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Magnetohydrodynamic Tube Waves: Waves in Fibrils*

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Abstract

A discussion of waves in magnetic flux tubes imbedded in magnetic unstratified surroundings is given. Seven types of non-leaky wave are identified. Many more leaky waves, tube oscillations which drive waves in the external medium and thereby lose energy to it, are also found. The particular example of longitudinal and transverse oscillations in chromospheric fibrils is examined in detail.

1. Introduction

Interest in waves in the solar atmosphere centres on two main aspects, (i) the mechanical transport of energy and its role in heating the chromosphere and corona, and (ii) the explanation of observed oscillations and wave motions; for example, the five-minute oscillations, three-minute oscillations in sunspot umbrae, running penumbral waves, etc.

Much progress has been made on the theory of waves in uniform or continuously varying media. In particular, a plane parallel vertically stratified model for the solar atmosphere is often adopted with either a horizontal uniform magnetic field (e.g. Nye and Thomas 1976), a vertical uniform field (e.g. Thomas 1978; Leroy 1981; Leroy and Schwartz 1982; and many other related papers), or a uniform inclined field (Schwartz and Bel 1984). A simple spreading field has also been investigated (Cally 1983; Schwartz *et al.* 1984).

However, the theory is now being forced to come to terms with observations. It has been known for some time that the outer layers of the Sun are far from the simple planar slabs pictured in the above references; fine structure, especially that associated with magnetic fields, proliferates and in many contexts cannot be ignored. Oscillations in such features as coronal loops, chromospheric fibrils, intense flux tubes at supergranule boundaries, ephemeral regions, etc. demand attention. A theory of waves in magnetic flux tubes is required as a step in this direction.

Some progress has been made, especially in the *thin flux tube approximation* (Defouw 1976; Roberts and Webb 1978, 1979; Roberts 1981; Spruit 1982) in which

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the gravitationally stratified atmosphere problem is tractable. Furthermore, in the absence of gravity, much has been learnt of waves in tubes of finite width (Meerson *et al.* 1978; Wentzel 1979; Wilson 1981; Edwin and Roberts 1983). The results of both approaches show that tube waves differ substantially from their better known homogeneous or slowly varying atmosphere cousins.

In such calculations, mathematically, the process of matching waves internal and external to the tube produces a complicated dispersion relation involving the longitudinal wavenumber k and the frequency ω linked through various rational and Bessel functions. For each given k there is a discrete spectrum of allowed ω values.

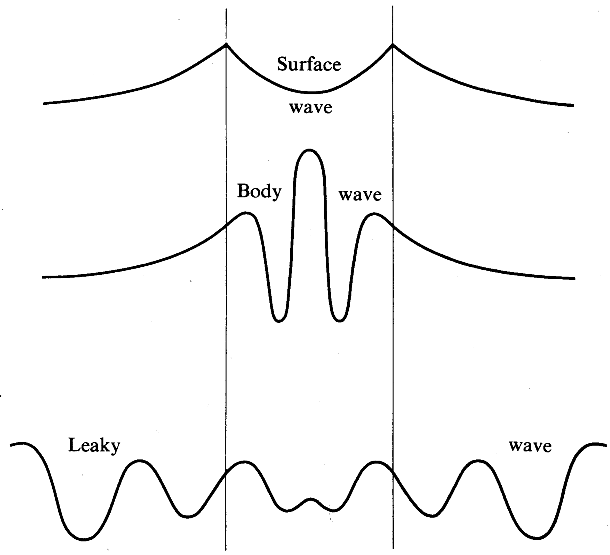

Fig. 1. Typical structures of the total pressure perturbation in the two types of non-leaky tube wave, surface (l imaginary) and body (l real), and in a leaky tube wave. In the last of these the structure inside the tube is not significant, different behaviours may occur, but the outward propagating external wave is its defining characteristic; the growth in the external solution with distance from the tube is due to wavefronts further out having been excited earlier, when the tube oscillation was stronger.

Most previous treatments have sought only real frequencies. These correspond to waves travelling along the tube without temporal decay, and are associated with an essentially exponential fall off of amplitude with distance from the tube boundary in the surrounding medium (see Fig. 1). However, the possibility of complex frequencies has been discussed, in the thin tube approximation by Roberts and Webb (1979) and Spruit (1982), and by Meerson *et al.* (1978) for finite tube width, but under some rather restrictive conditions. A complex ω value, with negative imaginary part ω_i, corresponds to leakage from the tube wave (Fig. 1); the internal motions drive an external wave which draws energy from them, and the mode decays in time as $\exp(\omega_i t)$. Wilson (1981) has also been concerned in large part with complex frequencies, treating a flux tube of finite width imbedded in a non-magnetic medium, though allowing for the possibility of an external parallel flow (with which we shall not be concerned).

