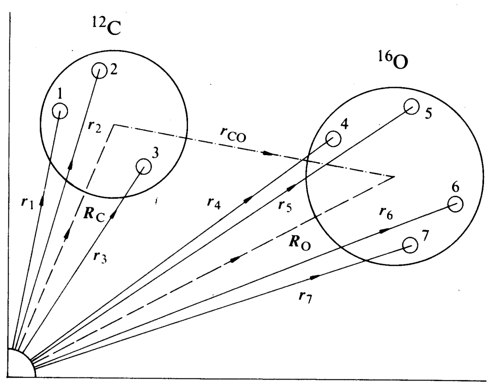


Fig. 1. Schematic diagram of the initial state. The vectors $\boldsymbol{r}_i$ ($i = 1$–7) are position vectors of seven α particles; $\boldsymbol{R}_C$ and $\boldsymbol{R}_O$ are the position vectors of the centres of mass of the ^{12}C and ^{16}O nuclei; $\boldsymbol{r}_{CO}$ is the vector between the centres of mass of the two nuclei.

The wavefunction for the initial state may be written as

$$\Psi_i = N_C N_O \exp\left(-\tfrac{1}{2}\alpha_C \sum_{i=1}^{3} \rho_{C_i}^2\right) \exp\left(-\tfrac{1}{2}\alpha_O \sum_{i=4}^{7} \rho_{O_i}^2\right) \exp(\mathrm{i}\, \boldsymbol{k}_i \cdot \boldsymbol{r}_{CO}), \tag{4}$$

where $\boldsymbol{k}_i$ is the initial relative momentum and $\boldsymbol{r}_{CO}$ is the vector distance between the centres of mass of ^{12}C and ^{16}O and is given by

$$\boldsymbol{r}_{CO} = -\boldsymbol{R}_C + \boldsymbol{R}_O \,. \tag{5}$$

The initial state is shown schematically in Fig. 1.

The ^{24}Mg nucleus is considered to be composed of two ^{12}C nuclei where each ^{12}C nucleus consists of three α particles. Hence the wavefunction for ^{24}Mg may be written as

$$\Psi_M = N_M \exp\left(-\tfrac{1}{2}\alpha'_C \sum_{i=1}^{3} \lambda^2_{C_i}\right) \exp\left(-\tfrac{1}{2}\alpha'_C \sum_{i=4}^{6} \lambda^2_{C_i}\right) \phi(\boldsymbol{R}_1 - \boldsymbol{R}_2) \,. \tag{6}$$

Here $\lambda_{C_i} = \boldsymbol{r}_i - \boldsymbol{R}_1$ (i = 1–3) and $\lambda_{C_i} = \boldsymbol{r}_i - \boldsymbol{R}_2$ (i = 4–6); $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the centre-of-mass position vectors of the first and second ^{12}C 'nucleus' in ^{24}Mg, respectively; $\phi(\boldsymbol{R}_1 - \boldsymbol{R}_2)$ is the relative wavefunction between the two carbon clusters, where

$$\phi(\boldsymbol{R}_1 - \boldsymbol{R}_2) \equiv \phi(\boldsymbol{R}) = \exp(-\tfrac{1}{2}\beta R^2) \,. \tag{7}$$

The width parameter β is chosen to yield the experimentally determined value (Hofstadter 1974) of 2·98 fm for the r.m.s. radius of ^{24}Mg taking α'_C to be equal to α_C. In this way we obtained a value for β of 0·13097 fm^{-2}.

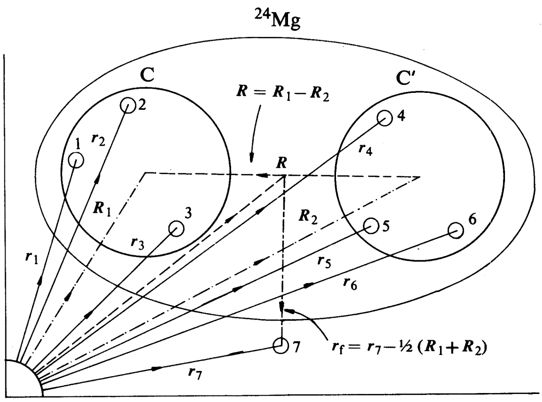


Fig. 2. Schematic diagram of the final state. The vectors $\boldsymbol{r}_i$ (i = 1–6) are the position vectors of six α particles of the ^{24}Mg nucleus and $\boldsymbol{r}_7$ is the position vector of the outgoing α particle; $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$ are the position vectors of the centres of mass of two ^{12}C clusters inside the ^{24}Mg nucleus; $\boldsymbol{R}$ is the position vector of the centre of mass of ^{24}Mg; $\boldsymbol{r}_f$ is the relative vector between ^{24}Mg and the emitted α particle.

