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GDR Contribution to Coulomb Excitation. I 1p Shell Nuclei

F. C. Barker

Department of Theoretical Physics, Research School of Physical Sciences, Australian National University, P.O. Box 4, Canberra, A.C.T. 2600.

Abstract

The contributions to the Coulomb excitation of the first excited states of ^{6}Li, ^{7}Li, ^{10}B and ^{12}C due to virtual excitation of the giant dipole resonance (GDR) are calculated, using shell model wavefunctions for the ground and first excited states. When the radial integrals are renormalized in order to fit the experimental B(E2) values for the transitions between these states, the calculated GDR contributions agree reasonably with the measured values in ^{6}Li, ^{7}Li and ^{10}B.

1. Introduction

In the preceding paper (Vermeer *et al.* 1982, present issue p. 283), the Coulomb excitation of the first excited state of ^{10}B has been measured. The calculated excitation probability depends essentially on the value of $B(\mathrm{E2}; 3^+ \rightarrow 1^+)$ for the transition from the 3^+ ground state to the 1^+ first excited state, and also on three higher order contributions, two being due to reorientation processes depending on the quadrupole moments Q_{3+} of the ground state and Q_{1+} of the first excited state, and the other to virtual excitation of the GDR. Measured values of $B(\mathrm{E2}; 3^+ \rightarrow 1^+)$ and Q_{3+} are available (Ajzenberg-Selove 1979). The measurements of Vermeer *et al.* then give a relation between the values of Q_{1+} and the GDR contribution. There are two published predictions of Q_{1+}: $-0{\cdot}8\ e\,\mathrm{fm}^2$ from shell model calculations (Barker 1981) and $-2{\cdot}2\ e\,\mathrm{fm}^2$ from a projected Hartree–Fock calculation (Bouten and Bouten 1981). The GDR contribution has not previously been calculated specifically for ^{10}B.

Somewhat similar studies involving the GDR contribution to Coulomb excitation of the 3^+ first excited state of ^{6}Li from the 1^+ ground state have been reported from Chalk River (Disdier *et al.* 1971; Häusser *et al.* 1973) and from Heidelberg (Scholz *et al.* 1977; Gemmeke *et al.* 1978). In the case of ^{7}Li, also studied at Heidelberg (Bamberger *et al.* 1972) and at Chalk River (Häusser *et al.* 1973), a reliable value of B(E2) connecting the $\frac{3}{2}^-$ ground state and $\frac{1}{2}^-$ first excited state is not available from other measurements, but the Q value of the excited state is necessarily zero. A measurement of the Coulomb excitation of the 2^+ first excited state of ^{12}C is in progress at Canberra (R. H. Spear, personal communication).

In this paper, shell model calculations of the GDR contributions in ^{6}Li, ^{7}Li, ^{10}B and ^{12}C are made.

2. Formulae

In the Coulomb excitation of the first excited state of a nucleus, the direct E2 excitation may be modified by interference with the two-step process involving virtual E1 excitation of high-lying states in the GDR region followed by their E1

decay to the first excited state. An expression for this E1 contribution in the case of arbitrary spins has been given by Häusser *et al.* (1973); it depends on the ratio

$$X \equiv S(\mathrm{E1})/\langle \mathrm{i} \,\|\, \mathscr{M}(\mathrm{E2}) \,\|\, \mathrm{f} \rangle , \tag{1}$$

where

$$S(\mathrm{E1}) = \sum_n W(11 I_\mathrm{i} I_\mathrm{f}, 2 I_n) \langle \mathrm{i} \,\|\, \mathscr{M}(\mathrm{E1}) \,\|\, n \rangle \langle n \,\|\, \mathscr{M}(\mathrm{E1}) \,\|\, \mathrm{f} \rangle / (E_n - E_\mathrm{i}) . \tag{2}$$

