Aust. J. Phys., 1983, **36**, 891–5

Radius Ratios of Argon Pinches

S. Lee

Plasma Research Laboratory, Physics Department,
University of Malaya, Kuala Lumpur 22–11, Malaysia.

Abstract

According to a new theory based on energy balance, the radius ratio r_p/r_0 of a constant-current plasma pinch depends only on the value of the effective specific heat ratio γ of the pinched plasma. In this paper the value of γ for argon is computed as a function of temperature. From these values the constant-current pinch ratios are computed. The results show argon pinch ratios of about $0\cdot08$ from $(2\text{–}4)\times10^6$ K, rising to $0\cdot18$ at $1\cdot1\times10^7$ K and to $0\cdot27$ at 10^8 K. There is good agreement between this theory and an Imperial College measurement of $r_p/r_0 \sim 0\cdot17$ for a constant-current argon pinch at an estimated temperature of 1 keV.

1. Introduction

It has been observed (Yong and Lee 1977) that an argon plasma focus undergoes more severe electromechanical effects than a pure deuterium plasma focus. An increase in compression has also been observed (Baldock *et al.* 1982) in the linear Z-pinch operated in argon when compared with one operated in hydrogen. It is proposed here that this effect can be explained solely on the basis of the difference in effective specific heat ratio γ of the argon plasma when compared with the γ of the deuterium or hydrogen plasma.

This proposal is a natural consequence of the recent energy balance theory by Lee (1981, 1983). Based on a fundamental consideration of energy and pressure balance, the pinch ratio r_p/r_0 for the case of a constant-current constant-length pinch is found to be

$$r_p/r_0 = \exp\{-\gamma/2(\gamma-1)\}, \tag{1}$$

where γ is the effective specific heat ratio of the hot plasma in quasi-equilibrium. For a high temperature hydrogen pinch $\gamma=\frac{5}{3}$ and this constant-current pinch ratio is $0\cdot29$, according to equation (1). This agrees, within experimental error, with the published result (Haines 1981) of Imperial College of a pinch ratio of approximately $\frac{1}{3}$ measured on a constant-current hydrogen pinch.

An extension of this theory to the deuterium plasma focus, taken as a pinch of constant current but of variable length (Lee 1983), gives a pinch radius ratio of $0\cdot14$ compared with an experimental value of $0\cdot13$.

Recently a pinch ratio of approximately $\frac{1}{6}$ has been reported (Baldock *et al.* 1982) for a constant-current argon pinch at an estimated temperature of 1 keV. This

0004-9506/83/060891$02.00

greater compression appears consistent with equation (1) since it is known that γ for argon drops to below $1{\cdot}20$ during its freely ionizing temperature range. As the temperature is increased above 4×10^6 K, argon becomes fully ionized and γ then rises towards $\frac{5}{3}$. It is the purpose of this paper to compute the value of γ as a function of temperature and hence to compute the value of r_p/r_0 for an argon pinch as a function of temperature.

2. Theory

The value of γ may be defined in terms of the enthalpy per unit mass h as

$$h = \frac{\gamma}{\gamma-1}\frac{R_0}{M}T\zeta, \tag{2}$$

where the departure coefficient is

$$\zeta = 1 + \sum_{r=1}^{r=n} r\alpha_r. \tag{3}$$

Here R_0 is the universal gas constant, M the molecular (or atomic) weight, T the temperature and α_r the fraction of the plasma which is ionized to the rth ionized state. The enthalpy of a plasma such as argon may also be written in terms of its ionization and excitation energies as

$$h = \tfrac{5}{2}(R_0/M)T\zeta + m^{-1}\sum_{r=1}^{r=n}\alpha_r I_r + m^{-1}\sum_{r=0}^{r=n}\alpha_r \bar{E}_r. \tag{4}$$

Here I_r is the total energy required to raise one ion from its unionized state to its rth ionized state and $\bar{E}_r$ is the average excitation energy per rth ionized ion. The ionization potentials are known quantities whilst the excitation energies are temperature dependent and are computed from the tabulated values of atomic and ionic energy levels and the statistical weights of these levels (Moore 1949), once a suitably converging summation scheme is adopted. The mass of the atom or ion is here denoted as m.

