Aust. J. Phys., 1986, **39**, 217–24

Enhancement of Parity Violation for Overlapping Nuclear Resonances

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Abstract

Proton scattering from a medium weight nucleus and the possibility of finding two resonances, with the same spin but opposite parity, overlapping is studied. As a guide to the experimental study of this possibility we have constructed a simple separable potential model involving spin $\frac{1}{2}$–spin 0 scattering that contains the required resonances for adjacent l values in the absence of a parity-violating (PV) interaction. A phenomenological PV interaction is then introduced and the magnitude of the longitudinal asymmetry is studied as a function of the model parameters. It is found that the magnitude of the asymmetry is greatest at zero separation for the case of narrow resonance levels.

1. General Remarks

Although the form of the parity-violating (PV) interaction between nucleons is of fundamental importance, the effect is very small. Recent attempts (Balzer *et al.* 1980) to study this effect by measuring the longitudinal asymmetry of protons in p–p scattering led to a longitudinal analysing power equal to almost twice that reported previously (Potter *et al.* 1974). However, if instead we study proton scattering from a medium weight nucleus, it is possible that the spectrum may have two resonances with the same spin but opposite parity overlapping. The amount of mixing should be inversely proportional to the energy separation, so that there may be considerable enhancement of the longitudinal asymmetry in the energy region of the resonances if their separation is small.

Stodolsky and Forte (1980) reported the first observations of neutron weak spin rotation. An effect was found which is much greater than that anticipated by conventional weak interaction theory. If the resonance-dominated-amplitude explanation is adopted, then the role of resonances might be important when they are very close together.

In this respect, Karl and Tadic (1977) suggested the possibility of exploring situations in which enhancement might occur in relation to the measurement of the spin rotation of a beam of polarized thermal neutrons. The prediction of a rotation angle ϕ $(=10^{-5}\ \mathrm{rad\,m^{-1}})$ in ^{209}Bi, using several theoretical and semi-empirical PV potentials, was given and discussed in detail by Tadic and Barroso (1978). However,

0004-9506/86/020217$02.00

as pointed out by Barroso and Margaca (1980), several approximations not always valid were used to predict the behaviour of ϕ near a resonance. Barroso and Margaca (1980) proved that, for a target with a p-wave resonance in the eV region, the rotation angle corresponding to neutrons with energy in the meV region is enhanced by quite a large factor; this was in good agreement with the calculation made by Tadic and Barroso (1978).

The present work, supporting previous results and offering an experimental guide to detect parity violation, introduces a somewhat detailed study of the case where cross sections, asymmetry, and maximum asymmetry are calculated for various overlapping resonances. For simplicity, we study the observable enhancement of polarization, which signals parity violation, by using a potential model involving overlapping resonances. The model involves spin $\frac{1}{2}$–spin 0 scattering and is constructed so that, in the absence of parity violation, two overlapping resonances are present with the same spin but opposite parity. A phenomenological PV interaction is then introduced that mixes the levels, and the resulting longitudinal asymmetry of the spin $\frac{1}{2}$ particle is studied as a function of the resonance separation and widths. Only relative effects are studied and no attempt is made to relate the strength of the PV potential to the magnitude of the weak N–N interaction. Possible experiments to detect PV effects in proton elastic scattering from medium weight nuclei are discussed, and the most general form in the case of elastic scattering is considered.

2. Spin $\frac{1}{2}$–Spin 0 Scattering with Parity Violation

In order to study spin $\frac{1}{2}$–spin 0 scattering without parity conservation but with the retention of time reversal and rotational invariance, we adopt the description by Taylor (1972) and consider a nucleon incident along the z axis with momentum $\boldsymbol{k}$ and with the outgoing nucleon of momentum $\boldsymbol{k}'$ lying in the x–z plane. With parity violation, the scattering amplitude may be written as

$$f(\theta) = a(\theta) + \mathrm{i}b(\theta)\boldsymbol{\sigma}.\,\hat{\boldsymbol{n}} + \mathrm{i}c(\theta)\boldsymbol{\sigma}.\,\hat{\boldsymbol{q}}, \tag{1}$$

where $\boldsymbol{n} = \boldsymbol{k}\times\boldsymbol{k}'$ and $\boldsymbol{q} = \boldsymbol{k}+\boldsymbol{k}'$, $\boldsymbol{\sigma}$ represents the Pauli matrices, $a(\theta)$ and $b(\theta)$ are the usual non-flip and flip amplitudes and the $c(\theta)$ term is parity violating but maintains time reversal and rotational invariance. In the scattering of an unpolarized beam, the $c(\theta)$ term gives rise to polarization in the scattering plane. Since the detection of such effects requires double scattering, we concentrate instead on the asymmetry in the scattering of longitudinally polarized nucleons.