The notation is that of Häusser *et al.* The reduced matrix elements are as defined by de-Shalit and Talmi (1963). In the electric multipole operators defined by

$$\mathscr{M}(\mathrm{E}\lambda, \mu) = \sum_{j=1}^{A} e_j r_j^\lambda \, \mathrm{Y}_{\lambda\mu}(\Omega_j) , \tag{3}$$

the origin of coordinates is here taken as the centre of mass of the nucleus in order to avoid problems with lack of translational invariance. The denominator in (1) is related to $B(\mathrm{E2})$ by

$$B(\mathrm{E2}; \mathrm{i} \to \mathrm{f}) = (2I_\mathrm{i} + 1)^{-1} \, |\langle \mathrm{i} \,\|\, \mathscr{M}(\mathrm{E2}) \,\|\, \mathrm{f} \rangle|^2 . \tag{4}$$

In order to simplify the calculation of $S(\mathrm{E1})$, we assume that all the E1 strength from the ground state is concentrated at the energy E_g. Actually, for the case $|I_\mathrm{i} - I_\mathrm{f}| = 2$, which includes ^{6}Li, ^{10}B and ^{12}C, only intermediate states n with $I_n = \frac{1}{2}(I_\mathrm{i} + I_\mathrm{f})$ contribute to $S(\mathrm{E1})$, and it is sufficient to assume that all the E1 strength to states with this value of I_n is concentrated at E_g; unless $I_\mathrm{i} = 0$, the GDR also contains contributions from states of other spins, but it is not necessary to make assumptions about these. Then equation (2) can be written

$$\begin{aligned} S(\mathrm{E1}) &= (E_\mathrm{g} - E_\mathrm{i})^{-1} \sum_n W(11 I_\mathrm{i} I_\mathrm{f}, 2 I_n) \langle \mathrm{i} \,\|\, \mathscr{M}(\mathrm{E1}) \,\|\, n \rangle \langle n \,\|\, \mathscr{M}(\mathrm{E1}) \,\|\, \mathrm{f} \rangle \\ &= 5^{-\frac{1}{2}} (E_\mathrm{g} - E_\mathrm{i})^{-1} \langle \mathrm{i} \,\|\, \mathscr{O}^2 \,\|\, \mathrm{f} \rangle , \end{aligned} \tag{5}$$

where $\mathscr{O}^2$ is a tensor operator of rank 2 defined by

$$\mathscr{O}^2(\mu) = \sum_\nu (11 \nu \mu - \nu \,|\, 2\mu) \, \mathscr{M}(\mathrm{E1}, \nu) \, \mathscr{M}(\mathrm{E1}, \mu - \nu) . \tag{6}$$

We now assume that the initial and final states belong to the 1p shell configuration, and write them in the LS coupling representation

$$\begin{aligned} |k) &\equiv |\, T_k M_T I_k M_k) \\ &= \sum_{[\lambda] S L} a_k([\lambda] S L) \,|\, 1\mathrm{s}^4 1\mathrm{p}^{A-4} [\lambda] T_k S L, M_T I_k M_k) \qquad (k = \mathrm{i}, \mathrm{f}) . \end{aligned} \tag{7}$$

Then we have

$$\begin{aligned} \langle \mathrm{i} \,\|\, \mathscr{O}^2 \,\|\, \mathrm{f} \rangle = & -(\sqrt{15}\, e^2/40\pi)(2I_\mathrm{i} + 1)^{\frac{1}{2}} (A-4) \sum_{[\lambda] S L [\lambda'] L'} a_\mathrm{i}([\lambda] S L) \, a_\mathrm{f}([\lambda'] S L') \\ & \times U(2L' I_\mathrm{i} S, L I_\mathrm{f}) \sum_{[\bar{\lambda}] \bar{T} \bar{S} \bar{L}} \langle 1\mathrm{p}^{A-4} [\lambda] T_\mathrm{i} S L \{| \, 1\mathrm{p}^{A-5} [\bar{\lambda}] \bar{T} \bar{S} \bar{L}, 1\mathrm{p} \rangle \\ & \times \langle 1\mathrm{p}^{A-4} [\lambda'] T_\mathrm{f} S L' \{| \, 1\mathrm{p}^{A-5} [\bar{\lambda}] \bar{T} \bar{S} \bar{L}, 1\mathrm{p} \rangle \, U(21 L \bar{L}, 1 L') \\ & \times \left([1 + \{(N-Z)/A\}^2] \delta_{T_\mathrm{i} T_\mathrm{f}} - 2\sqrt{3} \{(N-Z)/A\} (T_\mathrm{f} \, 1 M_T 0 \,|\, T_\mathrm{i} M_T) \right. \\ & \left. \times U(1\tfrac{1}{2} T_\mathrm{i} \bar{T}, \tfrac{1}{2} T_\mathrm{f}) \right) (\langle 1\mathrm{p} : r^2 : 1\mathrm{p} \rangle - \tfrac{5}{3} \langle 1\mathrm{s} : r : 1\mathrm{p} \rangle^2) . \end{aligned} \tag{8}$$