The wavefunction for the final state becomes

$$\Psi_f = N_M \exp\left\{-\tfrac{1}{2}\alpha_C\left(\sum_{i=1}^{3} \lambda^2_{C_i} + \sum_{i=4}^{6} \lambda^2_{C_i}\right)\right\} \exp(-\tfrac{1}{2}\beta R^2) \exp(-\mathrm{i}\, \boldsymbol{k}_f \cdot \boldsymbol{r}_f) \,, \tag{8}$$

where k_f is the final relative momentum and r_f is the vector distance between the centres of mass of ^{24}Mg and the α particle. A schematic diagram of the final state is shown in Fig. 2.

The differential cross section is then given by (Goldfinger *et al.* 1977)

$$\mathrm{d}\sigma/\mathrm{d}\Omega = \mu_i \mu_f k_f^C |T|^2/(4\pi^2 k_i^C), \tag{9}$$

where μ_i and μ_f are the reduced masses of the initial and final states and k_i^C and k_f^C are the centre-of-mass momenta of the initial and final states and T is the transition amplitude given by

$$T = \langle \Psi_f | V_{C\alpha}(r_{C\alpha}) + V_{CC'}(r_{CC'}) | \Psi_i \rangle. \tag{10}$$

The two potentials $V_{C\alpha}$ and $V_{CC'}$ arise because we have taken the direct mode:

$$^{12}\mathrm{C} + (^{16}\mathrm{O} = \alpha \oplus {}^{12}\mathrm{C}) \rightarrow (^{24}\mathrm{Mg} = {}^{12}\mathrm{C} \oplus {}^{12}\mathrm{C}) + \alpha .$$

In (10) the vectors are defined as

$$r_{C\alpha} = \tfrac{1}{3}(r_1 + r_2 + r_3) - r_7, \tag{11}$$

$$r_{CC'} \equiv R = \tfrac{1}{3}(r_1 + r_2 + r_3) - \tfrac{1}{3}(r_4 + r_5 + r_6). \tag{12}$$

For both the carbon–α and carbon–carbon interaction we have used real potentials of the form

$$V(r) = W_{Oi}(1 + A_i r^2) \exp(-B_i r^2), \tag{13}$$

where $i = 1$ for the carbon–α interaction and $i = 2$ for the carbon–carbon interaction, W_{Oi} is the potential depth, A_i and B_i are adjustable parameters as suggested by Hussein and Zohni (1976), and r represents $r_{C\alpha}$ and $r_{CC'}$ in the carbon–α and carbon–carbon interactions. It is readily shown then that the expressions for $r_{C\alpha}$ and $r_{CC'}$ in equations (11) and (12) become

$$r_{C\alpha} = -\tfrac{4}{7} r_{CO} - \tfrac{6}{7} r_f, \qquad r_{CC'} = -\tfrac{8}{7} r_{CO} + \tfrac{2}{7} r_f. \tag{14a, b}$$

Siemssen (1970) suggested that all the real potentials are energy dependent for this type of reaction. The energy dependence may be taken into account by using a potential depth of the form

$$W_{Oi} = -(W'_{Oi} + c_i T_L), \tag{15}$$

where $i = 1$ for the carbon–α interaction and $i = 2$ for the carbon–carbon interaction, T_L is the incident energy of the projectile in the laboratory system and c_i is a constant. Using equations (4), (8), (14) and (15) the expressionfor the transition amplitude becomes

