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Comparison of DWBA and Adiabatic Treatments of Deuteron Breakup Angular Correlations*

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Abstract

Angular correlations calculated by the adiabatic method are shown to agree better with experiment than ones calculated by the prior-form DWBA. The problems of prior-form DWBA and its relationship to other methods of calculation are discussed.

1. Introduction

Distorted wave Born approximation (DWBA) calculations for direct reactions are based on matrix elements between product wavefunctions in the entrance and exit channels. However, the exit channel for a breakup reaction contains three interacting unbound particles: the target nucleus and the two fragments of the incident projectile. We therefore have a choice of product expressions for such exit channels (Henley and Lacy 1967); for example, in a (d, pn) reaction the possible product approximations are

$$\psi^{(-)}(\boldsymbol{r}_{\mathrm{p}}, \boldsymbol{r}_{\mathrm{n}}) \approx \chi_{\mathrm{p}}^{(-)}(\boldsymbol{r}_{\mathrm{p}})\, \chi_{\mathrm{n}}^{(-)}(\boldsymbol{r}_{\mathrm{n}}), \tag{1}$$

or

$$\psi^{(-)}(\boldsymbol{r}_{\mathrm{p}}, \boldsymbol{r}_{\mathrm{n}}) \approx \chi_{\mathrm{d}}^{(-)}(\boldsymbol{R})\, \phi^{(-)}(\boldsymbol{r}), \tag{2}$$

where the target nucleus is assumed to be at rest at the origin and where $\boldsymbol{R} = \frac{1}{2}(\boldsymbol{r}_{\mathrm{p}}+\boldsymbol{r}_{\mathrm{n}})$ and $\boldsymbol{r} = \boldsymbol{r}_{\mathrm{p}}-\boldsymbol{r}_{\mathrm{n}}$.

The operator in the DWBA matrix element is a residual interaction from the Hamiltonian H, being either the interaction not used to construct the entrance channel wavefunction (prior) or the interaction not used to construct the exit channel wavefunction (post). Here again, our choice of exit channel approximations leads to a choice of residual interactions. As it happens, the post interaction for approximation (2) is the same as the prior interaction, so we finally distinguish only two non-equivalent expressions for the DWBA matrix element, which are denoted by

$$T_{\mathrm{post}} = \langle \chi_{\mathrm{p}}^{(-)}(\boldsymbol{r}_{\mathrm{p}})\, \chi_{\mathrm{n}}^{(-)}(\boldsymbol{r}_{\mathrm{n}}) \,|\, V_{\mathrm{pn}} \,|\, \chi_{\mathrm{d}}^{(+)}(\boldsymbol{R}, \boldsymbol{k}_{\mathrm{d}})\, \phi_{\mathrm{d}}(\boldsymbol{r}) \rangle, \tag{3}$$

$$T_{\mathrm{prior}} = \langle \chi_{\mathrm{d}}^{(-)}(\boldsymbol{R}, \boldsymbol{k}'_{\mathrm{d}})\, \phi^{(-)}(\boldsymbol{r}) \,|\, U_{\mathrm{p}}(\boldsymbol{r}_{\mathrm{p}}) + U_{\mathrm{n}}(\boldsymbol{r}_{\mathrm{n}}) \,|\, \chi_{\mathrm{d}}^{(+)}(\boldsymbol{R}, \boldsymbol{k}_{\mathrm{d}})\, \phi_{\mathrm{d}}(\boldsymbol{r}) \rangle. \tag{4}$$

The usual post–prior equivalence of DWBA matrix elements does not relate these expressions, because they use different approximations in the exit channel. The

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discrepancy between these two expressions gives an indication of the errors in the exit channel wavefunctions.

Many modern breakup calculations apply the post-form DWBA, using a contour rotation integration technique (Vincent and Fortune 1970) such as, for example, studies of (d, pn) (Baur and Trautmann 1976; Baur *et al.* 1980), (^{3}He, dp) (Shyam *et al.* 1980) or α-particle breakup (Budzanowski *et al.* 1979). However, the prior-form DWBA has attractive features (Rybicki and Austern 1972) such as rapid convergence over the proton–neutron relative angular momentum and a well-behaved integrand. Unfortunately, the first application of this method to deuteron breakup (Rybicki and Austern 1972) disagreed with the experimental p–n angular correlations obtained for heavy target nuclei. This disagreement was attributed to an inaccurate treatment of Coulomb distortions. Somewhat later the method was applied to ^{3}He breakup at 90 MeV (Matsuoka *et al.* 1980), where Coulomb distortion is less important. At this energy the prior-form DWBA gave good agreement with experimental data for the d–p angular correlation at forward angles (Udagawa and Tamura 1980; Goto *et al.* 1980). Thus the prior-form DWBA did seem to be a practical means to analyse projectile breakup reactions if Coulomb distortion is not important.