We describe the density matrix for the incident beam that is longitudinally polarized along the $+z$ axis as

$$\rho_{\mathrm{in}} = \tfrac{1}{2}(1 + P_z\sigma_z), \tag{2}$$

where P_z is the degree of polarization of the beam. The outgoing density matrix is then

$$\rho_{\mathrm{out}} = f(\theta)\rho_{\mathrm{in}}f^{\dagger}(\theta). \tag{3}$$

Substituting (1) and (2) into (3) and omitting traceless terms we obtain

$$\rho_{\mathrm{out}} = \tfrac{1}{2}[|a|^2 + |b|^2 + |c|^2 + 2P_z\{\mathrm{Im}(ac^*)\cos\tfrac{1}{2}\theta\,\mathrm{Im}(bc^*)\sin\theta\sin\tfrac{1}{2}\theta\}]. \tag{4}$$

We note that if the direction of the incident nucleon polarization is changed from $+z$ to $-z$, the algebraic sign of P_z changes. If we define

$$\sigma_{\pm}(\theta) = \mathrm{Tr}\{\rho_{\mathrm{out}}(\pm z)\}\,, \tag{5}$$

then the asymmetry $A(\theta)$ may be given by

$$A(\theta) = \frac{1}{|P_z|}\left(\frac{\sigma_+(\theta)-\sigma_-(\theta)}{\sigma_+(\theta)+\sigma_-(\theta)}\right). \tag{6}$$

Evaluating (6) we obtain

$$A(\theta) = \frac{2\{\mathrm{Im}(ac^*)\cos\tfrac{1}{2}\theta + \mathrm{Im}(bc^*)\sin\theta\sin\tfrac{1}{2}\theta\}}{|a|^2+|b|^2+|c|^2}\,. \tag{7}$$

We note from this expression that asymmetry is linear in c and that it arises from interference with the parity conserving flip and non-flip amplitudes.

We proceed to make a partial analysis of $f(\theta)$ according to

$$\langle S_z|f(\theta)|S_z'\rangle = 4\pi \sum_{l,m,l',m'} \langle lm\tfrac{1}{2}S_z|JM\rangle\langle l'm'\tfrac{1}{2}S_z'|JM\rangle$$

$$\times f^{j}_{ll'}\, Y^{*m'}_{l'}(\hat{k}')\, Y^{m}_{l}(\hat{k}) \tag{8}$$

and first concentrate on the parity conserving (PC) ($l = l'$) part of $f(\theta)$. By letting

$$f_{+\pm}(\theta) = \langle\tfrac{1}{2}|f(\theta)|\pm\tfrac{1}{2}\rangle, \tag{9}$$

where the first subscript $(+)$ implies that the incident nucleon is polarized along the positive direction and the second subscript ($+$ or $-$) refers to $J = l\pm\frac{1}{2}$, we obtain the standard expressions

$$f^{\mathrm{PC}}_{++}(\theta) = a(\theta) = \sum_{l=0}^{\infty}\{(l+1)f^{(+)}_{ll} + lf^{(-)}_{ll}\}P_l(\cos\theta), \tag{10}$$

$$f^{\mathrm{PC}}_{+-}(\theta) = b(\theta) = \sin\theta\sum_{l=0}^{\infty}(f^{(+)}_{ll} - f^{(-)}_{ll})P_l(\cos\theta). \tag{11}$$