From equations (2) and (4) we may compute the value of γ by writing

$$\frac{\gamma}{\gamma-1} = \tfrac{5}{2} + \left(m^{-1}\sum_{r=1}^{r=n}\alpha_r I_r + m^{-1}\sum_{r=0}^{r=n}\alpha_r \bar{E}_r\right)\Big/(R_0/M)T\zeta, \tag{5}$$

where the α_r may be computed for any given temperature by the use of Saha's equations. We note that equation (5) gives $\gamma = \frac{5}{3}$ for two cases:

(*a*) when the excitation and ionization modes are negligible at low T;

(*b*) when the temperature is high enough for the plasma to be fully ionized, the numerator of the second term in equation (5) reaches its maximum value. If T is increased further this second term in equation (5) becomes correspondingly smaller and becomes negligible when compared with the first term at a sufficiently high temperature so that $\gamma\to\frac{5}{3}$. For argon, as will be seen, the value of γ is $1{\cdot}569$ at 4×10^7 K.

3. Computation Procedure

In several argon computations (Lee 1969) for relatively low temperatures the values of γ have been computed by a full procedure involving the evaluation of the

α_r and the $\bar{E}_r$ up to 5×10^4 K at which point $\alpha_4 \approx 1$. In a more recent full computation (Yong and Lee 1977), on the trajectory of an argon plasma focus, the values of the ionization fractions have been determined up to full ionization with $\alpha_{18} = 1$ being achieved at 4×10^6 K.

We discuss here a simple procedure for estimating the values of γ from the available curves of the α. To adopt this procedure we note that as the curve for a particular ionization fraction, α_s say, rises to its maximum value of $\alpha_s \leqslant 1$, at that point the second term of equation (5) is dominated by the ionization energy. At this point, if we assume that $\alpha_s \approx 1$, then we have

$$\sum_{r=1}^{r=n} \alpha_r I_r + \sum_{r=0}^{r=n} \alpha_r \bar{E}_r \approx I_s. \tag{6}$$

Hence, estimates of the values of γ may be readily made at 17 points of temperature, each point at successively higher temperatures corresponding to the maximum values of α_1 to α_{17}, by use of the approximate formula

$$\frac{\gamma}{\gamma-1} \approx \tfrac{5}{2} + \frac{I_s/m}{(R_0/M)T(1+s\alpha_s)}. \tag{7}$$

Here we emphasize that the approximation (7) is used only at the successive temperature corresponding to the maximum of each of the α versus T curves.

As T is raised further and α_{18} approaches 1, the approximation (7) becomes exact and remains exact for all higher temperatures of T. Thus for all values of T higher than that temperature at which $\alpha_{18} = 1$ we have

$$\frac{\gamma}{\gamma-1} = \tfrac{5}{2} + \frac{I_{18}/m}{19(R_0/M)T}. \tag{8}$$

Table 1. Data for computation of γ

s	T (K)	$\zeta \sim 1+s\alpha_s$	I_s (eV)	s	T (K)	$\zeta \sim 1+s\alpha_s$	I_s (eV)
1	$1\cdot5\times10^4$	2	15·8	10	$3\cdot4\times10^5$	11	1479
2	$2\cdot3\times10^4$	3	43·4	11	$3\cdot8\times10^5$	12	2009
3	$3\cdot3\times10^4$	4	84·3	12	$4\cdot6\times10^5$	13	2669
4	$5\cdot0\times10^4$	5	144	13	$5\cdot5\times10^5$	14	3394
5	$6\cdot3\times10^4$	6	219	14	$6\cdot0\times10^5$	15	4209
6	$8\cdot5\times10^4$	7	310	15	$6\cdot5\times10^5$	16	5159
7	$9\cdot5\times10^4$	8	434·4	16	$1\cdot8\times10^6$	17	6189
8	$2\cdot1\times10^5$	9	577·9	17	$2\cdot7\times10^6$	18	10889
9	$3\cdot0\times10^5$	10	999	18	$4\cdot0\times10^6$	19	15989