It is the purpose of the present paper to discuss the fibril waves observed by Giovanelli (1975) in the context of tube waves. To this end, sufficient of the general theory of Cally (1985) is introduced to allow a qualitative and quantitative exposition. This general theory differs from the more advanced previous treatments, by Edwin and Roberts (1983) and by Wilson (1981) in particular, in a number of important respects:

(i) It treats both leaky and non-leaky waves extensively in flux tubes surrounded by a background field. Edwin and Roberts were concerned only with non-leaky waves, whilst Wilson's flux tubes were magnetically isolated, thus allowing only acoustic waves to carry energy into the surrounding medium rather than the two (fast and slow) magnetohydrodynamic (MHD) modes which give rise to entirely new modes of oscillation (both leaky and non-leaky) and do not merely modify the existing ones.

(ii) It provides a simple and *complete* graphical classification (cf. Figs 2 and 3 and Table 1) of non-leaky modes, which allows for instant recognition of all such tube waves in any situation. It is hoped that it will therefore be of substantial practical utility, firstly in understanding observed oscillations (as with the fibril waves considered here), and secondly in using these observations as a diagnostic tool to reveal details of the medium (e.g. field strength, density, etc.).

(iii) It provides an almost complete asymptotic treatment of leaky waves (only part of which is reproduced here, cf. the 'Trig' modes of Section 3), which allows for a near complete rough sketch of the dispersion diagram pertaining to any situation to be drawn with little difficulty. Wilson's treatment was incomplete in this respect, even allowing for the neglect of external fields (see Section 3).

2. Model and Equations

A uniform magnetic flux tube of radius R consisting of straight field lines $\boldsymbol{B} = (0, 0, B)$ is embedded in a homogeneous atmosphere permeated by a parallel field $\boldsymbol{B}_{\mathrm{e}} = (0, 0, B_{\mathrm{e}})$. Cylindrical coordinates (r, θ, z) are adopted throughout. The internal density ρ, Alfvén speed $a = (4\pi\rho)^{-\frac{1}{2}} B$, and sound speed c, and their external counterparts ρ_{e}, a_{e} and c_{e}, specify the basic system. The density contrast $D = \rho/\rho_{\mathrm{e}}$ is determined, however, by the pressure equilibrium condition across R:

$$\rho/\rho_{\mathrm{e}} = (c_{\mathrm{e}}^2/\gamma_{\mathrm{e}} + \tfrac{1}{2} a_{\mathrm{e}}^2)/(c^2/\gamma + \tfrac{1}{2} a^2), \tag{1}$$

where γ is the ratio of specific heats inside the tube, and γ_{e} that outside.

The following mathematical steps are taken:

(i) Perturbation quantities in the linearized adiabatic MHD equations are assumed proportional to $\exp\{\mathrm{i}(kz + m\theta - \omega t)\}$, where m is an integer, and only the r dependence remains to be determined.

(ii) The resulting Bessel equation (Wentzel 1979) for the total pressure perturbations p' and p_{e}',

$$\left[\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}}{\mathrm{d}r}\right) + \left\{\begin{pmatrix} l^2 \\ l_{\mathrm{e}}^2 \end{pmatrix} - \frac{m^2}{r^2}\right\}\right]\begin{pmatrix} p' \\ p_{\mathrm{e}}' \end{pmatrix} = 0 \qquad \begin{pmatrix} r < R \\ r > R \end{pmatrix}, \tag{2}$$

is solved subject to the conditions that the internal solution be well behaved at $r = 0$, whilst the external solution must either vanish as $r \rightarrow \infty$ (ω real) or represent an outward propagation of energy (ω complex). Here l and l_e are respectively the effective internal and external radial wavenumbers:

$$l = \left(\frac{(\omega^2 - a^2 k^2)(\omega^2 - c^2 k^2)}{(a^2 + c^2)(\omega^2 - c_T^2 k^2)} \right)^{\frac{1}{2}}, \qquad -\tfrac{1}{2}\pi < \arg l \leqslant \tfrac{1}{2}\pi; \tag{3}$$

$$l_e = \left(\frac{(\omega^2 - a_e^2 k^2)(\omega^2 - c_e^2 k^2)}{(a_e^2 + c_e^2)(\omega^2 - c_{Te}^2 k^2)} \right)^{\frac{1}{2}}, \qquad -\tfrac{1}{2}\pi < \arg l_e \leqslant \tfrac{1}{2}\pi; \tag{4}$$

where $c_T = ac/(a^2 + c^2)^{\frac{1}{2}}$ is the so-called *tube* or *cusp* speed, and its external counterpart is $c_{Te} = a_e c_e/(a_e^2 + c_e^2)^{\frac{1}{2}}$ (note that the tube speeds are both subsonic and sub-Alfvénic). The resulting solutions are

$$p' \propto J_m(lr) \exp\{i(kz + m\theta - \omega t)\}, \tag{5}$$

$$p_e' \propto H_m^{(q)}(l_e r) \exp\{i(kz + m\theta - \omega t)\}, \tag{6}$$

where $H_m^{(q)}$ is a Hankel function, and

$$q = \begin{cases} 1, & \text{if } \operatorname{Re}\{\omega l_e/(\omega^2 - a_e^2 k^2)\} \geqslant 0 & \text{(7a)} \\ 2, & \text{otherwise.} & \text{(7b)} \end{cases}$$

The condition on q specifies the solution with outgoing energy flux as $r \rightarrow \infty$. An outgoing phase velocity corresponds to $q = 1$, but this is not always the appropriate choice.