In the case of the PV contribution, two terms arise for a given l, one with $l' = l+1$, $J = l+\frac{1}{2}$ and one with $l' = l-1$, $J = l-\frac{1}{2}$. The PV contributions in (8) then emerge as the following spin non-flip and flip terms

$$f^{\mathrm{PV}}_{++}(\theta) = -\sum_{l=0}^{\infty}\{(l+1)f^{(+)}_{l,l+1}\,P_{l+1}(\cos\theta) + lf^{(-)}_{l,l-1}\,P'_{l-1}(\cos\theta)\}\,, \tag{12}$$

$$f^{\mathrm{PV}}_{+-}(\theta) = \sin\theta\sum_{l=0}^{\infty}\{f^{(-)}_{l,l-1}\,P'_{l-1}(\cos\theta) - f^{(+)}_{l,l+1}\,P'_{l+1}(\cos\theta)\}\,. \tag{13}$$

From (12), (13) and (1) we obtain the expression

$$c(\theta) = 2\mathrm{i}\cos\tfrac{1}{2}\theta\sum_{l=0}^{\infty}\{f^{(+)}_{l,l+1}\,P'_{l+1}(\cos\theta) - f^{(-)}_{l,l-1}\,P'_{l-1}(\cos\theta)\}\,. \tag{14}$$

This completes the analysis necessary to give the asymmetry in terms of the PC and PV scattering amplitudes.

3. Potential Model

In order to study the asymmetry as a function of resonance position, we use a potential model in which resonances may be formed and their positions adjusted in adjacent partial waves. This would be difficult with the usual optical potentials since the partial waves are correlated. We choose instead a set of separable potentials so that the resonance positions can be adjusted independently in each partial wave. For a given J we have a two-channel scattering problem to solve in order to obtain all necessary scattering amplitudes. We introduce the partial wave t-matrix according to ($\hbar = 1$)

$$t_{ll'} = (-2\pi/m) f_{ll'}, \tag{15}$$

where m is the reduced mass. The necessary scattering amplitudes are obtained as solutions of the two-channel Lippmann–Schwinger equation

$$T_{l'l}(k', k, E) = V_{l'l}(k', k) + \frac{1}{2\pi^2} \sum_{l''} \int_0^\infty \frac{n^2 \, \mathrm{d}n \; V_{l'l''}(k', n) \; T(n', k, E)}{E - (n^2/2m) + \mathrm{i}\epsilon}. \tag{16}$$

The further advantage of choosing a separable form for V is that (16) then has an algebraic solution. We choose the potential to be of the form

$$V_{l'l}(k', k) = \lambda^J_{l'l} \, v_{l'}(k') \, v_l(k), \tag{17}$$

thereby defining both the PC ($l = l'$) and the PV ($l \neq l'$) parts of the interaction. We consider only the partial waves l and $l' = l+1$ both coupled to $J = l+\frac{1}{2}$ (we suppress J). By letting

$$I_{ij} = \frac{1}{2\pi^2} \int_0^\infty \frac{k^2 \, \mathrm{d}k \; v_i(k) \, v_j(k)}{E - k^2 + \mathrm{i}\epsilon}, \tag{18}$$

we obtain the on-shell amplitudes

$$T_{ll}(k) = \frac{\lambda_{ll} \, v_l^2(k)}{\Delta} \left\{ 1 + I_{l+1,l+1} \left(\frac{\lambda^2_{l,l+1}}{\lambda_{ll}} - \lambda_{l+1,l+1} \right) \right\} \tag{19}$$

and a similar expression for $T_{l+1,l+1}(k)$ with the replacements $l \to l+1$ in (19). The PV amplitude is

$$T_{l,l+1}(k) = \Delta^{-1} \{ \lambda_{l,l+1} \, v_l(k) \, v_{l+1}(k) \}. \tag{20}$$

These solutions are dependent on the Fredholm determinant given, in this case, by

$$\Delta = 1 - (\lambda_{ll} I_{ll} + \lambda_{l+1,l+1} I_{l+1,l+1}) + I_{ll} I_{l+1,l+1} (\lambda_{ll} \lambda_{l+1,l+1} - \lambda^2_{l,l+1}). \tag{21}$$