$$T = N_M N_C N_O \int \exp[-\tfrac{1}{2}\alpha'_C\{\lambda_{C1}^2+\lambda_{C2}^2+(\boldsymbol{\lambda}_{C1}+\boldsymbol{\lambda}_{C2})^2$$
$$+\lambda_{C4}^2+\lambda_{C5}^2+(\boldsymbol{\lambda}_{C4}+\boldsymbol{\lambda}_{C5})^2\}]$$
$$\times \exp(-\tfrac{1}{2}\beta r^2)\exp(-\mathrm{i}\,\boldsymbol{k}_f\cdot\boldsymbol{r}_f)$$
$$\times[W_{O1}\{1+A_1(\tfrac{4}{7}\boldsymbol{r}_{CO}-\tfrac{6}{7}\boldsymbol{r}_f)^2\}\exp\{-B_1(-\tfrac{4}{7}\boldsymbol{r}_{CO}-\tfrac{6}{7}\boldsymbol{r}_f)^2\}$$
$$+W_{O2}\{1+A_2(-\tfrac{8}{7}\boldsymbol{r}_{CO}+\tfrac{2}{7}\boldsymbol{r}_f)^2\}\exp\{-B_2(-\tfrac{8}{7}\boldsymbol{r}_{CO}+\tfrac{2}{7}\boldsymbol{r}_f)^2\}]$$
$$\times\exp[-\tfrac{1}{2}\alpha_C\{\rho_{C1}^2+\rho_{C2}^2+(\boldsymbol{\rho}_{C1}+\boldsymbol{\rho}_{C2})^2\}]$$
$$\times\exp[-\tfrac{1}{2}\alpha_O\{\rho_{O4}^2+\rho_{O5}^2+\rho_{O6}^2+(\boldsymbol{\rho}_{O4}+\boldsymbol{\rho}_{O5}+\boldsymbol{\rho}_{O6})^2\}]$$
$$\times\exp(\mathrm{i}\,\boldsymbol{k}_i\cdot\boldsymbol{r}_{CO})\,\mathrm{d}\lambda_{C_i}\,\mathrm{d}\boldsymbol{\rho}_{Ci}\,\mathrm{d}\boldsymbol{r}_{CO}\,\mathrm{d}\boldsymbol{r}_f\,\mathrm{d}\boldsymbol{R}. \qquad (16)$$

In equation (16) use has been made of the conditions for the centre of mass, for example

$$\boldsymbol{\lambda}_{C1}+\boldsymbol{\lambda}_{C2}+\boldsymbol{\lambda}_{C3}=0, \qquad \boldsymbol{\lambda}_{C4}+\boldsymbol{\lambda}_{C5}+\boldsymbol{\lambda}_{C6}=0, \qquad (17a, b)$$

$$\boldsymbol{\rho}_{C1}+\boldsymbol{\rho}_{C2}+\boldsymbol{\rho}_{C3}=0, \qquad \boldsymbol{\rho}_{O4}+\boldsymbol{\rho}_{O5}+\boldsymbol{\rho}_{O6}+\boldsymbol{\rho}_{O7}=0. \qquad (17c, d)$$

3. Results and Discussions

Transition amplitudes have been calculated using equation (16), where all the parameters except W_{O1}, W_{O2}, A_1, A_2, B_1 and B_2, and also c_1 and c_2, are known. Values of these parameters which give the best fit to the experimental data are

$$W_{O1} = 60\cdot0\ \mathrm{MeV}, \qquad W_{O2} = 85\cdot0\ \mathrm{MeV}, \qquad (18a, b)$$

$$A_1 = 0\cdot12331\ \mathrm{fm}^{-2}, \qquad A_2 = 0\cdot05499\ \mathrm{fm}^{-2}, \qquad (18c, d)$$

$$B_1 = 0\cdot22092\ \mathrm{fm}^{-2}, \qquad B_2 = 0\cdot05499\ \mathrm{fm}^{-2}, \qquad (18e, f)$$

$$c_1 = c_2 = 1\cdot30. \qquad (18g)$$

Our value of A_1 is slightly different from that of Hussein and Zohni (1976) but our value of B_1 is approximately the same as that calculated by them. The parameters A_2 and B_2 are chosen in such a way that when they are used in equation (13) a potential of the shape similar to that given by the real part of the Woods–Saxon type of potential (Michaud and Vogt 1972) is generated. From the transition amplitude the differential cross sections at seven different energies of the projectile were calculated. The results of the present analysis together with experimental data (Treu *et al.* 1978) are shown in Fig. 3. It can be seen from Fig. 3 that for the energies investigated agreement between the present calculations and experimental results is satisfactory.