Here we write

$$\langle nl:r^q:n'l'\rangle = \int_0^\infty u_{nl}(r)\,u_{n'l'}(r)\,r^q r^2\,\mathrm{d}r, \qquad \int_0^\infty u_{nl}^2(r)\,r^2\,\mathrm{d}r = 1, \tag{9a, b}$$

where $u_{nl}(r)$ is the single-particle radial wavefunction with r measured from the centre of the potential well, and the expression (8) involves fractional parentage coefficients and angular momentum recoupling coefficients. For each of the cases of interest here, $T_\mathrm{i} = T_\mathrm{f} = T$ say. Then the details of the nuclear wavefunctions, which are contained in the expansion coefficients $a_k([\lambda]SL)$, can be expressed in terms of spectroscopic amplitudes from the state k to the parent state $|\,n\bar{T}\bar{M}_T\bar{I}\,\rangle$ of the nuclei with $A-1$ nucleons, plus a p-wave nucleon, with channel spin s (Lane 1960):

$$\begin{aligned}\mathscr{S}^{\frac{1}{2}}_{k,n\bar{T}\bar{M}_T\bar{I}}(s) = {} & (A-4)^{\frac{1}{2}}(\bar{T}\tfrac{1}{2}\bar{M}_T\,M_T-\bar{M}_T\,|\,TM_T)\sum_{[\lambda]SL[\bar{\lambda}]\bar{S}\bar{L}} a_k([\lambda]SL)\,b_{n\bar{T}\bar{I}}([\bar{\lambda}]\bar{S}\bar{L})\\ & \times(-1)^{\bar{I}-S-\bar{S}+s}\,U(\bar{L}\bar{S}s\tfrac{1}{2},\bar{I}S)\,U(S\bar{L}I_k\,1,sL)\\ & \times\langle 1\mathrm{p}^{A-4}[\lambda]TSL\{|\,1\mathrm{p}^{A-5}[\bar{\lambda}]\bar{T}\bar{S}\bar{L},1\mathrm{p}\rangle,\end{aligned} \tag{10}$$

where the parent state is written

$$|\,n\bar{T}\bar{M}_T\bar{I}\bar{M}) = \sum_{[\bar{\lambda}]\bar{S}\bar{L}} b_{n\bar{T}\bar{I}}([\bar{\lambda}]\bar{S}\bar{L})\,|\,1\mathrm{s}^4 1\mathrm{p}^{A-5}[\bar{\lambda}]\bar{T}\bar{S}\bar{L},\bar{M}_T\bar{I}\bar{M}). \tag{11}$$

Then it follows that

$$\begin{aligned}\sum_{n\bar{M}_T\bar{I}s} & U(I_\mathrm{i}s21,1I_\mathrm{f})\,\mathscr{S}^{\frac{1}{2}}_{\mathrm{i},n\bar{T}\bar{M}_T\bar{I}}(s)\,\mathscr{S}^{\frac{1}{2}}_{\mathrm{f},n\bar{T}\bar{M}_T\bar{I}}(s)\\ & = (A-4)\sum_{[\lambda]SL[\lambda']L'} a_\mathrm{i}([\lambda]SL)\,a_\mathrm{f}([\lambda']SL')\,U(2L'I_\mathrm{i}S,LI_\mathrm{f})\\ & \quad\times\sum_{[\bar{\lambda}]\bar{S}\bar{L}}\langle 1\mathrm{p}^{A-4}[\lambda]TSL\{|\,1\mathrm{p}^{A-5}[\bar{\lambda}]\bar{T}\bar{S}\bar{L},1\mathrm{p}\rangle\langle 1\mathrm{p}^{A-4}[\lambda']TSL'\{|\,1\mathrm{p}^{A-5}[\bar{\lambda}]\bar{T}\bar{S}\bar{L},1\mathrm{p}\rangle\\ & \quad\times U(21L\bar{L},1L').\end{aligned} \tag{12}$$

