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Some Aspects of the U(1) Problem and the Pseudoscalar Mass Spectrum

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Abstract

A complete discussion of the nonet pseudoscalar mass spectrum and related topics is given. The U(1) problem is briefly reviewed and the large-N point of view is considered. The necessity for a vector ghost gluonic particle, in order to resolve the U(1) problem, is stressed. Scalar ghosts and the Kogut–Susskind mechanism are shown to be insufficient. An analysis of the η–η' mixing problem, with and without PCAC corrections, is made and it is suggested that the singlet decay constant may be nearly twice as large as the other octet decay constants. A general discussion of PCAC corrections is given and the remaining pseudoscalars are considered. As byproducts quark mass values are given and the m_q dependence of $\langle\bar{q}q\rangle$ is elucidated. In the usual scheme, it is found that $\langle\bar{\mathrm{s}}\mathrm{s}\rangle \approx 1\cdot48\langle\bar{\mathrm{u}}\mathrm{u}\rangle$. The $\eta \to \pi\pi\pi$ decays are also discussed.

1. The U(1) Problem(s)

Recently the large-N point of view (Witten 1979; Veneziano 1979; Di Vecchia and Veneziano 1980; Witten 1980) has provided a consistent picture for handling the U(1) problem (Weinberg 1975; Crewther 1977, 1979, 1980*a*, 1980*b*). In particular, the connection with phenomenology has been made (Veneziano 1979; Kawarabayashi and Ohta 1980, 1981; Di Vecchia *et al.* 1981). In this paper we examine some aspects of the pseudoscalar mass spectrum. The rest of this section is devoted to highlighting the U(1) problem and the large-N point of view. Section 2 is concerned with an application of these ideas to the η–η' mixing problem, while partially conserved axial-vector current (PCAC) corrections are considered in Section 3.

By assuming there is no anomaly, to the leading order of chiral symmetry breaking, the (mass)^2 matrix for the η–η' system (in the octet–singlet basis) can be written as (for a review of PCAC see for instance Pagels 1975)

$$\begin{pmatrix} \frac{4}{3}m_{\mathrm{K}}^2-\frac{1}{3}m_\pi^2 & -\frac{2}{3}\surd2\,(m_{\mathrm{K}}^2-m_\pi^2) \\ -\frac{2}{3}\surd2\,(m_{\mathrm{K}}^2-m_\pi^2) & \frac{2}{3}m_{\mathrm{K}}^2+\frac{1}{3}m_\pi^2 \end{pmatrix}, \tag{1}$$

where we have set $m_1 = m_2$, $F_\pi \approx F_{\mathrm{K}} \approx F_8 \approx F_{\mathrm{s}}$ and $\langle\bar{\mathrm{u}}\mathrm{u}\rangle \approx \langle\bar{\mathrm{d}}\mathrm{d}\rangle \approx \langle\bar{\mathrm{s}}\mathrm{s}\rangle$. The eigenvalues and eigenvectors of (1) are

$$M_1^2 = m_\pi^2 = O(m_{1,2}), \qquad |1\rangle = \surd\tfrac{1}{3}(|8\rangle+\surd2\,|s\rangle); \tag{2a}$$

$$M_2^2 = 2m_{\mathrm{K}}^2-m_\pi^2 = O(m_3), \qquad |2\rangle = \surd\tfrac{1}{3}(\surd2\,|8\rangle-|s\rangle). \tag{2b}$$

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As $|1\rangle$ is predominantly singlet and $|2\rangle$ predominantly octet it is usual to associate them with $|\eta'\rangle$ and $|\eta\rangle$ respectively. Defining the mixing angle as

$$|\eta\rangle = \cos\theta\,|8\rangle + \sin\theta\,|s\rangle, \qquad |\eta'\rangle = -\sin\theta\,|8\rangle + \cos\theta\,|s\rangle \tag{3a, b}$$

results in

$$\tan\theta = -\sqrt{\tfrac{1}{2}} \quad (\theta \approx -35{\cdot}3^\circ). \tag{4}$$

This situation is known as ideal mixing.

The U(1) problem concerns itself with the absence of any light isoscalar particle with (mass)^2 proportional to m_π^2. Such a particle is simply not observed experimentally. The existence of such a light isoscalar particle can also be connected with substantial isospin violations (Gross *et al.* 1979), e.g. for π^0–η mixing of $O(\varepsilon/m_0)$, with $m_0 = \frac{1}{2}(m_1+m_2)$ and $\varepsilon = \frac{1}{2}(m_2-m_1)$. Otherwise they are $O(\varepsilon/m_3)$.

The U(1) problem can further be connected with the suppression of $\eta \to 3\pi$ decays in the soft π^0 limit. Amusingly, this is a case where the predicted isospin violation is not as large (actually vanishing) as what is observed experimentally. The amplitudes for the $\eta \to 3\pi$ decays are proportional to

$$\langle \pi^0\pi\pi \,|\int_x \tfrac{1}{2}(m_1-m_2)\bar{q}\lambda_3\, q(x)\,|\,\eta\rangle,$$

where the operator inside the matrix element is the isospin violating part of the interaction Hamiltonian. Using a soft pion theorem this can be reduced to

$$\frac{\mathrm{i}}{F_\pi}\langle \pi\pi\,|\,\tfrac{1}{2}(m_1-m_2)\int_x \bar{q}\gamma_5 I^{(2)} q\,|\,\eta\rangle; \qquad I^{(2)} = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}, \tag{5}$$

$$= \frac{\mathrm{i}}{F_\pi}\langle \pi\pi\,|\,\tfrac{1}{2}(m_1-m_2)\int_x \left(\frac{1}{2\mathrm{i}(m_0^2-\varepsilon^2)}(m_0\,\partial^\mu J^2_{\mu 5,\mathrm{sym}} + 2\varepsilon\partial^\mu \mathscr{F}^3_{\mu 5})\right)|\,\eta\rangle, \tag{6}$$

where $J^2_{\mu 5,\mathrm{sym}}$ is the gauge variant* U(1) axial current (with a soft divergence) corresponding to the 'operator' $\bar{q}\gamma_\mu\gamma_5 I^{(2)} q$ and $\mathscr{F}^3_{\mu 5} = \bar{q}\gamma_\mu\gamma_5\frac{1}{2}\lambda_3\, q$.