Recently the prior-form DWBA has been applied again to analyse deuteron breakup at the relatively high energy of 56 MeV where Coulomb distortions should be small (Matsuoka *et al.* 1982). The prior-form DWBA successfully reproduced the experimental data for the p–n angular correlation in the region where the protons are emitted to the opposite side of the beam from the neutrons. However, it significantly overestimated the p–n angular correlation in the region where the protons are emitted to the same side of the beam as the neutrons. Coulomb distortion effects were examined carefully and were indeed fairly small at 56 MeV, even for the heavy ^{118}Sn target. It was concluded that the inaccurate treatment of the Coulomb distortion in the prior-form DWBA is irrelevant to the disagreement with the experimental data. Different parameters were introduced into the distorting potentials, without substantial improvement.

Meanwhile, other evidence for the inadequacy of the DWBA in breakup reactions has been found in coupled-channels calculations (Farrell *et al.* 1976; Rawitscher and Mukherjee 1980). It is interesting to determine whether the disagreement between the prior-form DWBA and experiment might really be caused by the omission of channel-coupling effects, rather than by incorrect Coulomb distortions.

Since straightforward numerical calculations of breakup by the coupled-channels method are still rather involved, it seems worthwhile to apply the adiabatic method (Johnson and Soper 1970) to test the importance of channel-coupling effects in the breakup angular correlation. The basic idea of this method is to neglect the excitation energy of the p–n relative motion in a three-body model of the system. In this approximation the p–n relative coordinate becomes a parameter, and the Schrödinger equation for the three-body system is reduced to a single-variable differential equation. The solutions of this equation for each value of r provide an r-dependent t-matrix $T(r)$. Naturally, such an approximation is expected to become better as the deuteron energy increases.

The adiabatic method was recently extended to the angular correlation in deuteron breakup (Amakawa and Tamura 1982). Breakup amplitudes are obtained by taking matrix elements of $T(r)$ between the initial deuteron state and final breakup states.

The accuracy of the adiabatic method has been examined previously for deuteron elastic scattering, for stripping reactions and for angle-integrated breakup cross sections (Rawitscher 1974, 1975*a*, 1975*b*; Farrell *et al.* 1976; Austern *et al.* 1978; Kawai *et al.* 1978; Amakawa *et al.* 1981; Yahiro *et al.* 1982; Iseri *et al.* 1983; Amakawa and Austern 1983). The accuracy was found to depend strongly on the type of reaction: although the adiabatic method gives a fairly accurate elastic component of the deuteron–nucleus wavefunction, the phases of the breakup components are incorrect at large distances. Since breakup components with different p–n relative momentum $\hbar k = \frac{1}{2}|\boldsymbol{p}_p - \boldsymbol{p}_n|$ contribute *coherently* to the stripping cross section, the adiabatic method is of marginal value for stripping reactions. On the other hand, breakup components with different k contribute *incoherently* to the p–n angular correlation. For this reason, despite the incorrect phases of the breakup components, the calculation of breakup correlations by the adiabatic method is expected to be practical, and to have acceptable accuracy. It seems worth while to apply the adiabatic approximation to analyse the angular correlation data and to compare with the prior-form DWBA.

2. Calculations and Analysis

Calculations and Comparison with Experiment

We calculate p–n angular correlations by the adiabatic method, for comparison with previous experimental data and prior-form DWBA calculations (Matsuoka *et al.* 1982). On the whole the adiabatic calculations use the formulation and the notation given already by Amakawa and Tamura (1982). Global optical potentials (Watson *et al.* 1969; Becchetti and Greenlees 1969) are used for the proton–target and neutron–target interactions, with parameters evaluated at half the incident deuteron energy. The spin–orbit potential is neglected. Coulomb breakup is omitted, since it is difficult to treat by the adiabatic method. (This is an inherent defect of the method.) We use exact kinematics, as in the corresponding DWBA calculations (equation 8 of Matsuoka *et al.* 1982), avoiding some previous simplifications (equation 16 of Amakawa and Tamura 1982). The s wave ($l = 0$) and the d wave ($l = 2$) of the p–n relative motion are included. The deuteron is assumed to be a pure s state ($l = 0$). The deuteron wavefunction $\phi_d(r)$ and the s-wave scattering wavefunction $\phi_0(kr)$ are generated by using a Yamaguchi-type separable potential with $\alpha = 0{\cdot}2316\,\mathrm{fm}^{-1}$ and $\beta = 1{\cdot}45\,\mathrm{fm}^{-1}$ (Yamaguchi 1954). These wavefunctions are almost identical to the ones used in the DWBA calculation (Matsuoka *et al.* 1982) and are not expected to disturb the comparison of the DWBA and adiabatic calculations. A spherical Bessel function is used for the d-wave scattering wavefunction $\phi_2(kr)$. The adiabatic coupled-channels equations are solved by using the computer code JUPITOR (Tamura 1967) with some modifications.