(iii) The internal and external solutions are matched at $r = R$ by requiring that the total pressure perturbation and the radial component of velocity be continuous across the boundary.

This last step leads to a dispersion relation which is best expressed in terms of the dimensionless velocities

$$A = a/c_e, \quad C = c/c_e, \quad A_e = a_e/c_e, \quad C_T = c_T/c_e, \quad C_{Te} = c_{Te}/c_e, \tag{8a}$$

the frequency

$$\Omega = \omega R/c_e, \tag{8b}$$

and the wavenumbers

$$K = kR, \qquad L = lR, \qquad L_e = l_e R. \tag{8c}$$

The dispersion relation is then

$$\phi \equiv \frac{L}{\Omega^2 - A^2 K^2} \frac{J_m'(L)}{J_m(L)} = \frac{D L_e}{\Omega^2 - A_e^2 K^2} \frac{H_m^{(q)\prime}(L_e)}{H_m^{(q)}(L_e)} \equiv \psi. \tag{9}$$

The primes denote differentiation with respect to the argument. Equation (9) is consistent with the dispersion relations obtained by many authors [see e.g. equation

(15) of Wentzel (1979), equation (22) of Wilson (1981) and equation (8) of Edwin and Roberts (1983)].

3. Some General Results

Two approaches to the dispersion relation suggest themselves. The first is to specify the wavenumber K and derive the frequency Ω, which if complex with $\operatorname{Im} \Omega < 0$ corresponds to temporal decay. This is generally appropriate for the initial value problem in which the tube suffers an initial perturbation along part of its length, due for example to random turbulent buffeting, and oscillates freely thereafter. For the boundary value problem though, for example when a wave is being driven by steady overstable convection in a thin subphotospheric layer, Ω should be prescribed and K deduced; complex K then describes spatial decay. For concreteness, the former approach will be adopted here; extension to the latter is straightforward (cf. Wilson 1981).

With the dimensionless longitudinal phase speed $V = \Omega/K$ and the wavenumber K both real, L and L_e are either real or purely imaginary (cf. equations 3 and 4). Consequently, ϕ is always real, whilst ψ is real only if L_e is imaginary (cf. Wilson 1981). Equation (9) therefore indicates that any *real* phase speed V must lie either below C_{Te} or between A_e and 1 [i.e. $\omega/k < c_{Te}$ or $\min(a_e, c_e) < \omega/k < \max(a_e, c_e)$]. Hence the upper bound on the longitudinal phase speed of a non-leaky tube wave is determined by the *external*, not the internal, plasma.

A number of other quite detailed conclusions concerning non-leaky waves may be drawn by sketching ϕ and ψ (see Figs 2 and 3). If one takes the appropriate part of Fig. 2 (i.e. 2*a* if $A < C$ and 2*b* otherwise), mentally adjusts the V axis scaling, and fits over it a similarly adjusted Fig. 3, the points of intersection $\phi = \psi$ correspond to the discrete *real* eigenfrequencies. Such a mental exercise allows the identification of seven types of non-leaky tube wave (see Table 1), each of which is denoted by a letter, either 'S' or 'B' depending on whether it is a surface (L imaginary) or body (L real) wave, a superscript '+' or '−' indicating that it is externally fast [$\min(A_e, 1) < V < \max(A_e, 1)$] or slow ($V < C_{Te}$), and a subscript '+' [$V > \min(A, C)$] or '−' [$V < \min(A, C)$] referring to the relation of the phase speed to the characteristic internal velocities.

However, this is by no means the full story. Many varieties of leaky mode may also exist. Of particular interest are the so called 'Trig' modes (Cally 1985), which in the high frequency limit $|\Omega| \gg K, AK, CK, A_eK, |L| \gg 1$ and $|L_e| \gg 1$ are given by

$$\Omega_n \sim (A^2+C^2)^{\frac{1}{2}}\left(\{m+2n+1+\tfrac{1}{2}\operatorname{sgn}(X-1)\}\tfrac{1}{2}\pi - \tfrac{1}{2}\mathrm{i}\ln\left|\frac{1+X}{1-X}\right|\right), \tag{10}$$

where n is an arbitrary integer consistent with $\operatorname{Re}\Omega > 0$ and $|\Omega|$ large, and

$$X = D\left(\frac{A^2+C^2}{A_e^2+1}\right)^{\frac{1}{2}}. \tag{11}$$

An infinite number of these modes *always* exists, though sometimes the first few eigenfrequencies, for which the asymptotic approximations above are not valid, may in fact be real. In this case (see Fig. 4) the real modes are the B_+^+ waves mentioned in