The expression (6), being the integral of a total divergence, is then expected to vanish because there should not be any (physical) zero mass particle around when the chiral symmetry is explicitly broken. If however one was to further reduce (5) by using soft π and η theorems (taking the $|\eta\rangle$ as transforming as an $|8\rangle$) a non-vanishing value is obtained,†

$$\mathrm{Amp}(\eta \to 3\pi) = \frac{m_2-m_1}{m_2+m_1}\frac{m_\pi^2}{\sqrt{3}F_\pi F_\eta}, \tag{7}$$

in fair agreement with experimental values if (our normalization corresponds to a value of $F_\pi \approx 93$ MeV)

* The gauge invariant current will be denoted by $J_{\mu 5}$. For the present discussion (no anomaly) there is no difference between $J_{\mu 5}$ and $J_{\mu 5,\mathrm{sym}}$.

† The amplitude for the charged η decay has an additional momentum dependent piece.

$$\frac{m_2-m_1}{m_2+m_1} \approx \begin{cases} 0\cdot 45 \pm 0\cdot 04 \\ 0\cdot 52 \pm 0\cdot 04 \end{cases}.$$

The upper value comes from the $\eta \to \pi^0\pi^0\pi^0$ data and the lower from the $\eta \to \pi^0\pi^+\pi^-$ data. The value of $(m_2-m_1)/(m_2+m_1)$ obtained in this way is consistent with that obtained by other means; for example, from the mesonic sector, cf. equation (27) in Section 3. However, at the present level of discussion, in which we have ignored the anomaly, self-consistency requires that we use the mass eigenstates of (2); doing this, a vanishing result is obtained in place of (7), which is consistent with (5) being a total divergence. When the anomaly is taken into account the correct result is given by (32), from which (7) follows on setting θ (the η–η' mixing angle) to zero. We should also remark that there is some conflict in the literature regarding equation (7). Some authors have an additional factor of $\frac{4}{3}$ (R.J. Crewther, personal communication) in the amplitude for $\eta \to \pi\pi\pi$ ($\frac{16}{9}$ in the rate). The origin of this factor (Langacker and Pagels 1974) seems to involve a careless manipulation of limits. Our result (7) follows directly from the chiral Ward identities and agrees with Weinberg (1975) and Di Vecchia *et al.* (1981).

The obvious and only way out of all these problems is to allow for the inclusion of effects of the anomaly and topological charge. In the large-N point of view* one assumes a non-vanishing positive value for (Witten 1979)

$$\langle\langle \nu^2 \rangle\rangle_{\mathrm{YM}} \equiv -\mathrm{i}\int_x \partial^x_\mu \partial_\nu \, T\langle K_\mu(x)\, K_\nu(0)\rangle_{\mathrm{YM}}$$

in the leading order of the $1/N$ expansion (i.e. pure Yang–Mills theory). This is done by introducing a vector ghost (Veneziano 1979) with propagator† $-\mathrm{i}g_{\mu\nu}/q^2$ that couples to K_μ. If we assume that there are no zero mass poles coupled to the gauge invariant $\mathrm{U}(1)_{\mathrm{ax}}$ current $J^L_{\mu 5}$ (where L is the number of flavours), then the anomalous Ward identity (Crewther 1979) (*in the chiral limit*)

$$0 = \int_x \partial^x_\mu \partial_\nu \, T\langle J^L_{\mu 5}(x)\, K_\nu(0)\rangle = 2L \int_x \partial_\mu \partial_\nu \, T\langle K_\mu(x)\, K_\nu(0)\rangle$$

implies that $\langle\langle \nu^2 \rangle\rangle_{\mathrm{QCD}}$ should vanish, i.e. when we include fermions. In order for the mesonic and the gluonic contributions to $\langle\langle \nu^2 \rangle\rangle$ to cancel it is necessary that they be of opposite signs, i.e. one of the contributing intermediate states must be of negative norm. This is the reason for introducing a vector gluonic *ghost*; this results in a positive value for $\langle\langle \nu^2 \rangle\rangle_{\mathrm{YM}}$. The correct sign propagator for the gluonic state would necessarily give the wrong sign shift in the singlet $(\text{mass})^2$.

The required cancellation of the gluonic and fermionic contributions to $\langle\langle \nu^2 \rangle\rangle_{\mathrm{QCD}}$ implies a nonzero ghost–singlet coupling, which is $O(1/\sqrt{N_c})$. This in turn provides a mechanism by which the η' can acquire an additional $(\text{mass})^2$ of $\sim 1/N_c$ in the chiral limit. This comes about through the resummation of the diagrams in the leading order of the topological expansion ($N \to \infty$, for L/N fixed) in which the flavour singlet mixes with the gluonic ghost (Venezanoi 1979). The essential feature of the large-N approach

* The usefulness of the large-N picture relies on the fact that as $N \to \infty$ the anomaly can be turned off.
† Although this appears to be a positive norm propagator it is necessarily an antihermitian field. The corresponding hermitian field is a real ghost.

to the U(1) problem is that the η' is considered as much a relic of U(1) axial symmetry as the pions are of SU(2) axial symmetry. This is because in the large-N limit the anomaly which is $O(g^2) = O(1/N_c)$ vanishes and we are left with L^2 genuine Nambu–Goldstone bosons. Furthermore, the assumption $\langle\langle v^2\rangle\rangle_{\rm YM} > 0$ can be reconciled with Crewther's anomalous Ward identities and the quark condensate* $\langle\bar{q}q\rangle \neq 0$. Instantons (gauge field configurations with integer topological charge) are unable to do this (Crewther 1977).