Fig. 1 shows the p–n angular correlations for deuteron elastic breakup on (*a*) ^{12}C and (*b*) ^{51}V targets at 56 MeV. The neutron detector is fixed at (*a*) 15° and (*b*) 17·5° in the lab frame for the ^{12}C and the ^{51}V targets. Solid and dashed curves, respectively, show the adiabatic (AD) and prior-form DWBA calculations without Coulomb breakup. Dotted curves show the results of a pure Coulomb breakup calculation by the prior-form DWBA method (Matsuoka *et al.* 1982), using p–n relative angular momenta $l = 0, 1, 2$. As mentioned in the Introduction, for both targets the prior-form

DWBA successfully reproduces the experimental angular correlation at negative proton angles, but it overestimates the correlation function at positive proton angles. The adiabatic method gives substantially better agreement at positive proton angles and it achieves an acceptable overall agreement with the experimental data. The disagreement between the adiabatic calculation and experimental data at $\theta_p^{lab} = -15°$ and $-20°$ can probably be attributed to Coulomb breakup, which is neglected in the adiabatic calculation.

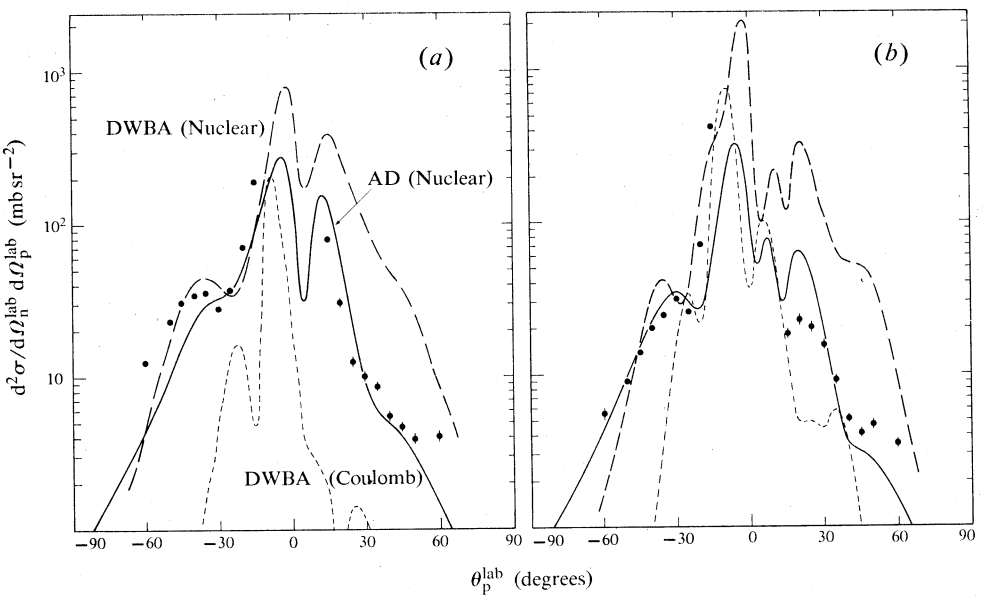

Fig. 1. Energy-integrated p–n angular correlations for the elastic breakup reaction (*a*) ^{12}C(d, pn)^{12}C and (*b*) ^{51}V(d, pn)^{51}V at $E_d = 56$ MeV as a function of proton angle. The neutron detector is fixed at (*a*) 15° and (*b*) 17·5°. The solid curves show the adiabatic calculation of breakup by the nuclear interaction. The dashed and dotted curves show prior-form DWBA calculations of breakup by the nuclear and Coulomb interactions respectively. The experimental data are taken from Matsuoka *et al.* (1982).

The overall agreement between theory and experiment is less satisfactory for the ^{12}C target than for ^{51}V. This may be because the calculations assume a very simple dynamical model, namely a three-body model in which the target nucleus is inert and the proton and neutron that constitute the deuteron interact with the target nucleus through local optical potentials. These basic assumptions may be less adequate for ^{12}C, since it has important collective excitations. Of course, there can be other considerations that affect the quality of the fit to experiment, such as the choice of optical potential parameters.