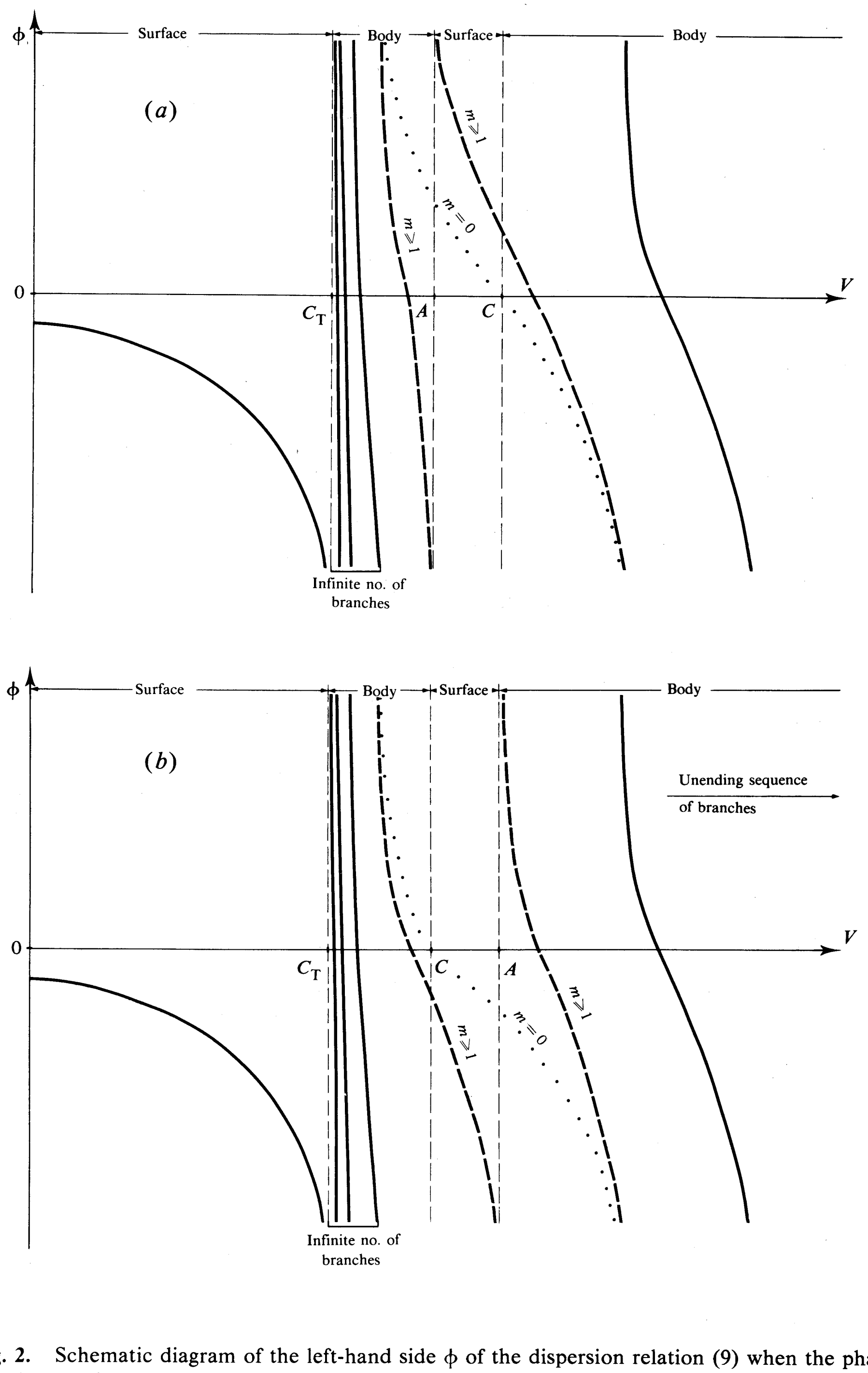

Fig. 2. Schematic diagram of the left-hand side ϕ of the dispersion relation (9) when the phase speed V and wavenumber K are both real (non-leaky waves): (*a*) for $A < C$ and (*b*) for $A > C$. Sausage ($m = 0$) and other ($m \geqslant 1$) modes are distinguished when their behaviours are qualitatively different, but in general the full curves represent arbitrary m.