It should be noted that the Kogut–Susskind (KS) (1975) mechanism is insufficient in resolving the U(1) problem. A particular version (Coleman 1979) of the KS mechanism consists of two massless scalar fields ϕ_+ and ϕ_-, with positive and negative norm respectively. Gauge invariant quantities are supposed to couple to the sum $\phi_+ + \phi_-$, which has zero propagator, while the gauge variant current $J_{\mu 5,\rm sym}$ is supposed to couple to the difference $\phi_+ - \phi_-$. Goldstone bosons can then contribute to matrix elements containing one $J_{\mu 5,\rm sym}$ and a string of gauge invariant operators. The problem with this particular version is that the total contribution of ϕ_+ and ϕ_- to

$$\int_x T\langle J_{\mu 5,\rm sym}(x)\, J_{\nu 5,\rm sym}(0)\rangle$$

vanishes. A consistent resolution to the U(1) problem requires (Crewther 1979) a non-vanishing positive value (in the *non*-chiral limit) for $\langle\langle v^2\rangle\rangle_{\rm QCD}$ that is $O(m)$ with respect to the light quark masses.† This requirement essentially follows from a Ward identity (Crewther 1977) which relates $\langle\langle v^2\rangle\rangle$ to $\langle m\bar{q}q\rangle$. Assuming no zero mass poles couple to $J^L_{\mu 5}$ [a necessary requirement to solve the U(1) problem] $\langle\langle v^2\rangle\rangle$ can be rewritten as

$$\langle\langle v^2\rangle\rangle = -\frac{\mathrm{i}}{4L^2}\int_x \partial^\mu_x \partial^\nu T\langle J^L_{\mu 5,\rm sym}(x)\, J^L_{\nu 5,\rm sym}(0)\rangle\,. \tag{8}$$

As already noted the particular version of the KS mechanism stated above cannot give (8), and hence $\langle\langle v^2\rangle\rangle$, a nonzero value and thus cannot solve the U(1) problem.

One may think of remedying this deficiency of the KS mechanism by allowing only ϕ_+ (or ϕ_-) to couple to $J_{\mu 5,\rm sym}$. This is still not enough, as by Lorentz invariance alone, a scalar pole cannot give (8) a nonzero value:

$$\begin{aligned}\langle\langle v^2\rangle\rangle_{\rm scalar} &\sim \lim_{q\to\infty}\int_x \exp(\mathrm{i}\,q\,.\,x)\partial^x_\mu \partial_\nu\, T\langle J_{\mu 5,\rm sym}(x)\, J_{\nu 5,\rm sym}(0)\rangle \\ &\sim \lim_{q\to\infty} q_\mu q_\nu q_\mu \frac{1}{q^2} q_\nu = 0\,.\end{aligned}$$

Furthermore, in the KS mechanism the scalars are supposed to remain massless even with explicit chiral symmetry breaking. In other words, they are 'trapped' particles. It is hard to imagine how then a genuine light Goldstone boson singlet can appear when the anomaly is turned off (by taking $N \to \infty$ say). The Veneziano (1979) mechanism appears then to be the only viable alternative.

* Recently Veneziano (1980) has argued that the assumption $\langle\langle v^2\rangle\rangle_{\rm YM} > 0$ not only provides a mechanism by which the η' can acquire an additional mass but also implies the spontaneous chiral symmetry breaking itself.

† The large-N picture and the Veneziano mechanism can be shown to remain consistent with the anomalous Ward identities in the non-chiral limit (Veneziano 1979).

2. The η–η′ Mixing

With the modifications of the previous section the η–η' (mass)^2 matrix can now be written* as (Veneziano 1979)

$$M^2 = \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\frac{2}{3}\surd 2(m_K^2 - m_\pi^2) \\ -\frac{2}{3}\surd 2(m_K^2 - m_\pi^2) & \frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2 + \chi^2/N \end{pmatrix}$$

$$\approx \begin{pmatrix} 0\cdot 321 & -0\cdot 214 \\ -0\cdot 214 & 0\cdot 170 + \chi^2/N \end{pmatrix}. \tag{9}$$

The extra χ^2/N contribution comes from the gluonic (ghost)-singlet coupling $\mathrm{i}\, q_\mu \chi/\surd N$ [χ is $O(1)$]. This corresponds to giving the singlet an additional mass through gluon annihilation diagrams. In writing (9) it has been assumed that $1/N$ corrections are small, but in fact they will turn out to be large. In essence an assumption about pole dominance with an η' has been made.

The inclusion of $\chi \neq 0$ effects now upsets ideal mixing. The eigenvalues of (9) are

$$m_{\eta',\eta}^2 = (m_K^2 + \chi^2/2N) \pm \tfrac{1}{2}\{(2m_K^2 - 2m_\pi^2 - \chi^2/3N)^2 + 8\chi^4/9N^2\}^{\frac{1}{2}}. \tag{10}$$

In equation (10) we have identified the larger of the two solutions with $m_{\eta'}^2$. Note that in the limit $\chi^2/N \to 0$ this solution becomes $2m_K^2 - m_\pi^2$, which corresponds to the solution of (1) that we previously identified with the η. In the above χ^2/N remains a free parameter. Taking the trace of (9) gives the relation

$$m_\eta^2 + m_{\eta'}^2 = 2m_K^2 + \chi^2/N. \tag{11}$$

From (9) and (10), the mixing angle θ (defined by equation 3) is found to be (note that equations 10–12 can be found in Veneziano 1979)

$$\tan\theta = \frac{(\frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 - m_\eta^2)}{\frac{2}{3}\surd 2(m_K^2 - m_\pi^2)}. \tag{12}$$

Following Veneziano (1979) χ^2/N can be determined from (11) by substituting the experimental value for the sum $m_{\eta'}^2 + m_\eta^2 \approx 1\cdot 2182\ \mathrm{GeV}^2$, giving the result

$$\chi^2/N \approx 0\cdot 727\ \mathrm{GeV}^2.$$

Inserting this back into equations (10) and (12) results in

$$m_{\eta'}^2 \approx 0\cdot 968\ \mathrm{GeV}^2, \qquad m_\eta^2 \approx 0\cdot 251\ \mathrm{GeV}^2, \qquad \theta = 18\cdot 4^\circ, \tag{13a, b, c}$$

compared with the experimental values of

$$= 0\cdot 9170\ \mathrm{GeV}^2, \qquad = 0\cdot 3012\ \mathrm{GeV}^2, \qquad = 10^\circ.$$

Veneziano (1979) quoted the values

$$\approx 0\cdot 951\ \mathrm{GeV}^2, \qquad \approx 0\cdot 267\ \mathrm{GeV}^2, \qquad \approx 14^\circ, \tag{14a, b, c}$$

* Valid to the leading order of chiral symmetry breaking and the next to leading order of the large N_c expansion (leading order of the topological expansion) with $m_q/N\Lambda_{\mathrm{QCD}} \sim m_{\pi,K,\eta}^2/m_{\eta'}^2$ fixed.

which differ from (13) because in his estimation only the largest terms were kept. The agreement with experiment is at best only encouraging. The important thing to note is that in order to get a reasonable fit for the pseudoscalar mass spectrum it is in practice necessary that $1/N$ corrections be large, at least for the pseudoscalar channel. That is, Zweig forbidden processes have to be violated by a large amount in this case. Large-N corrections have to play a dual role: a balance of being small in principle (to justify η' pole dominance in obtaining equation 9) and large in practice (to account for the large η' mass in the expression 13a).