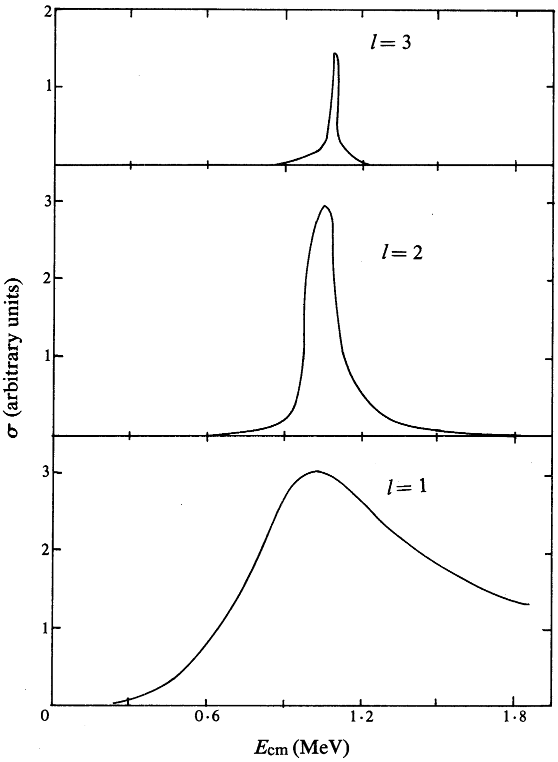

Fig. 1. Total cross section against E_{cm} for single l-waves in the PC and spinless case.

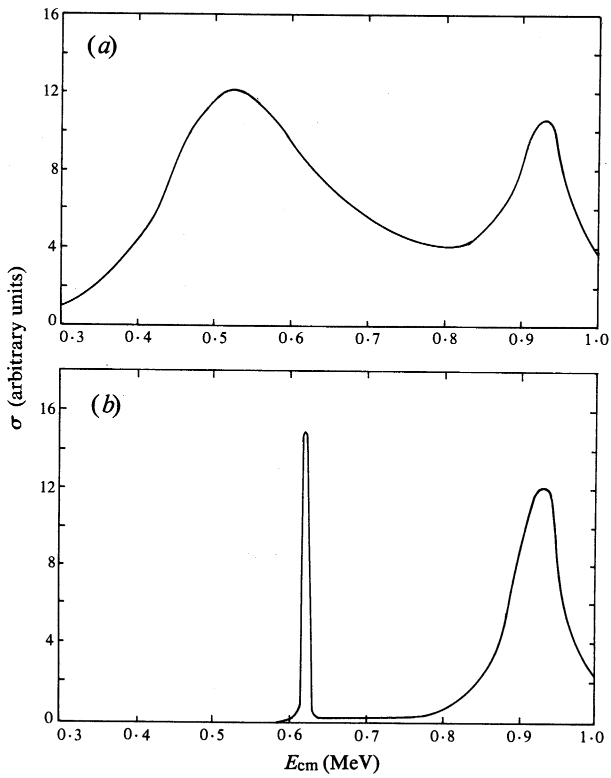

Fig. 2. Total cross section against E_{cm} for the PC cases (a) $J = 3/2$ and (b) $J = 5/2$.

Since we will be dealing with low energy resonances we can adjust the coupling constant in either of the partial waves relative to that necessary for a zero energy bound state. We take

$$\lambda_{ll} = \eta_l \lambda_0 , \tag{22}$$

where λ_0 is that value of coupling necessary for a zero energy bound state in the absence of parity violation and η is near unity. For the lth partial wave we choose the form factor to be

$$v_l = \beta^{3l} k^l / (k^2 + \beta^2)^{2l} . \tag{23}$$

The resonance position is adjusted by varying η_l in (22). For our form factor (23), λ_0 is given by

$$\lambda_0 = -4\pi / m\beta \tag{24}$$

and for $0{\cdot}9 < \eta_l < 1$ we obtain low energy resonances. In our subsequent calculation we choose $\beta = 2$ and fix the masses appropriate to nucleon scattering on a mass four system. In some calculations we have also added a PC s-wave interaction to make the calculation more 'realistic'.

