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A Note on Lau's Generalised Theory of Gravitation

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Abstract

Lau has recently proposed a generalised theory of gravitation incorporating time-dependent cosmological and gravitational terms which is consistent with Dirac's large numbers hypothesis. It is shown that all vacuum solutions in this theory are identical to those of Einstein's theory with a cosmological constant. In particular, solar system experiments cannot be used to test the theory.

An interesting generalisation of Einstein's theory of gravitation has recently been proposed by Lau (1985) with time-dependent cosmological and gravitational parameters, which is consistent with Dirac's (1938) large numbers hypothesis. The field equations in this theory are

$$R_{ab}-\tfrac{1}{2}Rg_{ab}+\Lambda g_{ab} = GT_{ab}, \tag{1}$$

where R_{ab} is the Ricci tensor, R is the Ricci scalar, g_{ab} is the metric tensor and T_{ab} is the energy–momentum tensor. The cosmological parameter Λ and the gravitational parameter G are functions of time. We are using units in which the speed of light is unity and the constant 8π has been absorbed into G. By adopting Dirac cosmology, Lau has found specific forms for Λ and G but their exact form does not concern us at this stage. The vacuum field equations are therefore

$$R_{ab}-\tfrac{1}{2}Rg_{ab}+\Lambda g_{ab} = 0. \tag{2}$$

If we take the divergence of (2) we obtain

$$(R^{ab}-\tfrac{1}{2}Rg^{ab})_{;b}+\Lambda_{,b}\,g^{ab}+\Lambda g^{ab}{}_{;b} = 0. \tag{3}$$

The first term on the left-hand side of equation (3) is zero by virtue of the Bianchi identities and the metric tensor has zero covariant divergence (see e.g. Adler *et al.* 1975); we are thus left with

$$\Lambda_{,b}\,g^{ab} = 0. \tag{4}$$

It follows immediately from this that Λ is a constant. Thus any vacuum solution in Lau's theory is identical to the corresponding general relativistic solution with a

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cosmological constant. In particular there does not exist a one-body solution in Lau's theory with a time-dependent Λ. This is in contrast to the scale covariant theory of Canuto *et al.* (1977), which is also consistent with Dirac's large numbers hypothesis and in which there exists a one-body solution with time-dependent Λ (Adams 1978). There is thus no possibility of verifying the theory by making use of solar system tests.

Lau assumed Dirac's (1937) condition $G \sim 1/t$ and derived the form $\Lambda \sim 1/t^2$. Thus major differences between Einstein's general relativity and Lau's theory are expected to arise only at early cosmological times. We are at present investigating nucleosynthesis and the cosmic microwave background radiation in Lau's theory. However, we would like to point out that Dirac (1937, 1978) has repeatedly stressed that the form $G \sim 1/t$ is only an asymptotic form valid for late cosmological times.

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