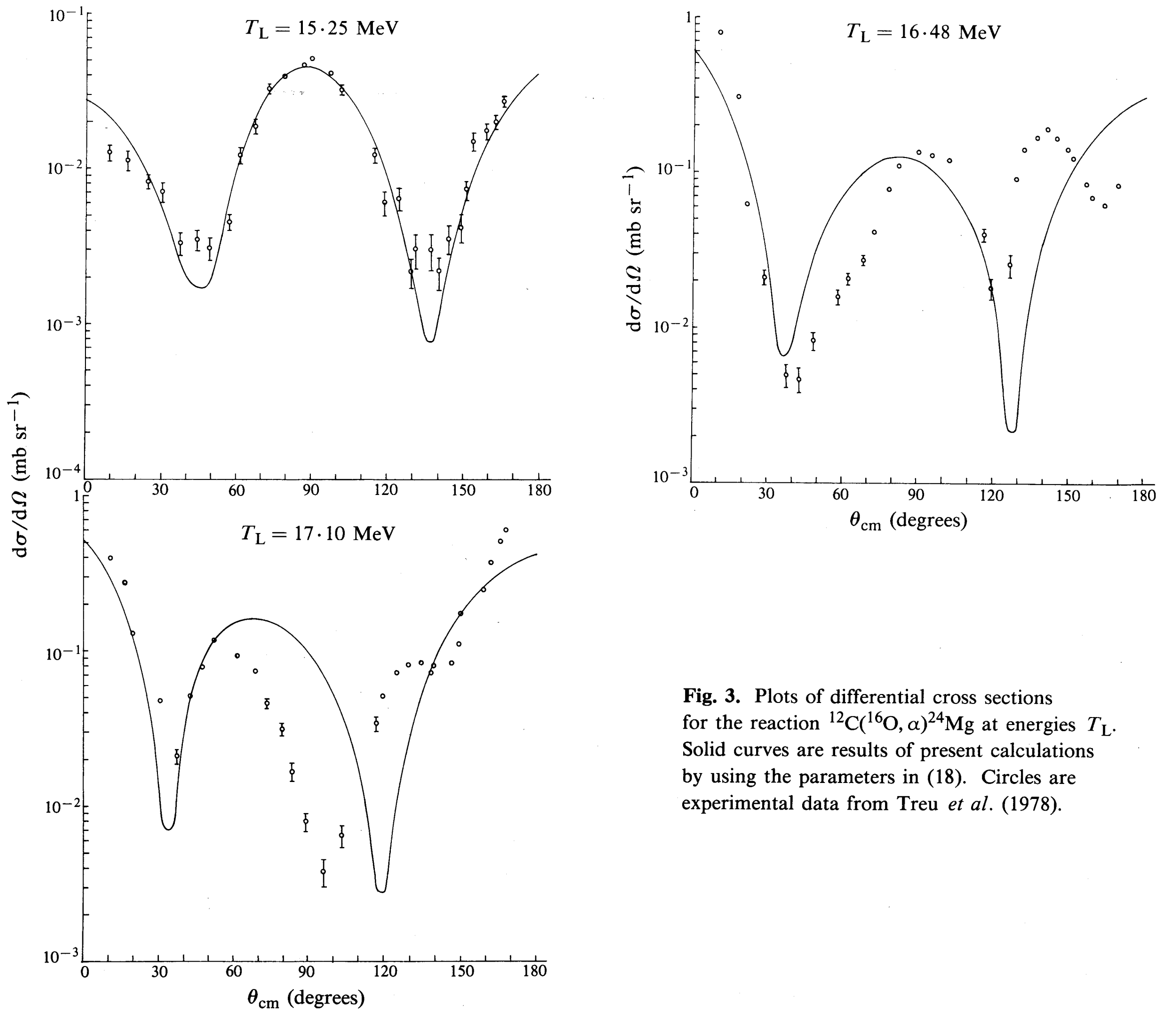
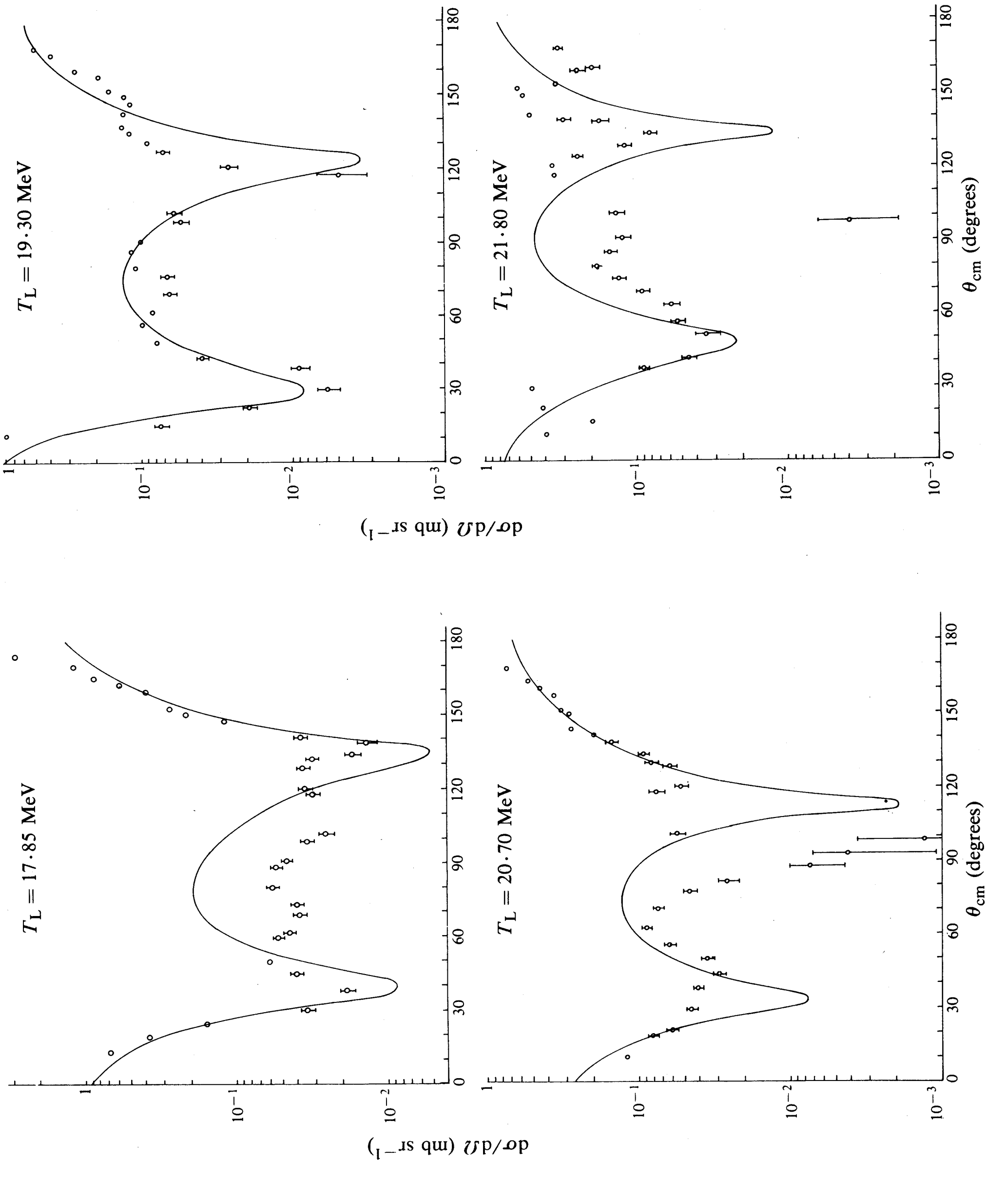