4. Results

The general features of the resonances produced with the separable potentials are shown in Fig. 1 where the total cross section against energy is plotted for typical $l = 1, 2, 3$ resonances with no PV interactions. Centrifugal barrier effects make the width strongly dependent on l and there is little freedom to vary the width by changing β. Fig. 2 shows the total cross sections obtained by including a p-wave resonance together with a higher energy d-wave state for $J = \frac{3}{2}$ and $\frac{5}{2}$, $l = 2$ and 3. Again, no parity violation has been included here.

With the magnitude of the PC interactions established we now turn to the PV interaction. A PC s-wave interaction has also been included to provide a more realistic background to the cross section. The resulting total cross sections against energy are shown in Fig. 3 for $J = \frac{3}{2}$ and $\frac{5}{2}$. The corresponding angular distributions of the longitudinal asymmetry are shown in Fig. 4. The energies chosen are close to the corresponding resonant peaks of the total cross section. In both cases the asymmetry is positive at the energy of the lower resonance but turns negative if the energy approaches or exceeds the higher level. Also, $A(\theta)$ vanishes at certain angles, independent of the energy. This follows from the simple angular dependence of $A(\theta)$ if only one PV resonant pair of amplitudes is present. Under these circumstances there are $J + \frac{1}{2}$ nodes in $A(\theta)$ and the PV amplitude $c(\theta)$ has the simple structure

$$c(\theta) \approx \cos \tfrac{1}{2}\theta \, \{ P'_{l+1}(\cos\theta) - P'_l(\cos\theta) \} , \tag{25}$$

where l refers to the lower angular momentum of the pair. The positions of the nodes are easily obtained from (25) and appear at $\theta = 70°$ and $180°$ for $J = \frac{3}{2}$ and at $\theta = 46°$, $107°$ and $180°$ for $J = \frac{5}{2}$.

In order to detect parity violation in subsequent proton–nucleus scattering experiments, it is of interest to study the magnitude of asymmetry as a function of resonance level separation. Such a study requires a definition of the resonance position. We use the pole position of the scattering amplitude as our definition since

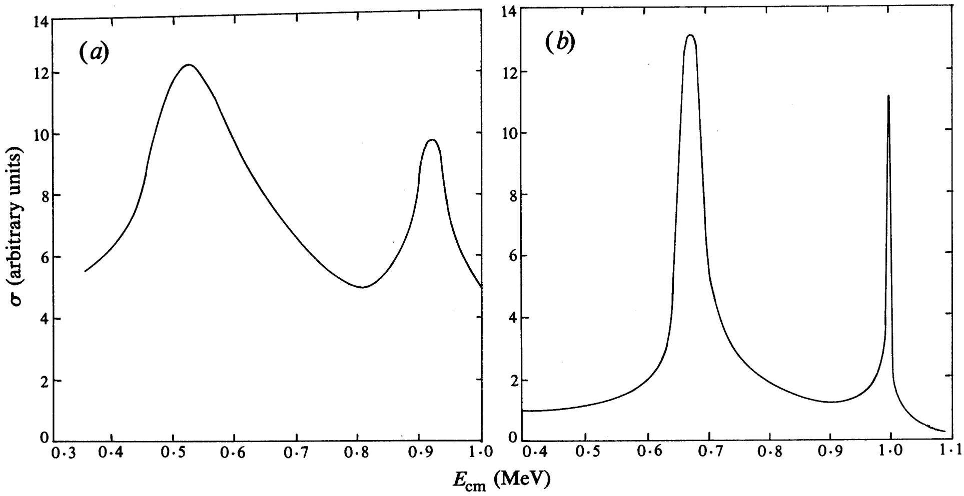

Fig. 3. Total cross section against $E_{\rm cm}$ for the PV cases (*a*) $J = \frac{3}{2}$ and (*b*) $J = \frac{5}{2}$.