Analysis

Let us examine the discrepancy between the DWBA and adiabatic calculations at positive proton angles. Since the neutron detector is defined to be fixed at a positive angle, a positive proton angle corresponds to a small value for the p–n relative momentum $\hbar k = \frac{1}{2}|\boldsymbol{p}_p - \boldsymbol{p}_n|$; a negative proton angle corresponds to a large value for k.

Therefore, the discrepancy between the DWBA and adiabatic correlation functions at positive proton angles must be associated with a discrepancy between the associated breakup matrix elements at small k. Since the breakup matrix element for a given k is directly related to the angle-integrated breakup cross section for that value of k, we can conveniently examine the integrated cross section.

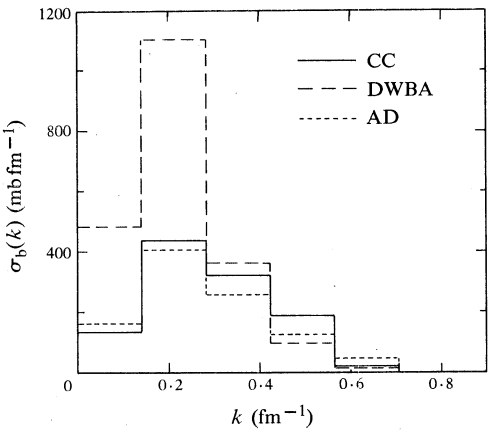

Fig. 2. Integrated s-wave breakup cross sections for ^{40}Ca(d, pn)^{40}Ca at $E_{cm} = 22{\cdot}9$ MeV as a function of p–n relative momentum. The solid, dashed and dotted lines are for the coupled-channels (Austern *et al.* 1978), prior-form DWBA (Farrell *et al.* 1976) and adiabatic (present work) calculations respectively. Breakup by the Coulomb interaction is omitted in these calculations.

Fig. 2 shows angle-integrated breakup cross sections $\sigma_b(k)$ calculated with relative angular momentum $l = 0$ for a d+^{40}Ca example at 22·9 MeV. Our adiabatic result is compared with the results of a full coupled-channels calculation (Austern *et al.* 1978) and of a corresponding prior-form DWBA calculation (Farrell *et al.* 1976). Although these three calculations take into account only the s-wave breakup channel, they should be sufficient to obtain a qualitative understanding. It is seen in Fig. 2 that at small k the integrated cross section obtained from the prior-form DWBA (dashed line) significantly exceeds that from the accurate coupled-channels calculation (dotted line), but at large k they approximately agree. Accordingly, we may conclude that the failure of the DWBA calculation at small k is the reason for the disagreement between the DWBA and the angular correlation data at positive proton angles.

In previous discussions (Farrell *et al.* 1976; Rawitscher and Mukherjee 1980) the failure of the DWBA to agree with coupled-channels calculations of breakup was attributed to continuum–continuum coupling among breakup channels, as well as to the tendency for the DWBA to use a misleading deuteron distorting potential (see Farrell *et al.* 1976; especially p. 363). In the prior-form DWBA the same deuteron optical potential is used for the breakup channels as for the elastic channel. However, this potential is much stronger than the one that would be appropriate at small k for good correspondence with the coupled-channels dynamics. This defect is very serious if the optical potential is weakly absorbing.

The adiabatic calculation of breakup is not subject to the above criticism, since it is based on individual neutron and proton optical potentials. No deuteron optical potential is ever used. We see in Fig. 2 that the breakup cross section yielded by the adiabatic approximation tends to agree with the coupled-channels result. It is no surprise that the adiabatic calculation tends to agree with the angular correlation data.

3. Conclusions

The adiabatic method gives an acceptable overall agreement with the angular correlation data for deuteron breakup on ^{12}C and ^{51}V targets. At the energies considered here, the failure of the prior-form DWBA must be attributed to strong channel-coupling effects and to an incorrect distorting potential for breakup channels with small p–n relative momentum, rather than to improper treatment of Coulomb distortion.

The prior-form DWBA was reported to give good agreement with the experimental angular correlation for ^3He breakup at forward angles (Udagawa and Tamura 1980; Goto *et al.* 1980). Strong binding energy of the ^3He and strong absorption in the ^3He optical potential seem to reduce the channel-coupling effect and the defect of the prior-form DWBA due to the incorrect distorting potential in the breakup channels.

It is interesting that the post-form DWBA tends to be more successful in reproducing the experimental angular correlations for low-energy deuteron breakup (Baur and Trautmann 1976; Baur *et al.* 1980). Of course, numerical calculations with the post-form DWBA are complicated by slow convergence. Nevertheless, this approach does not rely on misleading deuteron optical potentials for distorted waves in breakup channels.

A recent paper by Thompson and Nagarajan (1983) presents a closely related study of ^7Li breakup.

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