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Analysis of Angular Distributions of Heavy-ion Inelastic Collisions

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Abstract

The relative contributions of nuclear and Coulomb excitation to inelastic collisions of heavy ions are examined, and it is shown how to distinguish their effects on the angular distribution. From the experimental distributions for Coulomb excitation it is possible to make an estimate of the effective range of the Coulomb interaction in the collision. The distributions for nuclear excitation are analysed in the same way as in our previous work on transfer reactions, and give very similar results for the greater nuclear penetration in such collisions as compared with elastic collisions.

1. Introduction

When pairs of heavy ions collide, the shells of the two nuclei are unable to penetrate each other, so that energy transfer and energy loss occur mainly for grazing collisions with a nuclear separation R, a critical angular momentum $l_c \sim kR$, and a critical scattering angle $\theta_c \sim 2\arctan(n/l_c)$. Hence the S matrix $S(l)$, which is the weighting factor for the lth term in the partial wave series for the scattering amplitude $f(\theta)$, is a maximum for $l \approx l_c$. The width of this l window is small for transfer reactions and for inelastic collisions involving nuclear excitation, because the range of the nuclear interaction is small, so the same approximations may be used in treating both types of collision. But for inelastic collisions involving Coulomb excitation, the range of the Coulomb interaction is much larger, so the l window is much wider, and the same approximations can no longer be used. We can, however, still assume that the excitation energy is small compared with the incident energy (we consider only excitation to the lowest 2^+ state), so that the terms in the partial wave series differ from those for elastic collisions only in the form of $S(l)$, and in the change of l during the collision.

2. Relative Importance of Nuclear and Coulomb Excitation

Since the collision energy in most experiments with heavy ions does not vary greatly, we expect the contribution to inelastic scattering from Coulomb excitation to depend mainly on $Z_1 Z_2$.

The form of $|S(l, l=I)|$, where I is the total channel spin, for excitation of the target nucleus to a 2^+ state, is known for three cases. There is a narrow peak for nuclear excitation (N) followed by a wide peak for Coulomb excitation (C), the peaks being at slightly different values of l. The two peaks are of comparable height for 100 MeV ^{18}O on ^{120}Sn (Glendenning 1975), and for 56 MeV ^{16}O on ^{74}Ge (Cobern *et al.* 1976), while for 56 MeV ^{16}O on ^{28}Si (Mermaz *et al.* 1979) the peak for C is less than one-quarter that for N.

For 50 and 42 MeV ^{10}B on ^{28}Si and ^{11}B on ^{30}Si (Parks *et al.* 1979) and for 127 and 71 MeV ^{12}C on ^{12}C (Stokstad *et al.* 1979) C is found to be negligible compared with N.

Consistent with the above is that the maximum value of $\sigma(\theta)$ is greater for C than for N for 104 MeV ^{16}O on ^{208}Pb (Becchetti *et al.* 1972), 70 MeV ^{12}C on ^{148}Nd and ^{144}Nd (Hillis *et al.* 1977), and 56 MeV ^{16}O on ^{74}Ge (Cobern *et al.* 1976).

As a result of absorption, the value of $|S(l)|$ falls from its maximum to a small value as l decreases, at about the same rate for N and C. For reasons which will become clearer later, $\sigma(\theta)$ decreases with increasing θ at a slightly slower rate for N than for C, and if the experimental values of $\sigma(\theta)$ for only the largest values of θ are taken, these can be used to provide information about nuclear excitation.

3. Nuclear Excitation

The analysis for nuclear excitation follows that for transfer reactions (Mohr 1980). The same form is taken for $S(l)$ with width parameter Δ. We make the same approximations: that the term $2l+1$ in the partial wave series has the constant value $2l_c+1$, and that the difference between successive Coulomb phases is $\arctan(n/l_c)$ instead of the exact value $\arctan(n/l)$. Then we have $\sin^{\frac{1}{2}}\theta\, f^{\pm}(\theta) \propto \exp(-\Delta\,|\,\theta \pm \theta_c\,|)$.