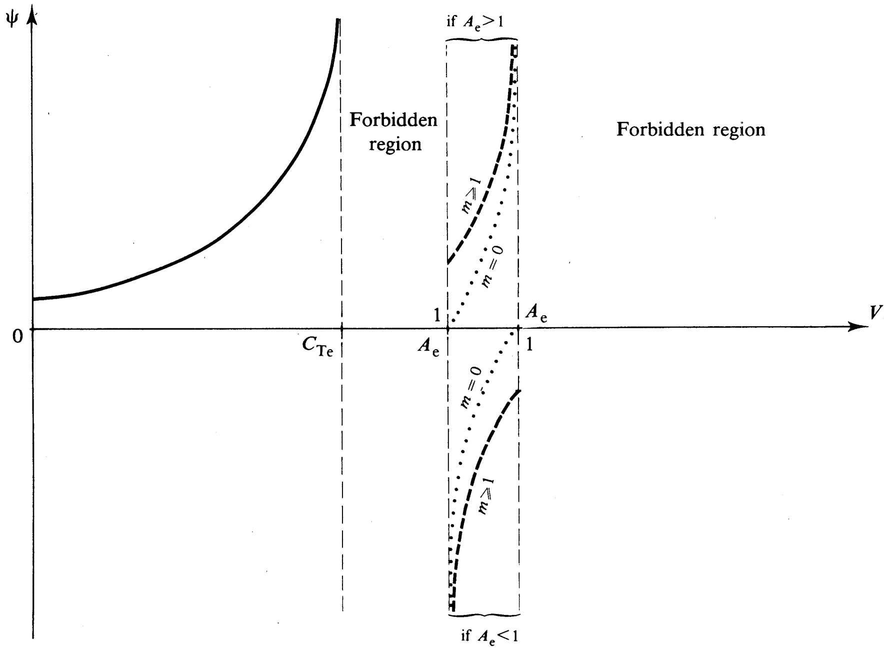

Fig. 3. As in Fig. 2, but for the right-hand side ψ of equation (9). Those phase speeds for which ψ is complex, and therefore $\phi = \psi$ is impossible, are marked *forbidden region.* If $A_e > 1$, the curves for $1 < V < A_e$ in the upper half plane are appropriate, whilst if $A_e < V < 1$ the lower curves apply. The behaviour for $V < C_{Te}$ is common to both cases.

Table 1; as n increases Ω_n moves off the real axis and tends asymptotically towards the horizontal line Im $\Omega = (\Omega_i)_{asym}$, the imaginary part of the right-hand side of equation (10). The corresponding asymptotic decay rate is (in s)

$$\tau_{asym} = 2R\left((a^2+c^2)^{\frac{1}{2}} \ln \left| \frac{1+X}{1-X} \right| \right)^{-1}, \tag{12}$$

indicating that thin tubes leak more profusely than do wide ones. It is also of interest that leaky waves of arbitrarily large phase speed exist, whereas non-leaky modes are restricted by $V < \max(A_e, 1)$.

In many other instances as well, when one of the real modes of Table 1 does not exist for a particular wavenumber or tube parameters, its counterpart may be found off the real axis. Yet other complex frequencies exist which do not appear to be linked with any erstwhile real mode. The analysis of fibril waves in Section 4 provides some examples of these behaviours.

Another asymptotic case which may be considered is the thin tube approximation, in which $|\Omega| \ll 1$ and $|K| \ll 1$ are assumed, leading to $|L|$, $|L_e| \ll 1$ in general (except if $V \sim C_T$ or C_{Te}). Wilson (1981) treated this case, assuming $A_e = 0$, and used it as the basis for a numerical search procedure for thick tube solutions. However, only certain modes may be found in this way (see modes a and b in Fig. 6 for example); the Trig modes and their offshoots, as well as other types of wave found in tubes other than fibrils (see Cally 1985) do not show up in the thin tube approximation, and so were missed by Wilson.

Table 1. Some characteristics of the seven types of non-leaky tube waves

Listed are the necessary conditions for existence, sufficient conditions (where these are simple), longitudinal phase speed range, and the *maximum* number of modes of each type which may exist for each m. All of these details may be extracted from Figs 2 and 3

Wave	Necessary condition	Sufficient condition	Phase speed	Max. no. of modes
S_-^+	$A_e < 1$ and $A_e < C_T$	$\leftarrow$ $(m = 0)$ $A_e < C_T \leqslant 1$ $(m \geqslant 1)$	$V < C_T$; $A_e < V \leqslant 1$	1
S_+^-	$A < C$ and $A < C_{Te}$	$A < C_{Te} < C$ $(m \geqslant 1)$	$C \leqslant V \leqslant A$; $V < C_{Te}$	1
S_+^+	*Either* $C < A$ and $A_e < 1$, and $(C, A) \cap (A_e, 1)$ is non-empty *or* $A < C$ and $1 < A_e$, and $(A, C) \cap (1, A_e)$ is non-empty	*Either* $C \leqslant A_e < 1 \leqslant A$ $(m = 0)$ $C \leqslant A_e < A \leqslant 1$ $(m \geqslant 1)$ *or* $A \leqslant 1 < A_e \leqslant C$ $(m = 0)$ $1 \leqslant A < A_e \leqslant C$ $(m \geqslant 1)$	V lies between A and C and between A_e and 1	1
B_-^+	$[C_T, \min(C, A)]$ intersects $[\min(A_e, 1), \max(A_e, 1)]$	$\min(A_e, 1) \leqslant C_T < \max(A_e, 1)$	$C_T < V < \min(C, A)$ $V > \min(A_e, 1)$ $V < \max(A_e, 1)$	∞
B_+^-	$\max(A, C) < C_{Te}$	$\leftarrow$ if $C \leqslant A$ and $m \geqslant 1$	$\max(A, C) < V < C_{Te}$	Finite though unbounded
B_+^+	$\max(A, C) < \max(A_e, 1)$	$\max(C, 1) \leqslant A < A_e$ $(m \geqslant 1)$	$V > \max(A, C)$ $V > \min(A_e, 1)$ $V < \max(A_e, 1)$	Finite though unbounded
B_-^-	$C_T < C_{Te}$	$\leftarrow$	$C_T < V < \min(A, C)$ $V < C_{Te}$	∞