In order to improve the analysis we now repeat the calculations with $F_\pi \approx F_8 \approx F_K$, but allow F_s to differ from them (see below). By defining $\xi \equiv F_8/F_s$, equations (9)–(12) are modified, with for example (9) replaced by

$$M^2 = \begin{pmatrix} \frac{4}{3}m_K^2 - \frac{1}{3}m_\pi^2 & -\xi\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) \\ -\xi\frac{2}{3}\sqrt{2}(m_K^2 - m_\pi^2) & \xi^2(\frac{2}{3}m_K^2 + \frac{1}{3}m_\pi^2) + \chi^2/N \end{pmatrix}. \tag{15}$$

Table 1. Determination of χ^2/N, $m_{\eta'}^2$, m_η^2 and θ for specific values of $\xi \equiv F_8/F_s$

ξ	χ^2/N (GeV^2)	$m_{\eta'}^2$ (GeV^2)	m_η^2 (GeV^2)	θ (deg.)
0	0·897	0·897	0·321	0
0·45	0·863	0·913	0·305	9·4
0·51	0·853	0·917	0·301	10·4
0·55	0·845	0·920	0·298	11·1
1·00	0·727	0·968	0·251	18·4

By fixing ξ at various values and following the same procedure as before the predictions of Table 1 are made. The best fit is found for

$$\xi \equiv F_8/F_s \approx 0{\cdot}51\,. \tag{16}$$

For consistency a large value for F_s is required.

Let us now explain why we chose to improve the η–η' analysis by allowing F_s to differ from the other octet of decay constants. In the leading order of $1/N$, F_s is expected to be the same as the other decay constants; $1/N$ corrections are expected to split nonet symmetry because now the singlet can proceed through gluonic intermediate diagrams while the flavour bearing mesons cannot. As $1/N$ corrections are necessarily large such corrections should be considered. In the effective Lagrangian treatments a term representing this is (Witten 1980; Di Vecchia *et al.* 1981)

$$(\alpha/2F_\pi^2)\mathrm{tr}(U\partial_\mu U^\dagger)\,\mathrm{tr}(U^\dagger\partial_\mu U)\,. \tag{17}$$

Such a term is down by an *additional* power of $1/N$ with respect to a usual 4-meson vertex because the double flavour trace requires two quark loops. Hence as F_π is $O(\sqrt{N})$, then α is $O(1/N)$. This new term modifies the U(1) axial current of the effective Lagrangian from*

$$\tfrac{1}{2}\mathrm{i}\{\mathrm{tr}(U^\dagger\partial_\mu U) - \mathrm{tr}(U\partial_\mu U^\dagger)\} \quad \text{to} \quad \tfrac{1}{2}\mathrm{i}\{1 + (\alpha/F_\pi^2)\mathrm{tr}(UU^\dagger)\}\{\mathrm{tr}(U^\dagger\partial_\mu U) - \mathrm{tr}(U\partial_\mu U^\dagger)\}\,.$$

* As derived from the lowest order kinetic term $\frac{1}{2}\mathrm{tr}(\partial_\mu U^\dagger\partial^\mu U)$ in $\mathscr{L}_{\mathrm{eff}}$.

In going over to the nonlinear representation (in which spontaneous chiral symmetry breaking has been assumed and massive scalars removed), that is

$$U = \sqrt{\tfrac{1}{2}}\,F_\pi \exp\!\left(\frac{\mathrm{i}\boldsymbol{\lambda}\,.\,\boldsymbol{\pi}}{F_\pi}\right); \qquad \lambda_9 = \sqrt{\tfrac{2}{3}}\begin{pmatrix}1 & & \\ & 1 & \\ & & 1\end{pmatrix}, \qquad \pi_9 = \eta_{\mathrm{s}},$$

it is clear that (17) has the effect of rescaling the U(1) axial current by $1+\frac{3}{2}\alpha$. Since the previous U(1) axial current, sandwiched between the vacuum and $|\eta_{\mathrm{s}}\rangle$, equals $\mathrm{i}q_\mu F_\pi$ (nonet symmetry) the new current gives a rescaled value of $F_{\mathrm{s}} = (1+\frac{3}{2}\alpha)F_\pi$. Comparing this with the estimate (16) gives $\alpha \approx 0{\cdot}63$.

Di Vecchia *et al.* (1981) have also improved the mixing analysis, but by employing a possible term in the effective Lagrangian $\sim(\partial_\mu K_\mu)\mathrm{tr}\{M(U-U^\dagger)\}$. After eliminating the K_μ field (by its equation of motion) they arrived at a term $\sim\mathrm{tr}(\boldsymbol{\lambda}\,.\,\boldsymbol{\pi})\,\mathrm{tr}\{M\exp(\boldsymbol{\lambda}\,.\,\boldsymbol{\pi})\}$. Such a term induces additional octet–singlet and singlet–singlet couplings. These corrections are $O(m_3/N)$ and may appear to be non-leading with respect to the $O(1/N)$ corrections to F_{s}. In the (mass)^2 matrix however we have

$$(F_8^2/F_{\mathrm{s}}^2)m_{\mathrm{K}}^2 \approx m_{\mathrm{K}}^2+O(m_3/N), \qquad \text{if} \quad F_{\mathrm{s}} = F_8\{1+O(1/N)\},$$

and we see that the two different sorts of corrections are in fact compatible. In the scheme of Di Vecchia *et al.* (1981) the (mass)^2 matrix (9) was supplemented by the following addition (in the notation of that work $N = 3$ and ε is a free parameter):

$$\begin{pmatrix} 0 & -\frac{1}{3}\sqrt{2}\,\varepsilon m_{\mathrm{K}}^2 \\ -\frac{1}{3}\sqrt{2}\,\varepsilon m_{\mathrm{K}}^2 & \frac{2}{3}\varepsilon m_{\mathrm{K}}^2 \end{pmatrix}.$$

If we compare this term with (15), the two are compatible if $\xi = 1+\frac{1}{2}\varepsilon$. In other words, the above corrections can be reabsorbed into a redefinition of F_{s}. The result found by Di Vecchia *et al.*, i.e. $\varepsilon \approx -1{\cdot}08$, corresponds to $\xi \approx 0{\cdot}46$ which is compatible with our value.