For the larger θ_c, f^+ is small compared with f^-, which is peaked at $\theta = \theta_c$. For θ_c small (higher energies), f^+ is larger and produces interference maxima and minima in $f = f^+ + f^-$. Proceeding as for transfer reactions we may obtain the value of Δ. The error in the value of Δ involved in taking the form of the total inelastic cross section to be the same as the cross section for nuclear excitation at the largest angles is no more than 20%.

4. Coulomb Excitation

We first consider the case of $Z_1 Z_2$ large, when θ_c is relatively large, and $\sin^{\frac{1}{2}}\theta\, f^-(\theta)$—like $S(l)$—is approximately symmetrical about its peak in the case of nuclear excitation. Then $S(l)$ for Coulomb excitation has a slightly broader peak, so that Δ is slightly larger, and therefore $\sin^{\frac{1}{2}}\theta\, f^-(\theta)$ falls off slightly more rapidly at larger θ. However, at much larger l, $S(l)$ falls off much more slowly, since the Coulomb field falls off slowly at larger distances, greatly increasing the scattering at smaller angles. The approximations used in Section 3 can no longer be used, but $\sin^{\frac{1}{2}}\theta\, f^-(\theta)$ is still the sum of the partial wave series with coefficients $l^{\frac{1}{2}} S(l)$, and there is a general relation between the forms of these functions of θ and l. These forms are shown in Fig. 1 for 56 MeV ^{16}O on ^{74}Ge (Cobern *et al.* 1976), and a general explanation of the relation between them will be given in Section 5. We note here that the sharp cutoff in $l^{\frac{1}{2}} S(l)$ just beyond the final drop at $l = l_c'$ is responsible for the drop in $\sin^{\frac{1}{2}}\theta\, f^-(\theta)$ as θ falls below θ_c'. There is a fairly flat portion of the curve, with some small oscillations, between $\theta = \theta_c'$ and $\theta = \theta_c$. These features appear also in Becchetti *et al.* (1972), but in Hillis *et al.* (1977) and in most other cases the experiments and calculation have not been carried out down to sufficiently small angles to obtain the fall below $\theta = \theta_c'$.

We now consider what happens as $Z_1 Z_2$ increases and θ_c decreases. Then for nuclear excitation, as for transfer collisions, the bell-shaped peak for $\sin^{\frac{1}{2}}\theta\, f^-(\theta)$

becomes more and more asymmetrical, finally moving in to such small angles that only the fall above $\theta = \theta_c$ is observed, with perhaps only a suggestion of the fall below $\theta = \theta_c'$. The latter fall may not be distinguishable from the fall below $\theta = \theta_c'$ for Coulomb excitation, which gets less important for small $Z_1 Z_2$. At the same time $\sin^{\frac{1}{2}}\theta f^+(\theta)$ becomes important and produces interference maxima and minima in $\sin^{\frac{1}{2}}\theta f(\theta)$.

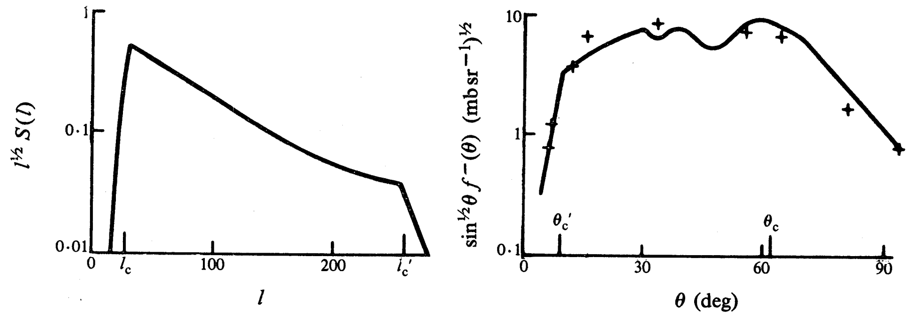

Fig. 1. Graphs of the related quantities $l^{\frac{1}{2}} S(l)$ and $\sin^{\frac{1}{2}}\theta f^-(\theta)$ as functions of l and θ respectively, for Coulomb excitation of ^{74}Ge by 56 MeV ^{16}O (see Section 5).

