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Interpretation of the Size Spectrum of Cosmic γ-ray Bursts in terms of an Idealized Galactic Distribution of Sources

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Abstract

A model for the size spectrum of cosmic γ-ray bursts is derived on the assumption that the sources of these bursts are distributed uniformly in space out to 0·3 kpc, thence uniformly in a disc (galactic plane) to 3 kpc, and thence uniformly in a line (galactic arm) to 27 kpc. Two forms are assumed for the distribution of the total energy release per burst: a gaussian with mean $\bar{s}$ and an exponential with characteristic s_0, such that $\bar{s} = s_0 = 10^{32}$ J. The derived size spectra are in agreement with results obtained from satellite and balloon observations, thus supporting both the galactic origin of the bursts and a representative total energy release for them of 10^{32} J.

1. Introduction

The cosmic γ-ray bursts discovered by the Vela satellites are a very interesting high-energy astronomical phenomenon. To date about 60 such bursts have been recorded in the energy range from a few keV to 1 MeV by the Vela satellites, the IMP-7, etc. (Strong *et al.* 1974; Cline and Desai 1976). The origin and emission mechanism of the bursts have been investigated by several authors (Lamb *et al.* 1973; Stecker and Frost 1973; Brecher and Morrison 1974; Colgate 1974; Pacini and Ruderman 1974; Woosley and Taam 1976) but these features are not well understood because, owing to positional uncertainties, individual bursts have yet to be identified with any known astronomical object. Thus intrinsic source luminosities are unavailable because source distances are unknown. This has prevented us from gaining a proper understanding of the phenomenon since intrinsic luminosities would provide an important clue to the nature of the emission mechanism, and source distances would resolve whether the bursts are galactic or extragalactic in origin.

In the absence of known identifications, an alternative means of gauging both source distances and intrinsic luminosities is provided by fitting a theoretical size spectrum (based on a joint spatial–luminosity distribution model) to the known size spectrum revealed by the available observations. The size spectrum of the γ-ray bursts is the cumulative distribution as a function of the observed energy density, or 'size', S of the number $N(S)$ of bursts per year having sizes greater than S. Study of the size spectrum of γ-ray burst sources is a potentially powerful means of investigating the nature and origin of these sources. For example, if the sources are distributed uniformly throughout space, the size spectrum follows the well-known $-1\cdot5$ power law: $N(S) \propto S^{-\alpha}$, where $\alpha = 1\cdot5$. For sources confined to an infinite thin sheet, approximating the shape of the galactic disc, we have a power law with

$\alpha = 1$. Finally, for sources strung out along an infinite line, approximating the shape of the galactic arm, we have $\alpha = 0{\cdot}5$.

The region of the size spectrum most sensitive to model fitting is the range $S < 10^{-8}\,\mathrm{J\,m^{-2}}$ (equivalent to $10^{-5}\,\mathrm{erg\,cm^{-2}}$), where few bursts have been observed owing to their low detection sensitivity. The bursts recorded by the Vela satellites mainly fall above this range. However, a few bursts with $S < 10^{-9}\,\mathrm{J\,m^{-2}}$ have been obtained by balloon-borne large-area detectors: Bewick *et al.* (1975) detected an *unconfirmed* burst of $1{\cdot}2\times10^{-10}\,\mathrm{J\,m^{-2}}$, while Nishimura *et al.* (1977) detected a confirmed burst of $6\times10^{-9}\,\mathrm{J\,m^{-2}}$. In addition, Herzo *et al.* (1976), Carter *et al.* (1976), Johnson *et al.* (1976) and Cline *et al.* (1977) have established upper limits to the annual frequency of bursts in the size range 10^{-10}–$10^{-11}\,\mathrm{J\,m^{-2}}$.

2. Origin of Bursts

There are three indications which support a galactic origin for the γ-ray bursts:

(1) The galactic latitude distribution of 16 γ-ray bursts discovered by the Vela satellites was found by Strong and Klebesadel (1974) to deviate significantly from that expected on the basis of an isotropic spatial distribution, suggesting that these sources concentrate near the galactic equator, although the number of bursts whose positions have been determined is too small for this to be conclusive.