3. PCAC Corrections

In this section we consider PCAC corrections generally and their effect on the work of the previous section. A brief discussion on current quark mass values is also given.

By considering the Ward identity (in the non-chiral limit)

$$\begin{aligned} 0 &= \int_x \partial_\mu T\langle \mathscr{F}^3_{\mu 5}(x)\,\partial_\nu \mathscr{F}^3_{\nu 5}(0)\rangle \\ &= \int_x T\langle \partial_\mu \mathscr{F}^3_{\mu 5}(x)\,\partial_\nu \mathscr{F}^3_{\nu 5}(0)\rangle + \langle [F^3_5,\, \partial_\nu \mathscr{F}^3_{\nu 5}]\rangle, \end{aligned} \tag{18}$$

where

$$\mathscr{F}^3_{\mu 5} \equiv \bar{q}\gamma_\mu\gamma_5\tfrac{1}{2}\lambda_3\, q, \qquad F^3_5 \equiv \int_x \mathscr{F}^3_{05}(x),$$

it is easy to show that*

* Here $F_{\pi 0}$ is defined by $\langle 0|\mathscr{F}^3_{\mu 5}|\pi^0\rangle = \mathrm{i}q_\mu F_{\pi 0}$, $\quad \partial^\nu \mathscr{F}^3_{\nu 5} = \mathrm{i}\,\bar{q}\gamma_5\begin{pmatrix} m_1 & \\ & m_2\end{pmatrix}q.$

$$m_{\pi^0}^2 F_{\pi^0}^2 = -\langle \bar{q} \begin{pmatrix} m_1 & \\ & m_2 \end{pmatrix} q \rangle . \tag{19a}$$

In obtaining (19a) we have evaluated the equal-time commutator in the second term on the right-hand side of (18) and have saturated the first term with *one* pion pole. The contribution coming from the 3-pion cut can easily be shown to be $O(m_\pi^4)$. Equation (19a) can thus be considered to be true with corrections up to $O(m_\pi^4 \ln m_\pi^2)$; that is, with $O(m_\pi^2 \ln m_\pi^2)$ corrections included in the symmetry values for F_{π^0}, $\langle \bar{q}q \rangle$ etc. We follow Langacker and Pagels (1973*a*) in keeping terms with logarithms in preference to terms without logarithms. They are obviously dominant near the chiral limit, although their importance in the real world is not immediately apparent. Equation (19a) should be considered as the leading chiral contribution to the π^0 $(\text{mass})^2$; electromagnetic corrections and contributions from π^0–η–η' mixing would have to be included as extra effects.*

Just as we derived equation (19a), the Ward identities for the other U(3) axial currents can be used to derive†

$$m_8^2 F_8^2 = -\tfrac{1}{3} \langle \bar{q} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & 4m_3 \end{pmatrix} q \rangle , \tag{19b}$$

$$m_s^2 F_s^2 = -\tfrac{2}{3} \langle \bar{q} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} q \rangle + \chi^2/N , \tag{19c}$$

$$m_{K^+}^2 F_{K^+}^2 = m_{K^-}^2 F_{K^-}^2 = -\tfrac{1}{2}(m_1 + m_3) \langle \bar{q} \begin{pmatrix} 1 & & \\ & 0 & \\ & & 1 \end{pmatrix} q \rangle , \tag{19d}$$

$$m_{K^0}^2 F_{K^0}^2 = m_{\bar{K}^0}^2 F_{\bar{K}^0}^2 = -\tfrac{1}{2}(m_2 + m_3) \langle \bar{q} \begin{pmatrix} 0 & & \\ & 1 & \\ & & 1 \end{pmatrix} q \rangle , \tag{19e}$$

$$m_{s8}^2 F_s F_8 = -\tfrac{1}{3}\sqrt{2} \langle \bar{q} \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & -2m_3 \end{pmatrix} q \rangle . \tag{19f}$$

* As the contributions from π^0–η–η' mixing to the π^0 $(\text{mass})^2$ are indeed small (Gross *et al.* 1979) we will totally ignore this in the subsequent discussion.

† The formulas for $m_{K^+}^2 F_{K^+}^2$ and $m_{K^0}^2 F_{K^0}^2$ are quite often misquoted in the literature, with e.g.

$$m_{K^+}^2 F_{K^+}^2 = -\langle \bar{q} \begin{pmatrix} m_1 & & \\ & 0 & \\ & & m_3 \end{pmatrix} q \rangle .$$

In writing equations (19) we have considered the η–η' system in the octet–singlet basis *prior* to mixing. With this clarification m_{s8}^2 should be understood as $\langle s\,|\,H'\,|\,8\rangle$, the off-diagonal (mass)^2 matrix element. The physical basis is always restored after diagonalization of the (mass)^2 matrix.

Defining

$$\alpha_u^s = \langle \bar{s}s\rangle/\langle \bar{u}u\rangle\,, \qquad \alpha_u^d = \langle \bar{d}d\rangle/\langle \bar{u}u\rangle\,,$$

equations (19) can be rewritten as (ξ stands for F_π/F_s here)

$$m_{\pi^0}^2 = (m_1 + \alpha_u^d m_2)\left(\frac{-\langle \bar{u}u\rangle}{F_\pi^2}\right) \approx (m_1+m_2)\left(\frac{-\langle \bar{u}u\rangle}{F_\pi^2}\right), \tag{20a}$$

$$m_8^2 = \frac{(m_1 + \alpha_u^d m_2 + 4\alpha_u^s m_3)m_{\pi^0}^2}{3(F_8/F_\pi)^2(m_1+m_2)}, \tag{20b}$$

$$m_s^2 = \frac{2(m_1 + \alpha_u^d m_2 + \alpha_u^s m_3)m_{\pi^0}^2}{3\xi^{-2}(m_1+m_2)} + \chi^2/N\,, \tag{20c}$$

$$m_{K^+}^2 - (\delta m_K^2)_{em} = \frac{(m_1+m_3)(1+\alpha_u^s)m_{\pi^0}^2}{2(F_K/F_\pi)^2(m_1+m_2)}, \tag{20d}$$

$$m_{K^0}^2 = \frac{(m_2+m_3)(\alpha_u^d+\alpha_u^s)m_{\pi^0}^2}{2(F_K/F_\pi)^2(F_{K^0}/F_{K^+})^2(m_1+m_2)}, \tag{20e}$$

$$m_{s8}^2 = \frac{\sqrt{2}\,(m_1 + \alpha_u^d m_2 - 2\alpha_u^s m_3)m_{\pi^0}^2}{3(F_8/F_\pi)\xi^{-1}(m_1+m_2)}. \tag{20f}$$

The contributions on the right-hand sides of equations (20) are chiral; for this reason an electromagnetic contribution has been subtracted from $m_{K^+}^2$. Electromagnetic contributions to the decay constants etc. are understood to be included.