Fig. 3. Plots of differential cross sections for the reaction $^{12}C(^{16}O, \alpha)^{24}Mg$ at energies T_L. Solid curves are results of present calculations by using the parameters in (18). Circles are experimental data from Treu *et al.* (1978).


$T_L = 17 \cdot 85$ MeV
$T_L = 19 \cdot 30$ MeV
$T_L = 20 \cdot 70$ MeV
$T_L = 21 \cdot 80$ MeV
$d\sigma/d\Omega$ (mb sr^{-1})
θ_{cm} (degrees)

We have shown here that by using a simple α particle model and a simple potential—namely the real part of a modified gaussian type potential and a PWBA calculation—it is possible to obtain surprisingly good agreement with experimental data in both the small and large angle region. Gross features of the experimental differential cross section are reproduced by this simplistic model calculation. Obviously this calculation cannot explain any fine structure and hence cannot account for some of the discrepancies at intermediate angles. The agreement at these angles could possibly be improved by using distorted waves rather than plane waves for the initial and final relative motions. This would probably cause the direct ^{12}C transfer angular distribution to be considerably damped at large angles. The large cross sections at backward angles would then have to be interpreted in terms of an exchange mechanism. Taking both the direct and exchange mechanisms into account and adding their amplitudes coherently may give rise to interference effects which can account for the more complex structure observed in the experimental angular distributions.

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