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Generalized Statistics and the Rishon Hypothesis

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Abstract

It is pointed out that the proposal of Harari and others, that leptons and quarks should be regarded as composites, consisting of rishons or quips, can be formulated as a field theory in terms of two fundamental spinor fields which satisfy a new generalization of quantum statistics. The requirement of macroscopic causality determines which of the many combinations of rishons may be observed as isolated particles.

One of the attractive features of the quark hypothesis, in its original formulation by Gell-Mann (1964) and Zweig (1964), was the possibility of reducing the kinds of fundamental particles to a small set. In recent years, the standard picture of fundamental quarks and leptons, together with bosons associated with their gauge transformations, has become less impressive with successive additions to the types of both quarks and leptons. For this reason, Harari (1979) and Shupe (1979) have been led to develop a composite model of quarks and leptons in which the fundamental building blocks are quips, or rishons. These are of two types: T particles, carrying a charge of $\frac{1}{3}e$, and V particles, with a zero charge. The electron is built from three T particles and the neutrino from three V particles, while quarks are built from different mixtures of T and V particles. In Harari's original formulation, the different colours of quarks of the same flavour were distinguished by different orderings of the T and V particles, thus: VVT, VTV and TVV, for quarks of charge $\frac{1}{3}e$, without commitment to the significance of the ordering.

More recently, various attempts have been made to develop a dynamical model by Henson (1982), Adler (1980) and Harari and Seiberg (1981), based on an $\mathrm{SU(3)_C \times SU(3)_H}$ symmetry; see also Gelmini (1981). It appears to us that, while the introduction of rather large groups of this type may be inevitable if the sub-particles are regarded as fermions, the effect is to detract from the simplicity of the original concept. An alternative, to which we wish to draw attention, is to suppose that the T and V particles are not fermions, but satisfy a generalized quantum statistics.

With a suitable generalized statistics, it is possible to represent the T and V particles by no more than two spinor field variables, $\psi(x)$ and $\chi(x)$ respectively. Since these do not anticommute in the usual way, the quarks associated with differently ordered products, for example $\psi(x_1)\psi(x_2)\chi(x_3)$, $\psi(x_1)\chi(x_2)\psi(x_3)$ and $\chi(x_1)\psi(x_2)\psi(x_3)$ corresponding to TTV, TVT and VTT, will differ in 'colour' as well as configuration.

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To ensure that macroscopic causality is preserved, it is necessary that products of field variables which do not commute or anticommute should correspond to groups of particles which are subject to confinement. It seems reasonable to suppose that, conversely, those products of field variables which do commute or anticommute with one another correspond to individually observable particles which are not subject to confinement. Thus, products such as $\psi(x_1)\psi(x_2)\psi(x_3)$ and $\chi(x_1)\chi(x_2)\chi(x_3)$, representing leptons which are observed in isolation, should anticommute with one another, but should not anticommute with $\psi(x_1)\psi(x_2)\chi(x_3)$ if we assume that quarks are always confined. Parastatistics, one of the two varieties of generalized statistics developed by one of the authors (Green 1953), is clearly excluded from this application by such considerations. However, it is possible to suppose that the T and V particles satisfy another variety of generalized statistics, related to what we have called modular statistics (Green 1975, 1976).

We assume that $\psi(x_1)$ and $\psi(x_2)$ do not anticommute with one another in the ordinary way, but satisfy the relations

$$\psi(x_1)\psi(x_2)\psi(x_3)\psi(x_4)+\psi(x_4)\psi(x_2)\psi(x_3)\psi(x_1)=0, \tag{1a}$$

$$\psi(x_1)\psi^*(x_2)\psi(x_3)+\psi(x_3)\psi^*(x_2)\psi(x_1)=\delta(x_1-x_2)\psi(x_3)+\delta(x_2-x_3)\psi(x_1), \tag{1b}$$

etc., which are characteristic of modular statistics of order 3. It will then follow (Green 1975, 1976) that if electrons are represented in terms of products of three of these ψ fields they will satisfy fermi statistics, as required for particles of spin half. Furthermore, provision is thereby made for two other kinds of charged leptons (μ and τ particles can be represented by products like $\bar{\chi}_1\psi_2\psi_3\psi_4\chi_1$). The field $\chi(x)$ representing the neutral V particles must be assumed to satisfy relations similar to (1). This leaves the rules for permuting the $\psi(x)$ and $\chi(x)$ fields together still to be decided. For this purpose, we propose a simple extension of the modular algebra.