Equations (20a), (20d) and (20e) can be used to solve for the quark mass ratios, giving

$$\frac{m_2-m_1}{m_2+m_1} = \frac{2(F_K/F_\pi)^2}{m_{\pi^0}^2}\left(\frac{(F_{K^0}/F_{K^+})^2 m_{K^0}^2}{\alpha_u^d+\alpha_u^s} - \frac{m_{K^+}^2-(\delta m_K^2)_{em}}{1+\alpha_u^s}\right), \tag{21}$$

$$\frac{2m_3}{m_1+m_2} = \frac{2(F_K/F_\pi)^2}{m_{\pi^0}^2}\left(\frac{(F_{K^0}/F_{K^+})^2 m_{K^0}^2}{\alpha_u^d+\alpha_u^s} + \frac{m_{K^+}^2-(\delta m_K^2)_{em}}{1+\alpha_u^s}\right) - 1\,. \tag{22}$$

Before proceeding we must estimate α_u^d and α_u^s.

Using the pseudoscalar density $i\bar{q}\gamma_5\lambda_3 q$ in place of $\partial^\nu \mathscr{F}_{\nu 5}^3$ in equation (18) and following the same procedure as before we obtain

$$F_{\pi^0} Z_{\pi^0}^{\frac{1}{2}} = -(\langle \bar{u}u\rangle + \langle \bar{d}d\rangle) \tag{23a}$$

up to $O(m_\pi^2 \ln m_\pi^2)$ corrections, where

$$Z_{\pi^0}^{\frac{1}{2}} \equiv \langle 0\,|\,i\bar{q}\gamma_5\lambda_3 q\,|\,\pi^0\rangle\,.$$

In a hopefully obvious notation the following additional equations can also be derived:

$$F_{\mathrm{K}^+} Z^{\frac{1}{2}}_{\mathrm{K}^+} = F_{\mathrm{K}^-} Z^{\frac{1}{2}}_{\mathrm{K}^-} = -(\langle \bar{\mathrm{u}}\mathrm{u}\rangle + \langle \bar{\mathrm{s}}\mathrm{s}\rangle), \tag{23b}$$

$$F_{\mathrm{K}^0} Z^{\frac{1}{2}}_{\mathrm{K}^0} = F_{\bar{\mathrm{K}}^0} Z^{\frac{1}{2}}_{\bar{\mathrm{K}}^0} = -(\langle \bar{\mathrm{d}}\mathrm{d}\rangle + \langle \bar{\mathrm{s}}\mathrm{s}\rangle). \tag{23c}$$

PCAC logarithm corrections to the F and the Z have been calculated by Langacker and Pagels (1973*a*) and Pagels (1975) with the following numerical results:

$$F_{\mathrm{K}} \approx 1\cdot 21\ F_\pi, \qquad Z^{\frac{1}{2}}_{\mathrm{K}} \approx 1\cdot 023\ Z^{\frac{1}{2}}_\pi, \tag{24a, b}$$

$$F_{\mathrm{K}^0} \approx 1\cdot 0065\ F_{\mathrm{K}^+}, \qquad Z^{\frac{1}{2}}_{\mathrm{K}^0} \approx 1\cdot 00072\ Z^{\frac{1}{2}}_{\mathrm{K}^+}, \tag{24c, d}$$

$$F_8 \approx 1\cdot 28\ F_\pi. \tag{24e}$$

A few remarks are in order. The minute difference between F_{K^0} and F_{K^+} (and between $Z^{\frac{1}{2}}_{\mathrm{K}^0}$ and $Z^{\frac{1}{2}}_{\mathrm{K}^+}$) is crucial in estimating the mass ratio m_2/m_1 (see equation 21). The above values follow from allowing an electromagnetic contribution to $m^2_{\mathrm{K}^+}$ which is roughly twice as large as the value predicted by Dashen's (1969) theorem.

Equations (23) can now be used to express the ratios of $\langle \bar{\mathrm{s}}\mathrm{s}\rangle$, $\langle \bar{\mathrm{u}}\mathrm{u}\rangle$ and $\langle \bar{\mathrm{d}}\mathrm{d}\rangle$ in terms of the F and Z, giving the result

$$\frac{2\langle \bar{\mathrm{s}}\mathrm{s}\rangle}{\langle \bar{\mathrm{u}}\mathrm{u}\rangle + \langle \bar{\mathrm{d}}\mathrm{d}\rangle} = \frac{Z^{\frac{1}{2}}_{\mathrm{K}^+} F_{\mathrm{K}^+} + Z^{\frac{1}{2}}_{\mathrm{K}^0} F_{\mathrm{K}^0} - Z^{\frac{1}{2}}_{\pi^0} F_{\pi^0}}{Z^{\frac{1}{2}}_{\pi^0} F_{\pi^0}}, \tag{25a}$$

$$\frac{\langle \bar{\mathrm{d}}\mathrm{d}\rangle - \langle \bar{\mathrm{u}}\mathrm{u}\rangle}{\langle \bar{\mathrm{d}}\mathrm{d}\rangle + \langle \bar{\mathrm{u}}\mathrm{u}\rangle} = \frac{Z^{\frac{1}{2}}_{\mathrm{K}^0} F_{\mathrm{K}^0} - Z^{\frac{1}{2}}_{\mathrm{K}^+} F_{\mathrm{K}^+}}{Z_{\pi^0} F_{\pi^0}}. \tag{25b}$$

Using the values in (24) we arrive at the estimates

$$\frac{2\langle \bar{\mathrm{s}}\mathrm{s}\rangle}{\langle \bar{\mathrm{u}}\mathrm{u}\rangle + \langle \bar{\mathrm{d}}\mathrm{d}\rangle} \approx 1\cdot 48, \qquad \frac{\langle \bar{\mathrm{d}}\mathrm{d}\rangle - \langle \bar{\mathrm{u}}\mathrm{u}\rangle}{\langle \bar{\mathrm{d}}\mathrm{d}\rangle + \langle \bar{\mathrm{u}}\mathrm{u}\rangle} \approx 0\cdot 0072, \tag{26a,b}$$

that is,

$$\langle \bar{\mathrm{s}}\mathrm{s}\rangle \approx 1\cdot 48 \langle \bar{\mathrm{u}}\mathrm{u}\rangle, \qquad \langle \bar{\mathrm{d}}\mathrm{d}\rangle \approx 1\cdot 0146 \langle \bar{\mathrm{u}}\mathrm{u}\rangle. \tag{26c, d}$$