If we write

$$F_{n\bar{T}\bar{M}_T\bar{I}} = \sum_s U(I_\mathrm{i}s21,1I_\mathrm{f})\,\mathscr{S}^{\frac{1}{2}}_{\mathrm{i},n\bar{T}\bar{M}_T\bar{I}}(s)\,\mathscr{S}^{\frac{1}{2}}_{\mathrm{f},n\bar{T}\bar{M}_T\bar{I}}(s), \tag{13}$$

then (8) can be written as

$$\begin{aligned}\langle \mathrm{i}\,\|\,\mathscr{O}^2\,\|\,\mathrm{f}\rangle = {} & -(\sqrt{15}\,e^2/40\pi)(2I_\mathrm{i}+1)^{\frac{1}{2}}\sum_{n\bar{T}\bar{M}_T\bar{I}} F_{n\bar{T}\bar{M}_T\bar{I}}\\ & \times[1+\{(N-Z)/A\}^2-2\sqrt{3}\{(N-Z)/A\}(T1M_T0\,|\,TM_T)\,U(1\tfrac{1}{2}T\bar{T},\tfrac{1}{2}T)]\\ & \times(\langle 1\mathrm{p}:r^2:1\mathrm{p}\rangle-\tfrac{5}{3}\langle 1\mathrm{s}:r:1\mathrm{p}\rangle^2).\end{aligned} \tag{14}$$

Similarly, we get

$$\begin{aligned}\langle \mathrm{i}\,\|\,\mathscr{M}(\mathrm{E2})\,\|\,\mathrm{f}\rangle = {} & -\{e/(8\pi)^{\frac{1}{2}}\}(2I_\mathrm{i}+1)^{\frac{1}{2}}\sum_{n\bar{T}\bar{M}_T\bar{I}} F_{n\bar{T}\bar{M}_T\bar{I}}\\ & \times[\{1-2N/A^2-(1-2/A)\sqrt{3}(T1M_T0\,|\,TM_T)\,U(1\tfrac{1}{2}T\bar{T},\tfrac{1}{2}T)\}\langle 10:r^2:1\mathrm{p}\rangle\\ & +\{2N/A^2-(2/A)\sqrt{3}(T1M_T0\,|\,TM_T)\,U(1\tfrac{1}{2}T\bar{T},\tfrac{1}{2}T)\}\tfrac{5}{3}\langle 1\mathrm{s}:r:1\mathrm{p}\rangle^2].\end{aligned} \tag{15}$$

For comparison with experimental results, the quantity X defined in equation (1) should be expressed in terms of the unit normally used in computer analyses of the experimental data (Disdier *et al.* 1971; Häusser *et al.* 1973; Fewell 1978). To obtain this unit, the quantity $S(\mathrm{E1})$ defined in (2) is written, for the case $I_i = 0$, in terms of the measurable quantity σ_{-2}, where σ_n is the nth moment of the photonuclear cross section, and a parameter η_0. The value of σ_{-2} is taken from a formula that is based on the hydrodynamic model and is empirically satisfied for heavier nuclei, $\sigma_{-2} = 3{\cdot}5A^{5/3}\,\mu\mathrm{b\,MeV}^{-1}$ (Levinger 1957), and η_0 is estimated from the hydrodynamic model (see de Boer and Eichler 1968). One finds that the unit is

$$X_0 = 0{\cdot}00058\,A/Z\ e\,\mathrm{MeV}^{-1}. \tag{16}$$

The ratio

$$k = X/X_0 \tag{17}$$

is used in fitting the experimental data.