5. Graphical Analysis

In our previous papers (Mohr 1979, 1980), much insight was provided by the amplitude–phase diagram and we now extend its use to Coulomb excitation.

First, however, we discuss the picture for nuclear excitations. For $S(l) = 1$, the diagram for $f^-(\theta)$ resembles a Cornu double spiral, which first turns around O ($l = 0$) and finally about O′ ($l = \infty$). As θ increases the spiral about O unwinds while the spiral about O′ winds up. Values of $S(l)$ differ appreciably from zero for only a small range of values of l, corresponding to a short section of spiral. The vector length of this section is a maximum (maximum $f^-(\theta)$) when the section is nearly straight, i.e. when about mid-way between O and O′ and then $\theta = \theta_c$. For smaller θ the section is curved, lying on the O′ spiral, for larger θ curved in the opposite sense, lying on the O spiral; and in both cases the vector length of the section is less than for $\theta = \theta_c$, so $|f(\theta)|^2$ is approximately symmetric about $\theta = \theta_c$ for a small l window.

For Coulomb excitation $S(l)$ is appreciably different from zero for a large range of values of l, and the flattish part of the curve of $\sin^{\frac{1}{2}}\theta f^-(\theta)$ corresponds to a long section of the double spiral extending over the straight part and part of both spirals. As θ increases above θ_c the section moves entirely onto the O′ spiral, and as θ decreases below θ_c' the section moves entirely onto the O spiral; and thus is explained the fall in $\sin^{\frac{1}{2}}\theta f^-(\theta)$ for $\theta > \theta_c$ and $\theta < \theta_c'$. Now $S(l_c) > S(l_c')$, but this is compensated for by the O spiral being larger than the O′ spiral (remember that the factor $l^{\frac{1}{2}}$ is larger for the series terms involved for $\theta = \theta_c'$ than the terms for $\theta = \theta_c$). The curve of $\sin^{\frac{1}{2}}\theta f^-(\theta)$ is thus fairly flat between θ_c' and θ_c as shown in Fig. 1 by the points indicated by the crosses, calculated numerically from the curve shown for $l^{\frac{1}{2}} S(l)$, using our diagram model as a guide.

The experimental and theoretical curves of $\sigma(\theta)$ versus θ have weak maxima and minima corresponding to the small oscillations shown in Fig. 1 for $\sin^{\frac{1}{2}}\theta f^-(\theta)$, these being largest near $\theta = \theta_c$ because the value of $l^{\frac{1}{2}} S(l)$ is larger at $l = l_c$ than at $l = l'_c$, so providing a 'sharper' edge for diffraction by the l window.

This near symmetry of $\sin^{\frac{1}{2}}\theta f^{-}(\theta)$ about $\theta = \frac{1}{2}(\theta'_c + \theta_c)$ for larger $Z_1 Z_2$ which derives from the near symmetry of the amplitude–phase diagrams for $\theta = \theta'_c$ and $\theta = \theta_c$ about the mid-point of OO′ suggests that there corresponds to the approximate relation $\theta_c = 2\arctan(n/l_c)$ the further relation $\theta'_c = 2\arctan(n/l'_c)$. Since l'_c is large we have $l'_c \approx kR'$, thus allowing us to deduce the cutoff radius of the Coulomb interaction R' from the experimental value of the cutoff angle θ'_c.

