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A Discussion of Laser Conditions and the Use of Polarized Nuclei in Grasers

H. Hora, G. V. H. Wilson and E. P. George

Department of Theoretical Physics, School of Physics,
University of New South Wales, Kensington, N.S.W. 2033.

Abstract

A high frequency limitation is derived for γ-ray lasers (grasers) from the conditions for the laser threshold. At low temperatures, the use of polarized nuclei should facilitate the production and observation of graser action using '*Q* switching' by magnetic rotation of the polarization axis.

Introduction

Stimulated emission of photons from excited states of nuclei to achieve γ-ray lasers or 'grasers' has been discussed by several authors (Vali and Vali 1963; Baldwin and Khokhlov 1975). One problem is the difficulty of realizing an optical cavity since, at high photon energies, reflectors (even grazing angle units) are difficult to realize. One proposed solution involved travelling wave pumping by laser compression (Baldwin and Khokhlov 1975) to achieve 10^5 times the solid state density (Hora 1975, 1976) at which the subsequent flux density of $\gtrsim 10^{26}$ α-particles $cm^{-2}\,s^{-1}$ from $H\,^{11}B$ reactions could be used for graser excitation (Hora 1974). The present paper discusses a new scheme aimed at achieving highly directed emission of grasers with low beam divergences, and also discusses controlled switching of the graser action. The scheme is based upon the anisotropic emission of γ-ray radiation from oriented nuclei.

In considering the properties of a graser incorporating oriented nuclei and magnetic switching we first discuss the question of a high frequency limit. Baldwin (1974) has pointed out that one limit is for photon energies to be $\lesssim 100$ MeV because of scattering by the Compton effect. Another limit to graser operation is associated with the inhomogeneous broadening of Mössbauer transitions. This has the effect of reducing the resonant cross section below the value required for sufficient graser amplification, and presently applies to nuclear transitions from states with half lives $\gtrsim 10^{-5}$ s. Baldwin and Khokhlov (1975) summarized the various broadening mechanisms. Some of these will vanish at low temperatures; others may be reduced by improvements in sample preparation and by the possible use of n.m.r. motional narrowing techniques.

High Frequency Limit

The criterion of the high frequency limit for grasers (or lasers generally) is based on the classical relation for the laser threshold under the condition of feasible densities of inverted nuclei. The relation for the threshold density n_2 of inverted states for a laser with a Lorentzian line profile is (Hora 1964, 1965*a*, 1965*b*)

$$n_2 > 4\pi^2 \Delta\nu\, \tau_n (1-R)\tilde{n}^2/\lambda_0^2 l, \tag{1}$$

where $\Delta\nu$ is the resonance linewidth of the emitting line, τ_n the spontaneous lifetime of the laser line, $1-R$ is the loss of the cavity by transmission through the end mirrors, $\tilde{n}$ is the refractive index of the active medium, λ_0 is the laser wavelength and l is the length of the active medium (assuming no further losses within the cavity). The use of the Lorentzian profile may be justified for grasers, since the line broadening will occur by radiation damping or temperature broadening in the crystal, as in the analogous case of solid state lasers. How useful the formulation (1) is can be seen from the fact that, for the laser action of homogeneous semiconductors excited by electron beams, the predicted current densities and necessary hole densities correspond very closely with the measured values (Hora 1964, 1965*a*, 1965*b*).

The fact that the resonance linewidth $\Delta\nu$ is larger than its minimum value $\Delta\nu^{\min}$,

$$\Delta\nu \gtrsim \Delta\nu^{\min} = 1/\tau_n, \tag{2}$$

is well known for normal lasers and has been discussed by Baldwin (1974) for grasers. Assuming that $\Delta\nu^{\min}$ can be used in the inequality (1), that in the case of γ-ray lasers there are no reflectors ($R = 0$) and that $l = 1$ cm and $\tilde{n} = 1$, then we can derive a minimum value of the laser wavelength $\lambda_0^{\min}$ from the condition (1) as

$$\lambda_0 \gtrsim \lambda_0^{\min} = (4\pi^2 \Delta\nu\, \tau_n \tilde{n}^2/l n_2)^{\frac{1}{2}} = 2\pi n_2^{-\frac{1}{2}} \;\; \mathrm{cm}. \tag{3}$$

For laboratory experiments we can assume that the density of inverted nuclei (or atoms) cannot be higher than the solid state density, given approximately by $n_2^{\max} = 10^{22}\,\mathrm{cm}^{-3}$. The possibility of terrestrial compression during very short times by lasers or by nuclear reactions (Hora 1975, 1976) can be excluded from the further considerations, as matter in this state will have a temperature far in excess of the low temperatures required for the nuclear polarization techniques suggested here. Using $n_2^{\max}$ in the relation (3) then results in $\lambda_0^{\min}$ and a corresponding maximum photon energy $\varepsilon^{\max} = hc/\lambda_0^{\min}$ given by

$$\lambda_0^{\min} = 6\cdot 28 \times 10^{-11} \;\; \mathrm{cm}, \qquad \varepsilon^{\max} = 1\cdot 98 \;\; \mathrm{MeV}. \tag{4}$$

Threshold for Graser Action

In the following graser model no reflecting cavity is assumed. Instead a travelling wave mechanism associated with the use of magnetic switching to produce an increased effective concentration of active nuclei will be used. Instead of using the threshold condition (1) of Schawlow and Townes (1958) we have the condition

$$|\alpha_{\mathrm{s.e.}}| > |\alpha_{\mathrm{abs}}|, \tag{5}$$

where $\alpha_{\mathrm{s.e.}}$ is the negative absorption constant for the stimulated emission and α_{abs} is the absorption constant for the laser due to mechanisms which are not related to transitions of the lasing energy levels. In order to eliminate the effects of frequency shifts by recoil during the emissions, Mössbauer conditions will be essential, i.e. there must be a high probability of zero phonon production during the transitions.