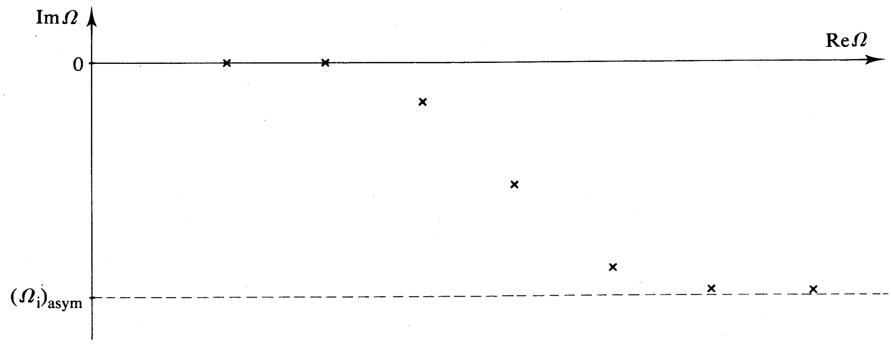

Fig. 4. Schematic diagram of the typical positioning in the complex frequency plane of the B_+^+ modes and their complex extension. The asymptotic behaviour for large Re Ω is given by equation (10).

4. Fibril Waves

Hα fibrils are ubiquitous on the solar disc. They are generally associated with active regions (in the penumbra and superpenumbra of sunspots in particular) and plagettes (magnetic field concentrations at supergranule boundaries), and are believed to be aligned with the magnetic field. They may extend almost vertically into the corona (the spicules when seen on the limb), or lie nearly horizontal in the chromosphere.

Motions within fibrils take two distinct forms. Firstly, longitudinal velocities are observed, vigorous in spicules, more gentle in chromospheric fibrils. These are possibly due to longitudinal (i.e. essentially acoustic) waves generated by convective buffeting at the base of the photosphere. In spicules, acoustic waves would shock and may then drive the observed flows of up to 30 km s^{-1}. Smaller amplitude back and forth oscillations in horizontal fibrils are possibly the unshocked 'sound' waves themselves.

Secondly, Giovanelli (1975) also observed basically transverse waves in chromospheric fibrils. These typically have phase speeds of around 70 km s^{-1} and wavelengths of about 12 Mm (i.e. 12×10^6 m). Giovanelli interpreted them as Alfvén waves and thereby inferred an Alfvén speed within the fibril of about 70 km s^{-1}. However, as the wave front appears to be essentially confined to the fibril, Wentzel (1979) and Edwin and Roberts (1983) have identified them as tube waves, and the Alfvén speed estimate must be regarded with suspicion.

A reliable model of fibrils, and their surroundings, is not currently available. It is widely believed though that they are cool dense structures with little if any difference in magnetic field strength across their boundaries. By assuming $B = B_e$ (around 10 G say), $c = \frac{1}{2}c_e$ and $\gamma = \gamma_e$, the density contrast $D = 4$ can be inferred, and consequently $a = \frac{1}{2}a_e$. Even in a relatively weak field, the Alfvén speed in the high chromosphere may be expected to comfortably exceed the sound speed, say $a_e = 7c_e$. The relevant parameters are then

$$A = 3\cdot5, \quad A_e = 7, \quad C = 0\cdot5, \quad C_T = 0\cdot495, \quad C_{Te} = 0\cdot990, \quad D = 4. \tag{13}$$

With an assumed fibril half-width of $R = 500$ km and the observed wavelength of

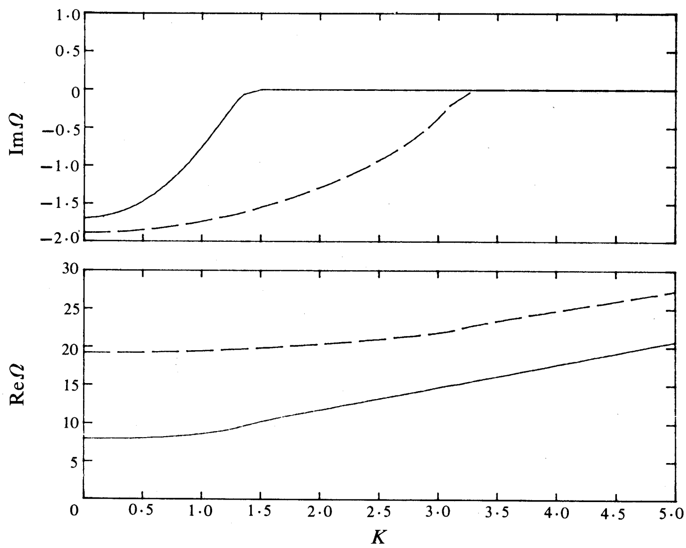

Fig. 5. Dispersion diagram for the first two B_+^+ sausage ($m = 0$) modes for the model fibril defined by equations (13). The real and imaginary parts of the first mode (solid curve) are plotted in the lower and upper graph respectively, and similarly for the second (dashed curve).