3. Calculated Values

With the simple assumption of harmonic oscillator single-particle wavefunctions, with length parameter b, one has

$$\langle \mathrm{1p}\,|\,r^2\,|\,\mathrm{1p}\rangle = \tfrac{5}{2}b^2, \qquad \langle \mathrm{1s}\,|\,r\,|\,\mathrm{1p}\rangle = (\tfrac{3}{2})^{\frac{1}{2}}b, \tag{18a, b}$$

so that $\langle \mathrm{i} \,\|\, \mathcal{O}^2 \,\|\, \mathrm{f}\rangle$ vanishes, leading to $S(\mathrm{E1}) = 0$ and $X = 0$. This result can alternatively be obtained from the sum-rule approach of Koo and Tassie (1976). They considered only isoscalar electric multipole operators and found that a perfect giant 2^l-pole state based on a 0^+ ground state cannot decay to a state with $J^\pi = (2l)^+$ by emitting a 2^l-pole photon. This result is obtained because the isoscalar operator has a vanishing double commutator with the potential term of the Hamiltonian. The same result is obtained for an isovector multipole operator if the potential has no charge-exchange term (L. J. Tassie, personal communication). Our assumptions of only one unfilled shell (the 1p shell), of harmonic oscillator single-particle wavefunctions, and of a perfect giant dipole state imply that the Hamiltonian is just the sum of harmonic oscillator single-particle Hamiltonians and therefore contains no charge-exchange term. The result for $I_i \neq 0$ follows from the more general sum-rule formulae of Koo and Tassie.

It is more reasonable to assume single-particle wavefunctions belonging to a finite-depth potential well. In such cases it is usual to choose the well depth by fitting the nucleon binding energy. We use the 1p nucleon binding energy appropriate to an 'average' parent state, obtained by using calculated values of the weighting factors $F_{n\bar{T}\bar{M}_T\bar{I}}$. We do not distinguish between nucleons originating in the ground state and in the first excited state of the nucleus under study, since the formulae for the Coulomb excitation probability including the GDR contribution neglect terms of order $(E_f - E_i)/(E_g - E_i)$ (de Boer and Eichler 1968); the 1p binding energies are therefore measured from the mean of the ground state and first excited state energies. The 1s proton binding energies are obtained from (e, e′p) or (p, 2p) experiments, the neutron values following after adjustments for Coulomb energy

differences. We use a real Woods–Saxon potential, with central and surface-peaked spin–orbit terms, plus the Coulomb potential of a uniformly charged sphere. Parameter values for such a potential for 1p shell nuclei have been given by Elton and Swift (1967) and by Gamba *et al.* (1973) from fitting elastic electron scattering form factors, single-particle binding energies from (e, e'p) and (p, 2p) experiments, and elastic proton scattering data. These values of the radius parameters, diffuseness and spin–orbit strength are used, and the central potential depth adjusted by fitting binding energies.

Table 1. Shell model interaction and potential parameter values

Case	Shell model interaction	Potential parameters (fm) r_0	R	a	Ref.
^{6}Li	A	1·45	2·48	0·65	D
^{7}Li	A	1·38	2·51	0·65	D
^{10}B	B	1·25	2·69	0·57	E
^{12}C	C	1·36	3·02	0·55	D

[A] Barker (1966).
[B] Barker (1981), set III.
[C] Cohen and Kurath (1965, 1967), (8-16)POT.
[D] Elton and Swift (1967).
[E] Gamba *et al.* (1973); note that Gamba *et al.* define r_0 by $R = r_0 A^{1/3}$ instead of the more usual $R = r_0(A-1)^{1/3}$.

Values of the spectroscopic amplitudes that enter the expression (13) for the weighting factors $F_{n\bar{T}\bar{M}_T\bar{I}}$ are taken for the shell model interactions as indicated in Table 1, which also gives the values of the potential parameters used. Table 2 gives the values of $F_{n\bar{T}\bar{M}_T\bar{I}}$ and of the corresponding 1p binding energies for both neutron and proton channels. The adopted average values of these binding energies, together with the 1s binding energies, are listed in Table 3, which also gives the resultant values of the radial integrals.