Inserting the experimental values for $m^2_{\mathrm{K}^0}$, $m^2_{\mathrm{K}^+}$ and $m^2_{\pi^0}$ and the chiral estimates from equations (24a), (24c), (26c) and (26d), together with $(\delta m^2_{\mathrm{K}^+})_{\mathrm{em}} \approx 2975\ \mathrm{MeV}^2$ $[(\delta m_{\mathrm{K}})_{\mathrm{em}} \approx 3\cdot 0\ \mathrm{MeV}]$, results in

$$(m_2 - m_1)/(m_2 + m_1) \approx 0\cdot 55, \tag{27}$$

$$2m_3/(m_1 + m_2) \approx 30\cdot 8. \tag{28}$$

These values essentially correspond to those found by Langacker and Pagels (1979).

The value for $(m_2 - m_1)/(m_2 + m_1)$ is fairly dependent on the precise choice of $(\delta m^2_{\mathrm{K}})_{\mathrm{em}}$. Table 2 gives an estimate of this dependence. The values given in equations

(27) and (28) and Table 2 should be compared with the lowest order estimates of Weinberg (1977):*

$$(m_2-m_1)/(m_2+m_1) \approx 0\cdot 287\,, \qquad 2m_3/(m_2+m_1) \approx 25\cdot 8\,,$$

$$m_3 \approx 150^{+30}_{-20}\ \mathrm{MeV} \quad \to \quad \begin{cases} m_1 \approx 4\cdot 2^{+0\cdot 8}_{-0\cdot 6}\ \mathrm{MeV} \\ m_2 \approx 7\cdot 5^{+1\cdot 5}_{-1\cdot 0}\ \mathrm{MeV} \end{cases}.$$

Table 2. Determination of light quark masses for certain values of $(\delta m_K^2)_{em}$, taking $m_3 = 150^{+30}_{-20}$ MeV

$(\delta m_K^2)_{em}$ (MeV^2)	$(m_2-m_1)/(m_2+m_1)$	m_1 (MeV)	m_2 (MeV)
0	0·312	$3\cdot 3^{+0.7}_{-0.4}$	$6\cdot 3^{+1.3}_{-0.8}$
1265[A]	0·412	$2\cdot 8^{+0.6}_{-0.3}$	$6\cdot 8^{+1.4}_{-0.9}$
2000	0·471	$2\cdot 6^{+0.5}_{-0.4}$	$7\cdot 1^{+1.4}_{-1.0}$
2975[B]	0·550	$2\cdot 2^{+0.4}_{-0.3}$	$7\cdot 5^{+1.4}_{-1.0}$
4000	0·633	$1\cdot 8^{+0.3}_{-0.3}$	$7\cdot 8^{+1.6}_{-1.0}$

[A] Dashen (1969). [B] Langacker and Pagels (1979).

Substituting equations (28), (26c) and (26d) into (20b), (20c) and (20f) gives (*with* PCAC corrections)

$$m_8^2 \approx 0\cdot 342\ \mathrm{GeV}^2\,, \tag{29a}$$

$$m_s^2 \approx 0\cdot 289\,\xi^2\ \mathrm{GeV}^2 + \chi^2/N\,, \tag{29b}$$

$$m_{s8}^2 \approx 0\cdot 299\,\xi\ \mathrm{GeV}^2\,. \tag{29c}$$

These should be compared with the previous values used in equations (9) and (15). Because there is a substantial difference between the values in (29) and (15) it is worth while repeating the previous analysis. The best fit to the η and η' masses is found for

$$\xi \approx 0\cdot 51 \quad (F_s = 1\cdot 96F_\pi) \quad \to \quad \begin{cases} m_\eta^2 = 0\cdot 301 \quad (\mathrm{exp:}\ 0\cdot 3012) \\ m_{\eta'}^2 = 0\cdot 917 \quad (\mathrm{exp:}\ 0\cdot 9170) \end{cases}.$$

The value for θ is however shifted to

$$\theta \approx 14\cdot 8^\circ\,.$$

The often quoted experimental value of $\theta \approx 10^\circ$ is obtained from (12) and relies on the standard form of the η–η' mixing matrix with $F_s = F_\pi$ (see for instance Binnie *et al.* 1979 and Abram *et al.* 1979). Determinations of θ_{exp} by other means† are not very accurate (C. H. Llewellyn Smith, personal communication).

We thus conclude that a value of F_s which is nearly twice as large as the other octet of decay constants is required in obtaining a reasonable fit to the η–η' mass spectrum.

* The Weinberg values follow from equations (21) and (22) with $F_K = F_\pi$, $(\delta m_K^2)_{em} = 1265$ MeV^2, $\alpha_u^d = \alpha_u^s = 1$ and $F_{K^0} = F_{K^+}$.

† For example, from the ratios $\Gamma(\pi^0\to\gamma\gamma):\Gamma(\eta\to\gamma\gamma):\Gamma(\eta'\to\gamma\gamma)$, $\Gamma(\psi\to\pi^0\gamma):\Gamma(\psi\to\eta\gamma):\Gamma(\psi\to\eta'\gamma)$ and $\Gamma(\psi'\to\psi\pi^0):\Gamma(\psi'\to\psi\eta)$.