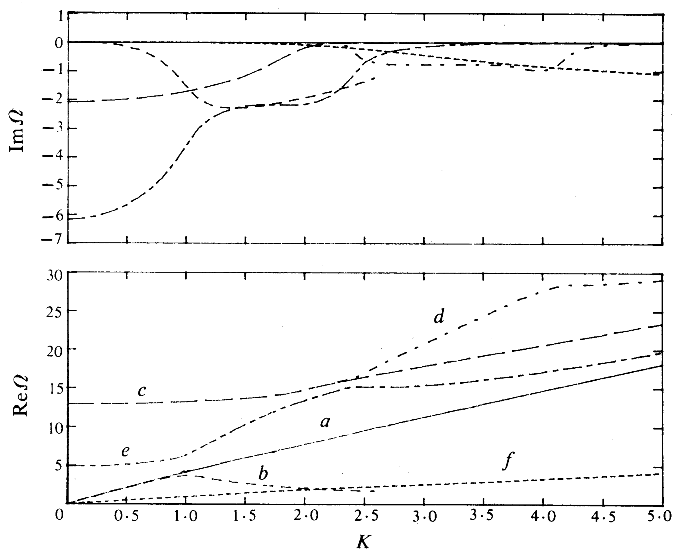

Fig. 6. Dispersion diagram of some kink modes ($m = 1$) for the model fibril. Each mode is referred to in Section 4 by its label *a*–*f*. Note that the first B_+^+ mode *a* is real for all K.

around 12 Mm, the wavenumber $K = 0{\cdot}25$ is appropriate. Even though these precise numbers cannot be regarded with any confidence, the qualitative behaviour of tube waves depends primarily on the ordering of the various speeds, not their exact values, and the assumption

$$C_{\mathrm{T}} < C < C_{\mathrm{Te}} < 1 < A < A_{\mathrm{e}} \tag{14}$$

is probably correct.

Of the non-leaky modes, a perusal of Table 1 shows that only B_{+}^{+} and B_{-}^{-} may exist under these conditions. Of these, the latter has a phase speed below the internal sound speed and therefore well under the observed value of $70\ \mathrm{km\,s^{-1}}$. Only the B_{+}^{+} mode, with $V > A$, provides a possible explanation of the transverse waves (this is the class of mode discussed in some detail in Section 3; see also Fig. 4).

Extensive numerical investigation allows the plotting of dispersion diagrams for the various waves. For the so-called sausage modes, $m = 0$, this is done for the first two B_{+}^{+} modes (see Fig. 5). The first has complex frequency for wavenumbers below about $K = 1{\cdot}5$ (i.e. wavelengths longer than around $2{\cdot}1$ Mm), and is real thereafter. The second is leaky below about $K = 3{\cdot}3$, and so on, each successive mode remaining leaky below progressively larger wavenumbers (smaller wavelengths). The value of Ω in the long wavelength limit $K = 0$ is given approximately by equation (10). Of particular interest is the fact that no B_{+}^{+} sausage mode is non-leaky at the wavenumber in question of $K = 0{\cdot}25$. However, they are strongly transverse, being almost pure pulsation modes with the longitudinal velocity being no more than about $0{\cdot}1\%$ of the radial velocity, either internally or externally.

The B_{-}^{-} modes are of a different character. As is obvious from Figs 2*b* and 3, there are an infinite number of B_{-}^{-} modes accumulated at $V = C_{\mathrm{T}}^{+}$. Outside the tube these have roughly equal longitudinal and transverse velocity amplitudes, but inside, at $r = R^{-}$, they are almost totally longitudinal with amplitudes some hundreds of times the internal and external transverse values. They represent therefore a type of slow compressional wave moving along the inside of the tube. As C_{T} is almost indistinguishable from C in a fibril, such oscillations might easily be mistaken for sound waves. It is possible that the $m = 0$ B_{-}^{-} modes are the back and forth oscillations observed in chromospheric fibrils.