6. Analysis of Angular Distributions for Nuclear Excitation

Heavy Nuclei Incident

Values of Δ obtained by the analysis are shown in Fig. 2, plotted as a function of $k - k_B$, where k_B is the value of k at the barrier energy.

The data points are arranged in groups for different incident nuclei. The squares are for ^{16}O and ^{18}O incident. The four values for very small $k - k_B$ are, in order of decreasing Δ, for target nuclei ^{74}Ge (Cobern *et al.* 1976), ^{120}Sn (Rehm *et al.* 1975), ^{28}Si and ^{24}Mg (Carter *et al.* 1978). The remaining six values are, in order of increasing $k - k_B$, for ^{92}Zr and ^{52}Cr (Essel *et al.* 1979), ^{48}Ca (Kovar *et al.* 1978), ^{28}Si (Mermaz *et al.* 1979), ^{120}Sn at 100 and 160 MeV (Glendenning 1975). The last two values are from theoretical angular distributions calculated for an optical model potential.

The four triangles are for 34, 42 and 50 MeV ^{10}B and ^{11}B on ^{28}Si (Parks *et al.* 1979) and 52 MeV ^{11}B on ^{40}Ca (Hnizdo *et al.* 1981).

The six open circles are, in order of increasing $k - k_B$, for ^{6}Li incident on ^{44}Ca and ^{40}Ca (Bohn *et al.* 1977), ^{26}Mg (Woods *et al.* 1980), ^{58}Ni for several 2^+ levels with almost the same Δ (Williamson *et al.* 1980), ^{60}Ni and ^{26,24}Mg (mean Δ) (Fulmer *et al.* 1981).

The five crosses are for ^{12}C on ^{12}C at five different energies (Stokstad *et al.* 1979).

Protons Incident

The ten dots for smallest $k - k_B$ are, in order of increasing $k - k_B$, for protons on ^{112}Cd (Stelson *et al.* 1968), ^{64,62}Ni (Dickens *et al.* 1963), Cd, Sn, Te (Makofske *et al.* 1968), ^{198,196,194}Pt (Deason *et al.* 1981), ^{56,54}Te, ^{58}Ni (Karban *et al.* 1971), ^{48}Ca at 35 MeV (Gruhn *et al.* 1972), ^{48}Ca at 40 MeV (Gruhn *et al.* 1972), ^{24}Mg (Zwieglinski *et al.* 1978), ^{42}Ca (Mani and Jacques 1971), and ^{58}Ni (Ingemarsson *et al.* 1979). Where isotopic or similar nuclei are grouped, the values of Δ are very close and the mean value is taken.

There remain the five dots for largest $k - k_B$. These are for groups of values for which the mean $k - k_B$ and Δ is taken. The dot at $\Delta = 7 \cdot 9$ is for 800 MeV protons on ^{12}C (Ray *et al.* 1978; Haji-Saeid *et al.* 1980) and for ^{24}Mg (Blanpied *et al.* 1979). The dot at $\Delta = 8 \cdot 2$ is for 800 MeV protons on five nuclei from ^{40}Ca to ^{90}Zr (Digiacomo *et al.* 1979; Ray *et al.* 1979; Adams *et al.* 1980). The dot at $\Delta = 10 \cdot 1$ is for 800 MeV protons on deformed nuclei ^{154}Sm and ^{176}Yb (Barlett *et al.* 1980). The dot at $\Delta = 9 \cdot 9$ is for 1040 MeV protons on ^{12}C (Bertini *et al.* 1973). The dot at $\Delta = 10 \cdot 0$ is for 1040 MeV protons on ^{58}Ni (Bertini *et al.* 1973), ^{44}Ca, ^{48}Ca and ^{48}Ti (Alkhazov *et al.* 1976). The slight difference in the values of $k - k_B$ is due to the effect of recoil in the collision.