(2) The two γ-ray bursts, events 71–2 and 72–2, each had one of its two possible source directions very close to that of Cyg X-1. Since Cyg X-1 is a powerful X-ray source and is believed to be a strong candidate for a black hole, it cannot be excluded as a possible source of γ-ray bursts. That one of the two bursts was observed in the middle of March 1971, when Cyg X-1 drastically changed its X-ray intensity, makes Cyg X-1 even more promising as a candidate for a source of γ-ray bursts.

(3) The X-ray bursts recently discovered by SAS-3 seem to be a high-energy astronomical phenomenon similar to the γ-ray bursts, although showing some differences with regard to their recurrence and their energy spectra (Lewin *et al.* 1977). Some of the sources of these X-ray bursts have been identified with galactic X-ray sources, and total energy releases for these sources have been estimated to be about 10^{31}–10^{32} J. The characteristics of the X-ray bursts have been gradually clarified, and these bursts are now interpreted in terms of a plasma turbulence in the magnetosphere of a neutron star. In addition, the hard X-ray bursts observed by Cosmos 428 possessed a time structure similar to that of the γ-ray bursts. Babushkina *et al.* (1975) noted that the location of the hard X-ray bursts in the celestial sphere coincided with that of the strong discrete sources of hard X-rays.

In view of the above indications we assume that the γ-ray burst sources are galactic in origin. Accordingly we propose a model spatial distribution for them which follows an idealized stellar distribution throughout the Galaxy, and a total energy release for them of 10^{32} J, which is well within the range of energies associated with stellar explosions, such as X-ray stars, flaring stars and X-ray burst stars. The proposed spatial distribution of the sources is uniform out to $0{\cdot}3$ kpc, which is the average thickness of the galactic disc, thence disc-like to 3 kpc, which is the distance to the galactic anticentre, and linear beyond 3 kpc. The distribution of total energy release per source is proposed to be either gaussian with mean $\bar{s}$ or exponential with characteristic s_0, where $\bar{s} = s_0 = 10^{32}$ J. The galactic origin model for γ-ray bursts is discussed from various points of view below.

3. Derivation of Model Size Spectrum

The size spectrum is derived here on the basis of the model proposed in Section 2. The distribution of total energy release s per burst is assumed to be either gaussian:

$$G(s) = \rho g(s) = \rho(k/\pi)^{\frac{1}{2}}\exp\{-k(s-\bar{s})^2\}, \tag{1}$$

or exponential:

$$E(s) = \rho\, e(s) = \rho\, s_0^{-1}\exp(-s/s_0), \tag{2}$$

where k is a parameter defining the sharpness of the gaussian distribution and ρ is a normalization constant chosen such that

$$\int_0^\infty G(s)\,\mathrm{d}s = \int_0^\infty E(s)\,\mathrm{d}s = \rho,$$

or

$$\int_0^\infty g(s)\,\mathrm{d}s = \int_0^\infty e(s)\,\mathrm{d}s = 1.$$

Thus, on the basis of the proposed spatial distribution and the luminosity distributions (1) and (2) we obtain the following two forms for the size spectrum:

$$N_{\mathrm{G}}(S) = \int_0^{R_1}\int_{4\pi r^2 S}^\infty \rho_1 4\pi r^2 g(s)\,\mathrm{d}s\,\mathrm{d}r + \int_{R_1}^{R_2}\int_{4\pi r^2 S}^\infty \rho_2 2\pi r\, g(s)\,\mathrm{d}s\,\mathrm{d}r + \int_{R_2}^{R_3}\int_{4\pi r^2 S}^\infty \rho_3\, g(s)\,\mathrm{d}s\,\mathrm{d}r \tag{3}$$

and

$$N_{\mathrm{E}}(S) = \int_0^{R_1}\int_{4\pi r^2 S}^\infty \rho_1' 4\pi r^2 e(s)\,\mathrm{d}s\,\mathrm{d}r + \int_{R_1}^{R_2}\int_{4\pi r^2 S}^\infty \rho_2' 2\pi r\, e(s)\,\mathrm{d}s\,\mathrm{d}r + \int_{R_2}^{R_3}\int_{4\pi r^2 S}^\infty \rho_3'\, e(s)\,\mathrm{d}s\,\mathrm{d}r. \tag{4}$$