Turning now to the kink modes, $m = 1$, we see that the B_{-}^{-} waves very much resemble their sausage counterparts. The situation otherwise, however, is quite different. A number of modes are illustrated by their dispersion curves in Fig. 6, and are discussed in turn:

(*a*) $\Omega = 1{\cdot}0944$ at $K = 0{\cdot}25$: The curve labelled *a* is real for all K. It represents the first B_{+}^{+} mode, and is almost totally transverse, having a longitudinal component of less than 1% at $K = 0{\cdot}25$. It is the mode identified by Wentzel (1979) and Edwin and Roberts (1983) as Giovanelli's transverse fibril wave. If this interpretation is correct, and the phase speed $\omega k = 70\ \mathrm{km\,s^{-1}}$ is assumed, the external sound speed in the model atmosphere is $c_{\mathrm{e}} = \omega K/\Omega k = 16\ \mathrm{km\,s^{-1}}$. Equations (13) then specify the other speeds: $a = 56\ \mathrm{km\,s^{-1}}$, $a_{\mathrm{e}} = 112\ \mathrm{km\,s^{-1}}$ and $c = 8\ \mathrm{km\,s^{-1}}$. In this case, a 20% error would result from identifying the Alfvén speed with the phase speed.

(*b*) $\Omega = 1{\cdot}0943 - 0{\cdot}019\,\mathrm{i}$ at $K = 0{\cdot}25$: This is a leaky wave which bifurcates from the *a* mode at $K = 0$, though the two remain close until $K \sim 0{\cdot}5$ say. For $K = 0{\cdot}25$, the decay of the wave due to leakage would be too small to be readily observable ($\tau \approx 27$ min compared with a wave period of 3 min). However, it rapidly becomes a more important effect as K is increased (e.g. $\tau = 14$ s at $K = 1{\cdot}5$, where the period is 73 s. The mode ceases to exist above about $K = 2{\cdot}58$.

(*c*) $\Omega = 13-2\cdot0\,\mathrm{i}$ at $K = 0\cdot25$: The second B_+^+ mode, this and the subsequent harmonics behave much as the B_+^+ sausage modes, i.e. Ω_n is complex at $K = 0$ (cf. equation 10), but approaches and eventually coalesces with the real axis as K increases, the critical K being larger for larger n. Again these modes are almost totally transverse.

(*d*) This mode does not exist at $K = 0\cdot25$. As with *b*, this mode bifurcates from a B_+^+ mode, this time the second, precisely at the critical wavenumber at which its parent eigenfrequency becomes real ($K = 2\cdot235$ in this case). It is transverse to within a few per cent. Beyond about $K = 4$, it tends asymptotically back towards the real axis, yet never reaches it.

(*e*) $\Omega = 4\cdot9-6\cdot0\,\mathrm{i}$ at $K = 0\cdot25$: Another transverse wave, this one is very leaky for large wavelengths ($\tau \sim 5$ s with a period of about 40 s), though it becomes asymptotically less so (in a rather erratic fashion) as the longitudinal wavelength decreases. The phase speed lies between those of the first two B_+^+ modes.

(*f*) $\Omega = 0\cdot247-5\times10^{-4}\,\mathrm{i}$ at $K = 0\cdot25$: At small K, this mode is strongly longitudinal and basically confined to the tube (there is a weak external motion which is primarily transverse). In this respect it resembles the B_-^- modes, though on the other hand it is distinguished by its greater phase velocity $V \approx C_{\mathrm{Te}}$ compared with C_{T}. It is peculiar that the phase speed of a wave which exists almost entirely within the fibril should be determined by the *external* tube speed. The *f* mode leaks substantially only for small wavelengths, for which its character is altered to become both longitudinal and transverse in roughly equal proportion ($K = 5$).

In light of the above remarks, the following suggestions may be proffered:

(i) The observed back and forth motions in chromospheric fibrils may be non-leaky B_-^- waves, probably $m = 0$ modes since for $m \geqslant 1$ there would simultaneously be flow in both directions in different sectors of the tube, which is not observed.

(ii) Giovanelli's (1975) transverse fibril waves are likely to consist of at least two modes, the non-leaky first B_+^+ mode *a* and its almost identical slightly leaky companion *b*, at least for the $m = 1$ component [the $m \geqslant 2$ components behave somewhat similarly; see Cally (1985)]. There is no $m = 0$ transverse wave with the required phase speed and wavelength.

(iii) A variety of transverse waves, for all azimuthal wavenumbers m, are found to be very leaky. (In particular, an infinity of long wavelength B_+^+ modes have decay times of around $\tau_{\mathrm{asym}} \approx 16$ s. It is not surprising that no longitudinal wave is found to leak significantly as it is the transverse pulsations at $r = R$ that excite waves in the external medium.) It seems reasonable to suggest that such waves excite sympathetic oscillations in neighbouring fibrils. Observations of this phenomenon would be of interest.

Of course, all these conclusions can only be tentative in the absence of a wave theory which includes gravity. A generalization to twisted and bent flux tubes would also be useful.

5. General Discussion

Some of the complexity of tube waves has been demonstrated, especially by the particular example in Section 4. The main lesson to be drawn from this work is that caution must be exercised when interpreting observations of waves in magnetically structured regions. It will almost certainly be misleading to identify them as sound, Alfvén, fast or slow waves of the types familiar in homogeneous atmospheres. It is also hoped that a more complete understanding of tube waves, especially when gravitational stratification is successfully incorporated into the theory, will allow the use of wave observations as a probe into the structure of many solar features.

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