A value of F_s which is larger than F_8 is not totally unreasonable. The decay constants are essentially a measure of the probabilities of finding a particle at the origin and should be larger, the larger the mass of the particle. It should then be expected that

$$F_\pi < F_K < F_8\,(F_\eta) < F_s\,(F_{\eta'}).$$

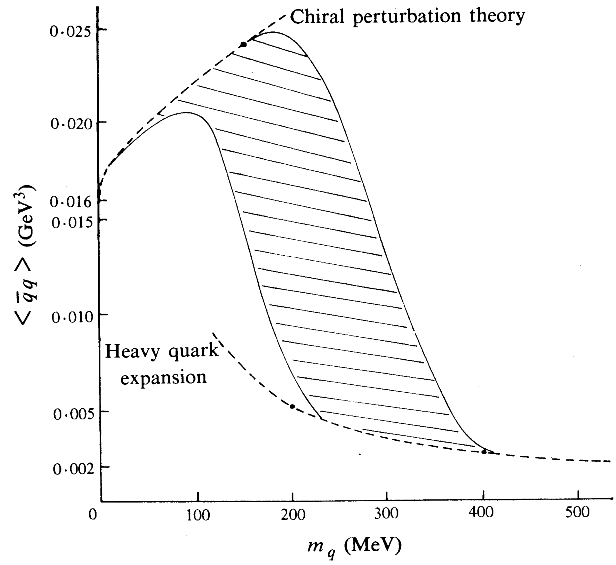

Fig. 1. Dependence of $\langle\bar{q}q\rangle$ on m_q.

As a byproduct of our analysis of PCAC corrections we are also able to make some general comments about the m_q dependence of $\langle\bar{q}q\rangle$. From equation (25a) it is clear that the reason why $|\langle\bar{s}s\rangle| > |\langle\bar{u}u\rangle|$ is simply because $F_K > F_\pi$. In general, chiral perturbation theory gives the result

$$\langle\bar{q}q\rangle_{m_q} = \langle\bar{q}q\rangle_{m_q=0}\{1 + cm_q\ln(\Lambda/m_q) + O(m_q)\}, \tag{30}$$

where $c > 0$. Then $|\langle\bar{q}q\rangle|$ increases from its symmetry value $|\langle\bar{q}q\rangle_0|$ with an infinite slope before approaching an essentially linear behaviour. For heavy quarks Shifman *et al.* (1979) have obtained the result

$$\langle\bar{q}q\rangle = -\frac{1}{12m_q}\langle\frac{\alpha_s}{\pi}F_{\mu\nu}\cdot F^{\mu\nu}\rangle + O(1/m_q^2). \tag{31}$$

The results (30) and (31) are represented pictorially in Fig. 1 together with a spread of where the actual curve may lie. Due to the successes of chiral perturbation theory (Langacker and Pagels 1973*a*; Pagels 1975), such as the predicted value of F_K, we are optimistic that the actual curve passes closely to our point given by (26c).

Before concluding we make a brief comment on the $\eta \to 3\pi$ decays. Using $|\eta\rangle = \cos\theta\,|\,8\rangle + \sin\theta\,|\,s\rangle$ and assuming the validity of soft octet and singlet theorems we can rewrite the expression (5) as

$$\frac{m_2-m_1}{m_2+m_1}\frac{m_\pi^2}{\sqrt{3}F_\pi}\left(\frac{\cos\theta}{F_8}+\frac{\sqrt{2}\sin\theta}{F_s}\right)\approx\begin{cases}0\cdot55\pm0\cdot05\\0\cdot64\pm0\cdot05\end{cases}. \tag{32}$$

The values given on the right-hand side of (32) are detemined from the experimental data coming from the $\eta\to3\pi^0$ decay (upper value) and the $\eta\to\pi^0\pi^+\pi^-$ decay (lower value).

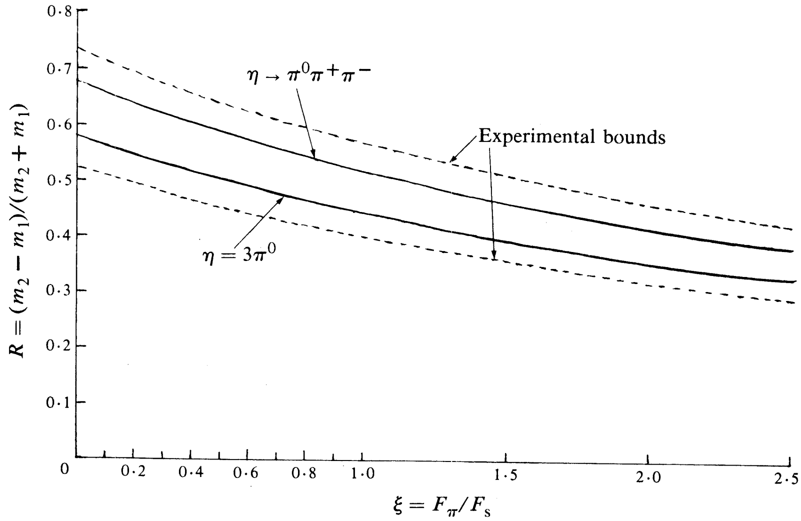

Fig. 2. The ratio $R\equiv(m_2-m_1)/(m_2+m_1)$ versus $\xi = F_\pi/F_s$ from the $\eta\to\pi\pi\pi$ data.

Inserting $F_8\approx1\cdot28F_\pi$, $\theta\approx10°$ and the values for m_π^2 and F_π ($\approx$93 MeV), we arrive at an equation relating the ratio $R\equiv(m_2-m_1)/(m_2+m_1)$ and $\xi\equiv F_\pi/F_s$:

$$R(0\cdot814+0\cdot246\xi)=\begin{cases}0\cdot47\pm0\cdot05\\0\cdot55\pm0\cdot05\end{cases}. \tag{33}$$

Equation (33) has been plotted in Fig. 2. The upper solid curve comes from the $\eta\to\pi^0\pi^+\pi^-$ data and the lower solid curve from the $\eta\to3\pi^0$ data. The dotted curves represent the experimental bounds. The following conclusions can be drawn, based on the crude approximations used to obtain (32) and the experimental uncertainties involved:

(1) $R\lesssim0\cdot73$; this means $m_2/m_1<5\cdot7$ (or $m_1>0\cdot18m_2$).

(2) $R\approx0\cdot56$; the best estimate we have (Langacker and Pagels 1979) implies $\xi\lesssim0\cdot7$. This at least confirms the compatibility of our result $\xi\approx0\cdot5$ and the Langacker–Pagels estimate of R (equation 27).

(3) $\xi\approx0\cdot5$; this implies $0\cdot45\leqslant R\leqslant0\cdot64$. It is interesting to note that this corresponds to a value of $(\delta m_K^2)_{em}$ in the range

$$1650\leqslant(\delta m_K^2)_{em}\leqslant4100\ \text{MeV}^2.$$

The value coming from a use of Dashen's (1969) theorem is just outside this range. Credence is given to the value anticipated by Langacker and Pagels (1973*b*, 1979).

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