4. Results

For the computation of γ by the above simplified procedure, we used the data of Yong and Lee (1977) in Table 1. These data were used with equations (7) and (8) to compute the values of γ as a function of T. The result is shown in Fig. 1 (curve A). It is seen that the value of γ has fallen to below 1·14 by 15000 K and remains at about 1·12 up to about 10^5 K when $\alpha_7 \approx 1$. Thus in this range of temperature (which has been called the freely ionizing range) the effective degree of freedom f of the plasma is approximately 17, since we may write $\gamma = (2+f)/f$. The next ionization involves a closed shell and thus between $1 \cdot 2$ and 2×10^5 K the plasma behaves energetically as a fully ionized gas so that over this range of temperature γ rises smoothly from $1 \cdot 12$ to $1 \cdot 20$. Then from 2 to $6 \cdot 5 \times 10^5$ K the value of γ again drops towards $1 \cdot 14$ as the ionization effects gain further dominance over the plasma enthalpy. From $6 \cdot 5 \times 10^5$ to $1 \cdot 8 \times 10^6$ K there is a further smooth rise of γ from $1 \cdot 14$ to $1 \cdot 26$, as over this range the plasma again behaves energetically as a fully ionized gas because of the closed-shell effect. Between $1 \cdot 8$ and 4×10^6 K the value of γ drops to $1 \cdot 24$; and above 4×10^6 K, with the gas fully ionized, there is a smooth and gradual increase of γ towards its eventual value of $\frac{5}{3} = 1 \cdot 667$. By 4×10^7 K its value has already increased to $1 \cdot 569$.

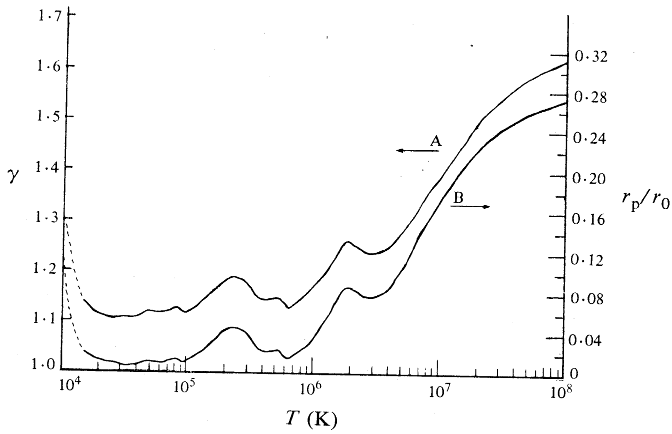

Fig. 1. Effective specific heat ratio γ (curve A), and constant-current pinch radius ratio r_p/r_0 (curve B) of argon as functions of temperature.

Curve B of Fig. 1 shows the effect of the variation of γ with temperature on the radius ratio of argon pinches. The two limits of the pinch ratios are noted from equation (1). For $\gamma = 1$, corresponding to a gas with $f = \infty$, the pinch radius ratio is 0; and, for $\gamma = \frac{5}{3}$, corresponding to an ideal gas with $f = 3$, the radius ratio is $0 \cdot 287$.