In equations (3) and (4), the first term is the contribution from a uniform spatial distribution of volume density ρ_1 or ρ_1' out to $R_1 = 0{\cdot}3$ kpc, the second term is the contribution from a disc-like spatial distribution of surface density ρ_2 or ρ_2' between R_1 and $R_2 = 3$ kpc, while the third term is the contribution from a linear distribution of line density ρ_3 or ρ_3' between R_2 and $R_3 = 27$ kpc. The ratios $\rho_1 : \rho_2$ and $\rho_2 : \rho_3$ are determined from the condition that the three regions join smoothly, i.e. the former is determined by the condition that at $S = \bar{s}/4\pi R_1^2$ we have

$$\int_0^{R_1}\int_{4\pi r^2 S}^\infty \rho_1 4\pi r^2 g(s)\,\mathrm{d}s\,\mathrm{d}r = \int_{R_1}^{R_2}\int_{4\pi r^2 S}^\infty \rho_2 2\pi r\, g(s)\,\mathrm{d}s\,\mathrm{d}r \tag{5}$$

and the latter is determined by the condition that at $S = \bar{s}/4\pi R_2^2$ we have

$$\int_{R_1}^{R_2}\int_{4\pi r^2 S}^\infty \rho_2 2\pi r\, g(s)\,\mathrm{d}s\,\mathrm{d}r = \int_{R_2}^{R_3}\int_{4\pi r^2 S}^\infty \rho_3\, g(s)\,\mathrm{d}s\,\mathrm{d}r. \tag{6}$$

Similar conditions are used to determine the ratios $\rho_1' : \rho_2'$ and $\rho_2' : \rho_3'$.