The values of the matrix elements and of B(E2) given in Table 4 are obtained from the values in Tables 2 and 3 by using equations (14), (15) and (4). The experimental values of B(E2) are also given. The discrepancies apparent between the calculated and experimental values of B(E2) are examples of a well-known phenomenon, which is usually attributed to the neglect of higher configurations in the assumed wavefunctions and remedied by the introduction of effective charges. In the $\mathscr{M}$(E2) operator, the charge e_j of the jth nucleon is replaced by $e_j+\varepsilon e$, where e is the proton charge and ε is usually about 0·5. It is not obvious, however, what effective charge should be used in the operator $\mathscr{O}^2$, which is needed in the calculation of S(E1); the only simple choices that avoid the introduction of additional arbitrary parameters are to use the bare charges ($\varepsilon = 0$) or the same effective charges as for $\mathscr{M}$(E2). Neither of these choices changes the value of the matrix element $\langle \mathrm{i} \| \mathscr{O}^2 \| \mathrm{f} \rangle$, since the additional charge εe is isoscalar and the operator $\mathscr{O}^2$ is derived from the E1 operator with the origin of coordinates at the centre of mass of the nucleus. An alternative way of fitting B(E2), which seems in the present cases to have as much justification as the use of effective charges, is to adjust the radial integrals.

Table 2. Values of weighting factors and 1p binding energies

^{6}Li	$\bar{T}\bar{I}^{n}$	$\frac{1}{2}\frac{3}{2}^{1}$								
	$F_{n\bar{T}\pm\frac{1}{2}\bar{I}}$	0·674								
	B_{1p}(neutron) (MeV)	4·57								
	B_{1p}(proton) (MeV)	3·50								
^{7}Li	$\bar{T}\bar{I}^{n}$	01^{1}	10^{1}	02^{1}	12^{1}	01^{2}	12^{2}	11^{1}	10^{2}	01^{3}
	$F_{n\bar{T}_0\bar{I}}$(neutron)	−0·618	−0·175	−0·021	−0·015	−0·012	−0·005	0·000	−0·006	0·000
	B_{1p}(neutron) (MeV)	7·01	10·57	11·58	12·37	12·80	14·82	15·66	17·32	18·64
	$F_{n\bar{T}_1\bar{I}}$(proton)		−0·349		−0·030		−0·010	0·000	−0·011	
	B_{1p}(proton) (MeV)		9·74		11·54		13·98	14·82	16·48	
^{10}B	$\bar{T}\bar{I}^{n}$	$\frac{1}{2}\frac{3}{2}^{1}$	$\frac{1}{2}\frac{5}{2}^{1}$	$\frac{1}{2}\frac{3}{2}^{2}$	$\frac{1}{2}\frac{5}{2}^{2}$	$\frac{1}{2}\frac{3}{2}^{3}$	$\frac{1}{2}\frac{5}{2}^{3}$	$\frac{1}{2}\frac{5}{2}^{4}$	$\frac{1}{2}\frac{3}{2}^{4}$	$\frac{1}{2}\frac{5}{2}^{5}$
	$F_{n\bar{T}\pm\frac{1}{2}\bar{I}}$	−0·391	0·052	0·016	−0·006	0·000	0·000	0·007	0·009	0·003
	B_{1p}(neutron) (MeV)	8·08	10·11	12·83	14·90	19·28	20·28	22·13	22·82	24·97
	B_{1p}(proton) (MeV)	6·23	8·26	10·98	13·05	17·43	18·43	20·28	20·97	23·12
^{12}C	$\bar{T}\bar{I}^{n}$	$\frac{1}{2}\frac{3}{2}^{1}$	$\frac{1}{2}\frac{1}{2}^{1}$	$\frac{1}{2}\frac{3}{2}^{2}$	$\frac{1}{2}\frac{3}{2}^{3}$	$\frac{1}{2}\frac{1}{2}^{2}$	$\frac{1}{2}\frac{1}{2}^{3}$	$\frac{1}{2}\frac{3}{2}^{4}$	$\frac{1}{2}\frac{3}{2}^{5}$	$\frac{1}{2}\frac{3}{2}^{6}$
	$F_{n\bar{T}\pm\frac{1}{2}\bar{I}}$	−0·837	−0·498	−0·226	−0·003	0·000	0·000	0·000	−0·001	0·000
	B_{1p}(neutron) (MeV)	16·50	18·21	21·90	27·93	28·11	28·51	29·86	29·95	31·54
	B_{1p}(proton) (MeV)	13·74	15·44	19·13	25·17	25·35	25·75	27·10	27·18	28·78