All but one of these high energy points correspond to angular distributions with marked maxima and minima, the depth of the minima usually being less for the experimental points used than for the optical model curves fitted to them. The point for 800 MeV protons on ^{12}C is for excitation to the 15·1 MeV state which is a 1^+ state, the angular distribution having no maxima or minima. Otherwise all the data used in this paper are for excitation to a 2^+ state, usually the lowest.

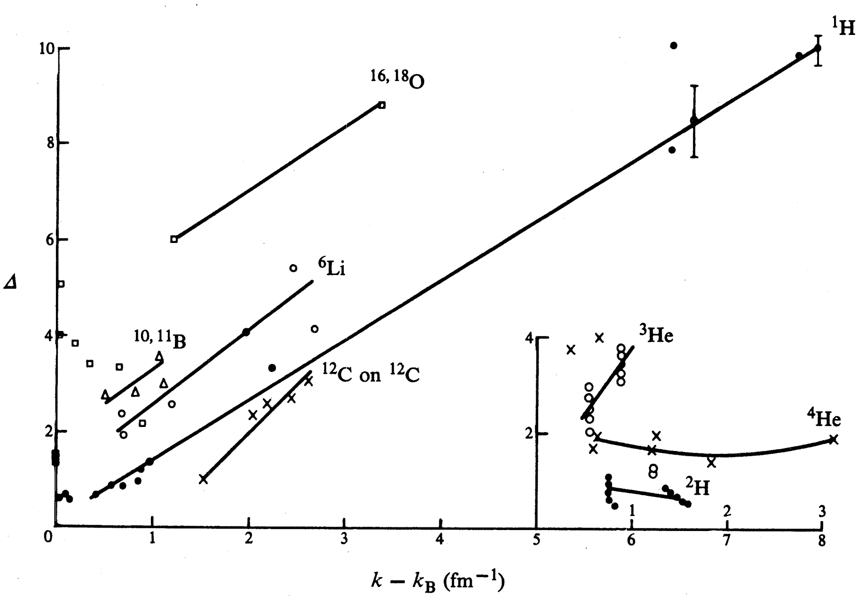

Fig. 2. Values for the width parameter Δ of the S matrix for nuclear excitation for different values of $k-k_B$. The lines link values for the same pair of nuclei at different energies (see Section 6).

The values of Δ for the lowest values of $k-k_B$ for ^{18}O and ^{16}O incident (squares in Fig. 2) show considerable variation with target nucleus, as happened also for transfer reactions, and is due partly to the inaccuracy in our calculated value of k_B for energies near the top of the barrier. From these points no useful conclusion can be drawn. Through the remaining points straight lines may be drawn as shown, giving approximately the same value for the slope of each line. The conclusion to be drawn will be discussed in Section 8.

^{2}H, ^{3}He and ^{4}He Incident

The data points for these nuclei are shown in the inset Fig. 2. The first group of five dots is for ^{2}H on ^{24}Mg (Nelson and Roberson 1972), ^{26}Mg, ^{56}Fe, ^{58}Ni and ^{65}Zn (Tjin *et al.* 1968). The group of dots at higher energy represents a selection of nuclei from ^{12}C to ^{64}Ni (Hinterberger *et al.* 1968).

The open circles, in order of increasing $k-k_B$, are for ^{3}He on five isotopes of Sm (Pálla and Pegel 1979), the same at a higher energy (Eagle *et al.* 1977), and on ^{28}Si and ^{30}Si (Fulmer *et al.* 1981).

The crosses, in order of increasing $k-k_B$, are for ^{4}He on ^{154}Sm (Hendrie *et al.* 1968), ^{208}Pb (Rutledge and Hiebert 1976), ^{58}Ni (Buck 1962), ^{40}Ca (Rutledge and Hiebert 1976), ^{58}Ni (Darriulat *et al.* 1963) and ^{20}Ne (Specht *et al.* 1970).