As a function of temperature, the argon pinch ratio drops to as low as $0 \cdot 006$ and remains around this value from 2×10^4 to 10^5 K, this being the freely ionizing range. Between 10^5 and 2×10^5 K, exhibiting fully ionized behaviour, the pinch ratio rises to $0 \cdot 04$, then falls again in the next freely ionizing range from 3 to $6 \cdot 5 \times 10^5$ K at which temperature α_{15} reaches a maximum. From $7 \cdot 0 \times 10^5$ to $1 \cdot 8 \times 10^6$ K the pinch ratio rises again to $0 \cdot 085$ in closed-shell behaviour with $\alpha_{16} \approx 1$ over this large range of temperature. Between 2 and 4×10^6 K the pinch ratio drops slightly to around $0 \cdot 08$ before rising towards the limit of $0 \cdot 29$ in the final fully ionized phase. This final rise starts at 4×10^6 K. The pinch ratio has increased to a value of $0 \cdot 17$ at 10^7 K, $0 \cdot 25$ at 4×10^7 K and $0 \cdot 27$ at 10^8 K.

Thus we note that for a constant-current argon pinch in the temperature range from 2 to 4×10^6 K, the pinch compression is more severe than in a hydrogen (or deuterium) pinch, with a pinch ratio of $0 \cdot 08$ compared with the hydrogen pinch ratio of $0 \cdot 29$. This difference in pinch ratios is still significant at 10^7 K with $0 \cdot 17$ for the argon pinch compared with $0 \cdot 29$ for the hydrogen pinch. At 10^8 K the difference has become insignificant.

5. Comparison with Experiment

As mentioned in the Introduction, the plasma focus in argon produces spectacularly more severe electromechanical effects than in a deuterium focus. This is indicated by the enhanced severity in current dip and voltage spike. This effect is consistent with a greater compression in argon.

Since the focus temperature is known to be in the region from $0 \cdot 5$ to $0 \cdot 7$ keV (Lee 1982), the compression in the argon focus, in terms of the pinch ratio, could be two to three times more severe than in the hydrogen focus according to our results as shown by curve B in Fig. 1. Thus for the plasma focus there is qualitative agreement between the above computation and experimental observations.

Finally we consider the Imperial College observation of a constant-current pinch ratio in argon of $\frac{1}{6} \approx 0 \cdot 17$ (Baldock *et al.* 1982) at an estimated temperature of ≈ 1 keV. This result compares with our computed value of $0 \cdot 18$ for argon at the corresponding temperature of $1 \cdot 16 \times 10^7$ K.

It would appear from this theory that if a high compression pinch is desired at a high temperature, say between 10^7 and 10^8 K, it would be advantageous to use krypton or xenon as, for these two gases, in this range of temperature the ionization is still not complete and the resultant low γ values would ensure correspondingly small pinch ratios. This effect could be important in the development of intense soft X-ray sources from pinch-type devices.

6. Conclusions

In this paper we have used a simplified method to compute the values of the specific heat ratio of argon as a function of temperature. From the energy balance theory we then computed the argon pinch ratio r_p/r_0 (which is γ-dependent) as a function of temperature. The results indicate agreement between our theory and a measurement of the argon pinch ratio made at Imperial College.

References

Baldock, P. F. M., *et al.* (1982). 'High Density High Temperature Plasma Production in Fast Z-pinch'. Ninth Ann. Conf. on Plasma Physics, Oxford.

Haines, M. G. (1981). *Phil. Trans. R. Soc. London* A **300**, 649.

Lee, S. (1969). Ph.D. Thesis, Australian National University.

Lee, S. (1981). *Bul. Fizik Malaysia* **2**, 240.

Lee, S. (1982). 'Fusion Energy–1981', IAEA-SMR-82, p. 288 (IAEA: Vienna).

Lee, S. (1983). *Plasma Phys.* **25**, 571.

Moore, C. E. (1949). 'Atomic Energy Levels', U.S. Bureau of Standards Cir. No. 467.

Yong, Y. C., and Lee, S. (1977). 'Multiple Ionization in an Argon Plasma Focus', Proc. Physics Symp. 1977, Kuala Lumpur, p. 149 (Univ. of Malaya: Kuala Lumpur).

Manuscript received 26 May, accepted 24 June 1983