Table 3. Average binding energies and radial integrals

Case	Neutron				Proton			
	B_{1s} (MeV)	B_{1p} (MeV)	$\langle 1p:r^2:1p\rangle$ (fm^2)	$\langle 1s:r:1p\rangle$ (fm)	B_{1s} (MeV)	B_{1p} (MeV)	$\langle 1p:r^2:1p\rangle$ (fm^2)	$\langle 1s:r:1p\rangle$ (fm)
^{6}Li	23·7	4·57	10·31	1·980	22·6[A]	3·50	10·85	1·980
^{7}Li	26·9	7·8	8·47	1·942	26·0[A]	10·1	7·52	1·959
^{10}B	39·4	7·4	8·39	1·784	37·5[A]	5·6	8·70	1·777
^{12}C	38·3	17·7	7·01	1·964	35·5[B]	14·9	7·20	1·976

[A] Nakamura *et al.* (1978). [B] Ajzenberg-Selove and Busch (1980).

Since the (final) term in (15) involving $\langle 1s:r:1p\rangle$ is of order A^{-1} relative to the $\langle 1p:r^2:1p\rangle$ term, and vanishes in the usual approximation of neglecting recoil, we renormalize only the radial integrals $\langle 1p:r^2:1p\rangle$, multiplying them by a factor f in order to fit $B(E2)$. Then these renormalized values of $\langle 1p:r^2:1p\rangle$ are also used in calculating $\langle i \| \mathcal{O}^2 \| f\rangle$. Values obtained in this way are given in Table 5.

Table 4. Calculated values of matrix elements and values of $B(E2)$

Case	$\langle i \| \mathcal{O}^2 \| f\rangle$ (e^2 fm^2)	$\langle i \| \mathcal{M}(E2) \| f\rangle$ (e fm^2)	$B(E2; i \to f)$ (e^2 fm^4) Calc.	Exp.
^{6}Li	−0·291	−4·61	7·08	25·6[A]
^{7}Li	0·125	2·48	1·54	8·3[B]
^{10}B	0·165	2·69	1·03	1·81[C]
^{12}C	0·061	4·40	19·4	38·8[D]

[A] Eigenbrod (1969). [B] Häusser *et al.* (1973). [C] Ajzenberg-Selove (1979). [D] Ajzenberg-Selove and Busch (1980).

Table 5. Renormalization factors for radial integrals and GDR contributions

Case	f	$\langle i \| \mathcal{M}(E2) \| f\rangle$ (e fm^2)	$\langle i \| \mathcal{O}^2 \| f\rangle$ (e^2 fm^2)	$E_g - E_i$ (MeV)	$S(E1)$ (e^2 fm^2 MeV^{-1})	X (e MeV^{-1})	k Calc.	Exp.
^{6}Li	2·01	−8·76	−1·060	24	−0·0198	0·00225	1·94	2·6–3·9
^{7}Li	2·44	5·76	0·948	24	0·0177	0·00307	2·27	2·3–3·6
^{10}B	1·35	3·56	0·314	28	0·0050	0·00141	1·22	1·3±0·3
^{12}C	1·45	6·23	0·369	29	0·0057	0·00091	0·77	

Values of $E_g - E_i$, which are needed in the calculation of $S(E1)$, may be estimated from photonuclear cross sections, although it should be noted that the GDR contains contributions from states with spins different from those contributing to $S(E1)$ (except for ^{12}C). For a perfect giant dipole state, one would have $E_g - E_i = \sigma_n/\sigma_{n-1}$, for any integer n. The approximation made in going from equation (2) to (5) should be most accurate, however, if we use $E_g - E_i = \sigma_{-1}/\sigma_{-2}$, since there is a close resemblance between the formula (2) for $S(E1)$ and that for σ_{-2}:

$$\sigma_{-2} = -(16\pi^3/9\hbar c)\{3/(2I_i+1)\}^{\frac{1}{2}} \times \sum_n W(11I_i I_i, 0I_n)\langle i \| \mathcal{M}(E1) \| n\rangle\langle n \| \mathcal{M}(E1) \| i\rangle/(E_n - E_i) \qquad (19)$$

(see Appendix J of Alder and Winther 1975). We make use of the values of σ_{-1} and σ_{-2} obtained by Ahrens *et al.* (1975) from their total nuclear photon absorption cross section measurements for Li, Be, C and heavier elements. Since the lower limit of their integrations was at 10 MeV photon energy, their values of σ_{-1} and σ_{-2} should not contain M1 contributions, which can be appreciable at lower energies (Knüpfer and Richter 1981). We use values with the upper limit of integration at 140 MeV. This gives $E_g - E_i$ = 24, 27 and 29 MeV for Li, Be and C respectively. We assume that the Li value is valid for both ^{6}Li and ^{7}Li, that the C value holds for ^{12}C, and that the appropriate value for ^{10}B is 28 MeV. These values, together

with the resultant values of $S(\mathrm{E1})$, X and k obtained from equations (5), (1) and (17) respectively are given in Table 5.