In all cases there is a marked increase in Δ with Z for the target nucleus, and this makes it impossible to draw lines through the points with any certainty, especially as the energy range is so restricted. We note also that the values for Δ for excitation to the 4^+ and 6^+ levels in ^{154}Sm (not shown in the figure) are progressively less than for the 2^+ level. These complicating effects may be due to the formation, near the barrier energy, of compound nuclear structures during the collision.

7. Analysis of Angular Distributions for Coulomb Excitation

From Section 5 we have the approximate relation $l_c'/l_c \approx \theta_c/\theta_c'$. This is satisfied approximately by the calculations of Cobern *et al.* (1976) for $l_c' = 290$, 250 and 140, if we take θ_c' to be the corresponding angle at which $\sigma(\theta)$ starts to fall from a maximum to small values. The value of l_c'/l_c is about 7, the largest ratio we have found in this work. Smaller values ranging down to about 3 are to be found in the work of Kovar *et al.* (1978), Mermaz *et al.* (1979), Eck *et al.* (1981), Hillis *et al.* (1977) and Glendenning (1975). As $Z_1 Z_2$ decreases, Coulomb excitation becomes less important and θ_c' moves closer to θ_c.

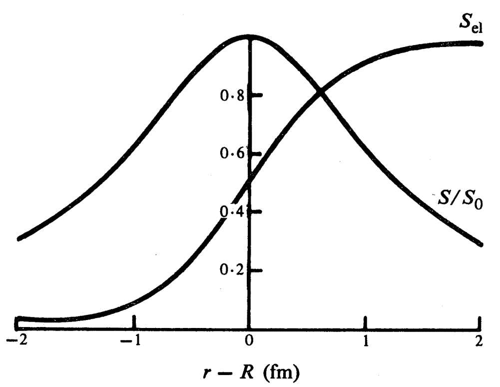

Fig. 3. Curves of S_{el} for elastic collisions, and S/S_0 for inelastic collisions involving nuclear excitation, as a function of $r-R$ (see Section 8).

8. Summary

The slope of the lines in Fig. 2 is about 1·3 fm. This is about the same as the average value for transfer reactions, where individual lines show slightly greater variation in slope; and it is to be compared with the slope of about 0·4 fm for elastic collisions (Mohr 1979) for which the form for S is different.

To summarize our results so far:

$$\text{for elastic collisions} \qquad S_{el} = \{1+\exp(-\lambda/\Delta_{WS})\}^{-1},$$

$$\text{for inelastic and transfer collisions} \quad S/S_0 = (1+\lambda^2/\Delta^2)^{-1},$$

where $\lambda/\Delta = (l-l_c)/\Delta = (r-R)(\Delta/k)^{-1}$, and similarly with Δ_{WS} in place of Δ.

This replacement of l by kr and l_c by kR is justified only for large l and l_c, when the colliding nuclei behave like particles. For small l and l_c, however, the nuclei behave like waves, requiring more complicated considerations of the collision process, and more careful calculations than ours for the effect of the Coulomb barrier, especially for large $Z_1 Z_2$ when the lines in Fig. 2 do not pass through the origin, as they would in an ideal plot.

If, therefore, we put $\Delta_{WS}/k = 0 \cdot 4$ fm and $\Delta/k = 1 \cdot 3$ fm in the above relations, we may plot S_{el} and S/S_0 as a function of $r-R$, giving the curves in Fig. 3. We note the greater penetration below the nuclear surface at $r = R$ for inelastic and transfer collisions.

The many experimental angular distributions of inelastic scattering referred to above were fitted by their authors with DWBA calculations with an optical model using values for the diffuseness a_{Re} ranging from 0·4–0·8 fm and a_{Im} ranging from 0·2–1·0 fm. This kind of wide range was found even for the same pair of colliding nuclei, and so can have no physical justification. Our approach avoids this large range of parameters, but to obtain more accurate values of the slopes of the lines for Δ will require more systematic experiments over a wide range of energies for each pair of colliding nuclei.

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