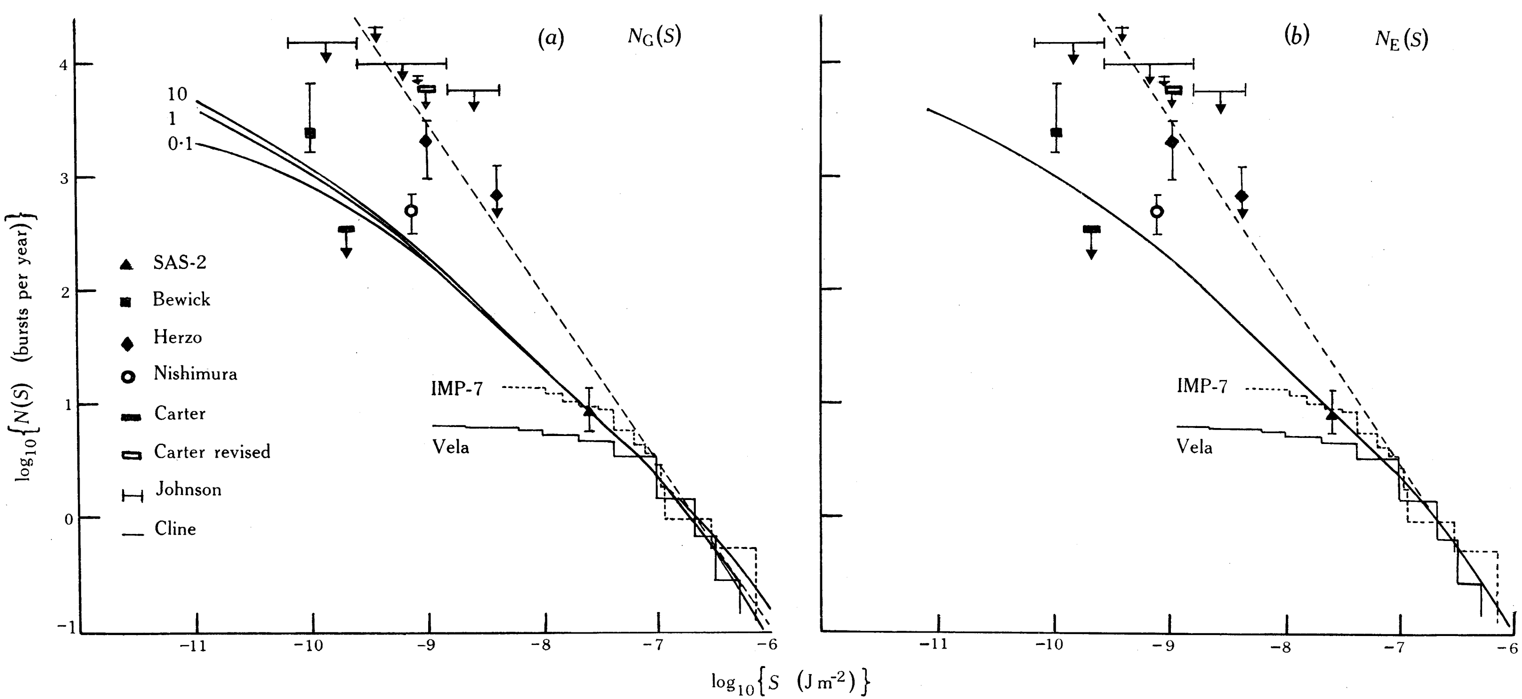

Fig. 1. Comparison with experimental results (points or stepped curves), experimental upper limits (bars) and the $S^{-1\cdot5}$ law spectrum (dashed straight line) of the model size spectra:

(*a*) $N_G(S)$, solid curves evaluated from equation (3) and parameterized according to the indicated values of $k\bar{s}^2 = 10$, 1 or 0·1;

(*b*) $N_E(S)$, solid curve evaluated from equation (4).

The experimental data are from SAS-2, Bewick *et al.* (1975), Herzo *et al.* (1976), Nishimura *et al.* (1977), Carter *et al.* (1976), Carter *et al.* as revised by Cline and Schmidt (1977), Johnson *et al.* (1976) and Cline *et al.* (1977).

Expressions (3) and (4) may now be evaluated using Galton's approximation

$$(2\pi)^{-\frac{1}{2}} \int_0^c \exp(-\tfrac{1}{2}x^2)\,\mathrm{d}x \approx \tfrac{1}{2}\{1-\exp(-2c^2/\pi)\}^{\frac{1}{2}}$$

and putting $\bar{s} = s_0 = 10^{32}$ J, and $k = 10\,\bar{s}^{-2}$, $1\cdot0\,\bar{s}^{-2}$ and $0\cdot1\,\bar{s}^{-2}$ in equation (3). On then normalizing both $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$ to the burst frequency of 2·3 bursts per year obtained by the Vela satellites for $S = 10^{-7}\,\mathrm{J\,m^{-2}}$, we obtain the three solid curves in Fig. 1*a* (parameterized as $k = 10\,\bar{s}^{-2}$, $1\cdot0\,\bar{s}^{-2}$ and $0\cdot1\,\bar{s}^{-2}$) for $N_\mathrm{G}(S)$ and the single solid curve in Fig. 1*b* for $N_\mathrm{E}(S)$. These curves are compared with experimental data obtained from observations by the Vela satellites, the IMP-7 and balloons, as well as with the $-1\cdot5$ power law.

4. Discussion

As shown in Figs 1*a* and 1*b*, $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$ are remarkably similar to one another. In addition, $N_\mathrm{G}(S)$ is fairly insensitive to variations in k over the range $0\cdot1\,\bar{s}^{-2}$ to $10\,\bar{s}^{-2}$, thereby being largely independent of the sharpness or broadness of the assumed luminosity distribution. It is thus apparent that the theoretical size spectrum is relatively unaffected by distributional assumptions concerning the intrinsic luminosity of bursts.