4. Experimental Values

Experimental values of k are available for ^{6}Li, ^{7}Li and ^{10}B.

For ^{6}Li, Disdier *et al.* (1971) fitted their experimental data with $k = 3{\cdot}9$. Häusser *et al.* (1973) fitted the same data with $S(\mathrm{E1}) = 0{\cdot}036\ e^2\,\mathrm{fm}^2\,\mathrm{MeV}^{-1}$; when combined with $B(E2;\ 1^+ \rightarrow 3^+) = 24\ e^2\,\mathrm{fm}^4$ (Disdier *et al.* 1971), this gives $k = 3{\cdot}6$. Gemmeke *et al.* (1978) obtained $k = 2{\cdot}6$.

For ^{7}Li, Häusser *et al.* (1973) fitted their own data with $S(\mathrm{E1}) = 0{\cdot}028\ e^2\ \mathrm{fm}^2\,\mathrm{MeV}^{-1}$ and $B(\mathrm{E2};\frac{3}{2}^- \rightarrow \frac{1}{2}^-) = 8{\cdot}3\ e^2\,\mathrm{fm}^4$, giving $k = 3{\cdot}6$; they also fitted various data of Bamberger *et al.* (1972) with values of $S(\mathrm{E1})$ and $B(\mathrm{E2};\frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ that correspond to $k = 3{\cdot}2$, $2{\cdot}3$ and $3{\cdot}4$.

In the preceding paper, Vermeer *et al.* (1982) found that a fit to their data for ^{10}B required $k > 0{\cdot}5$, for any value of Q_{1+}. For Q_{1+} in the range $-0{\cdot}8$ to $-2{\cdot}2\ e\,\mathrm{fm}^2$, as suggested by the calculations, their fits require $k = 1{\cdot}3 \pm 0{\cdot}3$.

These experimental values of k are included in Table 5.

5. Discussion

Too close an agreement between the calculated and experimental values of k should not be expected, in view of the approximations made in the calculations and uncertainties in the analysis of the experiments.

In the ^{6}Li measurements, it is not obvious that nuclear interference effects are insignificant in the 24 MeV Chalk River data and the 23 MeV Heidelberg data at the larger angles; the fact that such effects were not observed in elastic scattering does not necessarily mean that they were negligible for inelastic scattering (Thomson *et al.* 1971; Feng *et al.* 1976). Also the experimental value of k is very sensitive to the assumed $B(\mathrm{E2})$ value; the ^{6}Li data of Gemmeke *et al.* (1978) would suggest a much smaller value of k if $B(\mathrm{E2};\ 1^+ \rightarrow 3^+)$ were taken as $21{\cdot}8\ e^2\,\mathrm{fm}^4$ (Yen *et al.* 1974) rather than the $25{\cdot}6\ e^2\,\mathrm{fm}^4$ (Eigenbrod 1969) that they used.

Considerable uncertainty in the calculated values can arise from the assumption that all the E1 strength from the ground state is concentrated at the energy E_g, with the consequent necessity of estimating the value of $E_g - E_i$, and also from the assumption that the wavefunctions of the ground and first excited states belong to the 1 p shell configuration, requiring a renormalization of the radial integrals in order to fit $B(\mathrm{E2})$ values (the customary procedure of fitting $B(\mathrm{E2})$ values by introducing effective charges does not offer a viable approach to the calculation of the quantity $S(\mathrm{E1})$). This renormalization procedure does, however, have the advantage of making the calculated value of k insensitive to the choice of shell model interaction, potential well parameters and $B(\mathrm{E2})$ value.

In view of these uncertainties, the agreement between the calculated and experimental values of the GDR contribution, as shown by the k values in Table 5, seems reasonable.

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