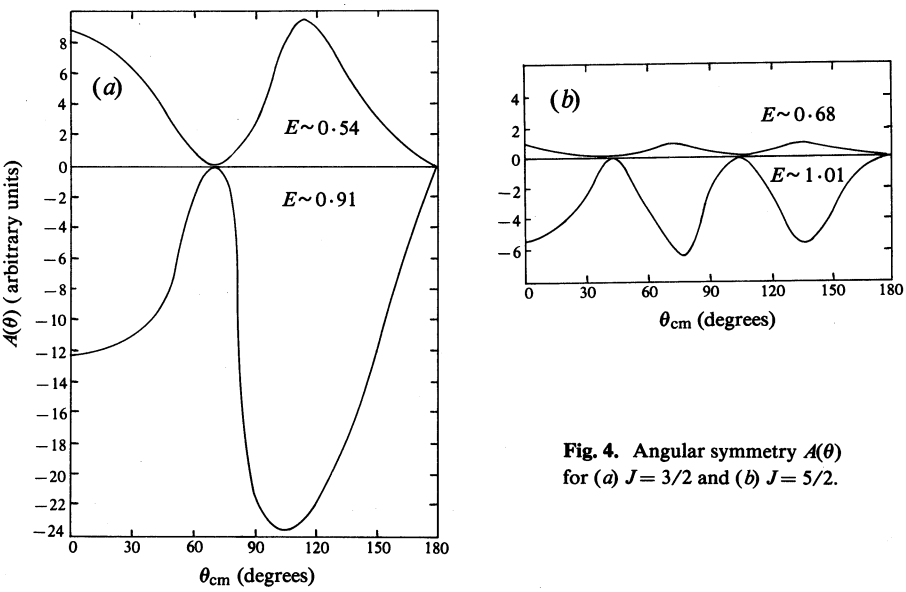

Fig. 4. Angular symmetry $A(\theta)$ for (*a*) $J = 3/2$ and (*b*) $J = 5/2$.

the simple expression for the Fredholm determinant allows analytic continuation. We then define the resonance separation as the difference between the real parts of the pole positions. To simplify the results we first position one of the resonances and study $A(\theta)$ as the second resonance is moved closer to the first. Fig. 5 shows the plot of maximum asymmetry appearing at any angle against the resonance separation for $J = \frac{3}{2}, \frac{5}{2}$ and $\frac{7}{2}$. We would expect $|A_{\rm max}|$ to be a maximum at $\Delta E = 0$ and fall to zero for large resonance separations. This is the case for $J = \frac{5}{2}$ and $\frac{7}{2}$ in which the resonances involved are quite narrow. For $J = \frac{3}{2}$ we do not obtain a maximum and the magnitude of $|A_{\rm max}|$ remains not peaked at zero. The behaviour is apparently related to the larger widths of the levels in the $J = \frac{3}{2}$ case.

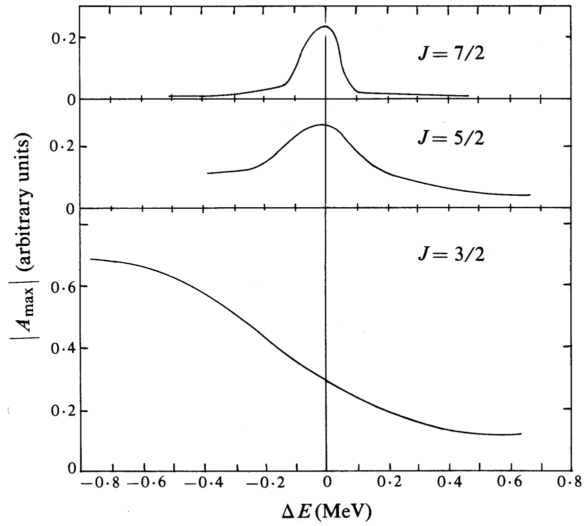

Fig. 5. Maximum asymmetry $A_{\max}$ against ΔE for $J = \frac{3}{2}, \frac{5}{2}, \frac{7}{2}$.

5. Discussion

We have shown by means of a simple potential model how the longitudinal asymmetry behaves in a situation with overlapping resonances together with parity violation. From the point of view of detecting parity violation in proton–nucleus scattering we found that the magnitude of the asymmetry is greatest at zero separation for the case of narrow resonance levels. We also found that the asymmetry has a simple nodal structure for the case of only a single J involved in the parity mixing.

Acknowledgments

I am grateful to Professor P. E. Shanley of the University of Notre Dame for his numerous comments and fruitful discussions.

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Manuscript received 3 May, accepted 10 October 1985