On the other hand, the theoretical size spectrum is much more sensitive to assumptions concerning the spatial distribution of sources. As can be seen in the figures, both $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$ follow the $S^{-1\cdot5}$ law between $S = 10^{-5}$ and $10^{-7}\,\mathrm{J\,m^{-2}}$, the $S^{-1\cdot0}$ law between $S = 10^{-7}$ and $10^{-9}\,\mathrm{J\,m^{-2}}$, and the $S^{-0\cdot5}$ law between $S = 10^{-9}$ and $10^{-11}\,\mathrm{J\,m^{-2}}$. This behaviour is constrained by our galactic distribution model: the $S^{-1\cdot5}$ spectrum being due to the uniform source distribution to 0·3 kpc, the $S^{-1\cdot0}$ spectrum being due to the disc-like distribution to 3 kpc and the $S^{-0\cdot5}$ spectrum being due to the linear distribution beyond 3 kpc. The theoretical size spectrum thus not only indicates a galactic structure within the γ-ray burst pattern, but also directly calibrates the distribution of intrinsic luminosity.

From Fig. 1 it can be seen that $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$ are in agreement with the results obtained by the Vela and IMP-7 satellites above about $10^{-7}\,\mathrm{J\,m^{-2}}$ and by Bewick *et al.* (1975) and Nishimura *et al.* (1977) below $10^{-9}\,\mathrm{J\,m^{-2}}$. In addition, $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$ are not inconsistent with the upper limits obtained by Carter *et al.* (1976), Herzo *et al.* (1976), Johnson *et al.* (1976) and Cline *et al.* (1977). On the other hand, it seems fairly evident that an $S^{-1\cdot5}$ law spectrum is incompatible with the experimental data so far obtained below $10^{-7}\,\mathrm{J\,m^{-2}}$, where there is an apparent deficiency of sources relative to the numbers predicted by the $S^{-1\cdot5}$ law. Hence, the present comparison seems to demonstrate that the γ-ray bursts are not of extragalactic origin but rather originate in our Galaxy, although the statistical reliability of this conclusion is poor.

Relative to $N_\mathrm{G}(S)$ and $N_\mathrm{E}(S)$, the size spectrum obtained by the Vela satellites shows an apparent bend below $10^{-7}\,\mathrm{J\,m^{-2}}$. But this flattening out is almost certainly an artefact of the low detection sensitivity of the Vela satellites, since the size spectrum obtained by the IMP-7 satellite, which has higher sensitivity, begins flattening out for somewhat smaller bursts.

Carter *et al.* (1976), who presented an upper limit to the frequency of bursts greater than 2×10^{-10} J m^{-2}, suggested that the data were consistent with an $S^{-0\cdot5}$ law spectrum, or a modified galactic distribution. However, Cline and Schmidt (1977) pointed out that their upper limit needed to be revised. In fact, Cline *et al.* (1977) suggested that the existing satellite and balloon data favoured an $S^{-1\cdot5}$ law spectrum. On the other hand, Manchanda and Ramsden (1977) noted that about 90% of bursts have sizes between 5×10^{-8} and 5×10^{-7} J m^{-2} and that, assuming a uniform intrinsic luminosity for the bursts, their spatial distribution is remarkably similar to that of galactic supernovae. This again suggests that the γ-ray bursts are of galactic origin.

5. Conclusions

The size spectrum of γ-ray bursts derived here on the basis of an idealized stellar distribution is in agreement with the measured frequency of bursts obtained from satellite and balloon observations, and is not inconsistent with experimental upper limits for the frequency of bursts below 10^{-9} J m^{-2} obtained from balloon observations. The available experimental data do tend to suggest that there is a deficiency of smaller bursts relative to that predicted by the $S^{-1\cdot5}$ law spectrum, thereby supporting a galactic origin for the bursts. The present analysis suggests that the experimental size spectrum so far obtained can be explained on the assumption of a galactic distribution of sources corresponding to that of stars of a known class and that the total energy release per burst is of the order of 10^{32} J.

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