It is known (Drühl *et al.* 1969, 1970) that the algebra of fields satisfying generalized statistics can be exhibited as a sub-algebra of fields satisfying fermi (or bose) statistics. In an appropriate extension of the algebra which contains the $\psi(x)$ field, generalized anticommutation relations of the type

$$\psi(x_1)S\psi(x_2)+\psi(x_2)S\psi(x_1)=0 \tag{2}$$

are satisfied, where S is a unitary operator satisfying $S^3=1$. Similar relations in which $\psi(x_1)$ and $\psi(x_2)$ are replaced by $S^p\psi(x_1)S^{-p}$ and $S^q\psi(x_2)S^{-q}$ are also satisfied (for details see Green 1975, 1976). In fact, linear combinations like

$$\sum_p \omega^{pq}S^p\psi(x)S^{-p},$$

where $q=0,1,2$ and $\omega=-\frac{1}{2}(1+\sqrt{3}\,\mathrm{i})$ is a complex cube root of unity, afford projections onto fields satisfying ordinary fermi (or bose) statistics in the extended algebra, as mentioned above; these would correspond to the 'colour components' in the usual formulation (Drühl *et al.* 1969, 1970).

If S' is the corresponding operator for the $\chi(x)$ field, it is natural to introduce an involution R such that $S'=RSR$ and $R^2=1$. Then R and S can be regarded as generators of a discrete group, whose order must clearly be some multiple of 6, or of 18 if S' is distinct from S. The simplest possibility, precisely of order 18, is the Z-metacyclic group $C_3 \times D_3$ (Coxeter and Moser 1957), whose defining relations are completed by specifying in addition that S and S' should commute:

$$C_3 \times D_3: \qquad S^3 = R^2 = 1, \quad SS' = S'S, \quad S' \equiv RSR. \tag{3}$$

Consistent with this, the question of the relative statistics of the $\chi(x)$ and $\psi(x)$ fields may be resolved by demanding that $\chi'(x) = R\chi(x)R$ should have the same Klein operator as $\psi(x)$, thus limiting the number of new 'colour components' which can be constructed in the extended algebra. Thus we have

$$\chi(x_1)\,S'\chi(x_2) + \chi(x_2)\,S'\chi(x_1) = 0, \tag{4a}$$

$$S\psi(x_1)\,S'\chi(x_2) + S'\chi(x_2)\,S\psi(x_1) = 0, \tag{4b}$$

and analogues where $\psi(x_1)$ and $\chi(x_2)$ are replaced by $S^p\psi(x_1)S^{-p}$ and $S'^q\chi(x)S'^{-q}$ respectively.

An *ansatz* may now be stated for the construction of the fields $\psi(x)$ and $\chi(x)$ in terms of anticommuting fermion fields $\alpha(x, U_c, U_h)$ and $\beta(x, U_c', U_h)$:

$$\psi(x) = S^2\alpha(x, U_c, U_h), \qquad \chi(x) = S'^2\beta(x, U_c', U_h), \tag{5a, b}$$

where the colour and hypercolour labelling operators U_c, $U_c' = RU_cR$ and U_h are supposed to commute with one another and to satisfy the relations

$$S^2U_cS = U_c, \qquad SU_c'S = \omega^2 U_c, \tag{6a, b}$$

$$S^2U_hS = \omega U_h, \qquad RU_hR = -U_h, \tag{6c, d}$$

when ω is a complex cube root of unity. These relations can be deduced from an equivalent requirement that $R\psi(x) + \chi(x)R$ and $\psi(x)R + R\chi(x)$ should be modular fields of order 6. Explicit representations of R, S, U_c, U_c' and U_h can be obtained by a simple extension of earlier work (Green 1975, 1976). It is important to observe, however, that the dynamical field theory can and should be formulated within the sub-algebra generated by $\psi(x)$ and $\chi(x)$ alone, without the explicit appearance of the Klein operators S and S', or the colour and hypercolour labels.

The implications of this proposal are in close correspondence with those of Harari's original scheme, with the addition of unambiguous restrictions on the combinations of T and V particles which are observable. In terms of the *ansatz*, these are precisely those which do not involve S and S'. Thus, the electron is the hypercolour singlet

$$\alpha(x_1, U_c, \omega U_h)\,\alpha(x_2, U_c, \omega^2 U_h)\,\alpha(x_3, U_c, U_h),$$

while the μ and τ are similar with U_c replaced by ωU_c and $\omega^2 U_c$ respectively. The next simplest combinations of V and T are of the type VTVTVT, which, by Harari's hypothesis, could represent the W particles. There are, in addition, various combinations of V, T, $\overline{\text{V}}$ and $\overline{\text{T}}$ which transform like gauge fields under $SU(3) \times SU(3)$ and provide material for other kinds of bosons. The quark model of the baryons needs little elaboration when the quarks are themselves regarded as composites. It has already been shown (Green 1975, 1976) how to construct a quantized field theory of interacting particles, consistent with macroscopic causality, when the fields satisfy modular statistics.

The scheme outlined above has a degree of elasticity associated with the discrete group $C_3 \times D_3$. The theory of discrete groups has not yet made much of an impression on theoretical physics, in spite of the facts that elementary particles manifestly constitute a discrete rather than a continuous set, and that Nature appears to favour a fixed rather than a variable gauge. Of course the discrete groups can be imbedded in continuous groups, and it is sometimes convenient to do this. But the application of generalized statistics to particle physics may have a salutary effect in placing emphasis on the underlying discrete symmetries.

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