However, because of the need to reduce resonant absorption, grasers will probably need to operate with Mössbauer transitions between two excited states rather than between an excited state and the ground state as in conventional Mössbauer spectroscopy.

The negative absorption constant for the stimulated emission is given by

$$\alpha_{\text{s.e.}} = \frac{2}{\Delta\nu}\frac{e^2}{mc}\left(n_1 - \frac{g_1}{g_2}n_2\right)f, \tag{6}$$

where $\Delta\nu$ is the linewidth of a Lorentz profile, n_1 is the density of states in the lower laser level, g_1 and g_2 are the degrees of degeneration of the laser states, e is the charge and m the mass of the electron, and c is the vacuum velocity of light. The oscillator strength of

$$f = \frac{mc}{8\pi^2 e^2}\frac{g_1}{g_2}\frac{\lambda_0}{\tau_n} \tag{7}$$

is determined by the spontaneous lifetime τ_n of the excited state, after neglecting n_1 by the choice of an excited state for the lower laser level.

From equations (6) and (7) we arrive at

$$n_2/|\alpha_{\text{abs}}| > 4\pi^2 \Delta\nu\, \tau/\lambda_0^2, \tag{8}$$

so that for photons of energy ε_ϕ it is necessary that

$$\varepsilon_\phi < \frac{hc}{2\pi}\left(\frac{n_2}{|\alpha_{\text{abs}}|\,\Delta\nu\,\tau}\right)^{\frac{1}{2}}. \tag{9}$$

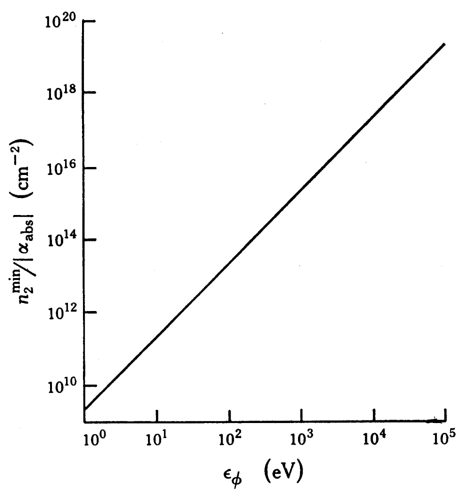

Fig. 1. Dependence upon the photon energy ε_ϕ of the ratio of the minimum density of inverted states n_2^{min} to the modulus of the absorption constant α_{abs} for the case $\Delta\nu\,\tau = 1$.

For short-lived nuclear states ($\tau \lesssim 10^{-5}$ s) it is reasonable to neglect broadening of the γ-ray line ($\Delta\nu\,\tau \approx 1$). However, for the more likely case of longer lived states it is probable that $\Delta\nu\,\tau \gg 1$. If it is assumed that $\Delta\nu\,\tau = 1$ then the absolute minimum density n_2^{min} is given by

$$n_2^{\text{min}}/|\alpha_{\text{abs}}| = 2{\cdot}56 \times 10^9\, \varepsilon_\phi^2, \tag{10}$$

with ε_ϕ in eV and $n_2^{\min}$ in cm^{-3}. This relationship is plotted in Fig. 1. Correction for values of $\Delta\nu\,\tau$ differing from unity may be made simply, since we have $n^{\min} \propto \Delta\nu\,\tau$. For various materials the value of $|\alpha_{\mathrm{abs}}|$ varies over several orders of magnitude. A very rough typical value would be $|\alpha_{\mathrm{abs}}| \approx 1$ cm^{-1}. For grasers of photon energies of $\gtrsim 10^4$ eV, remarkably high densities of inverted states are necessary. The generation of inverted isomeric nuclei of densities of $\gtrsim 10^{18}$ cm^{-3}, however, is not too optimistic. The pumping of the nuclei to short-lived states above the upper laser level might be realized by putting the material with the interacting nuclei into high flux reactors. For example, if we start with ^{113}In, neutron capture leads to ^{114}In levels with upper laser level lifetimes of 50 days. Neutron capture causes a chemical rupture of the target molecules, so that the excited nuclei may be isolated and processed to produce n_2 values up to solid state densities. Apart from ^{114}In, Baldwin (1974) has suggested more than 50 other possible isomeric nuclei having lifetimes of more than 100 s with photon energies, for transitions between the two 'laser' levels, ranging from 1 keV to 1 MeV. The values used here for the density of inverted nuclei and their dependence on the parameters included are the necessary conditions for the feasibility of the Q-switching process described in the following section.

Anisotropic γ-ray Emission from Oriented Nuclei

Orientation of nuclei may be achieved using either static or dynamic techniques. Most static techniques are based on thermal polarization of the nuclei in large magnetic fields at low temperatures. To obtain large degrees of orientation, magnetic field to temperature ratios of $\gtrsim 1000$ T K^{-1} are required. In the past, nuclear orientation research has made use almost solely of one-shot cooling by means of adiabatic demagnetization of paramagnetic salts to temperatures ~ 10 mK. However, it is now becoming routinely possible to obtain temperatures ~ 10–50 mK with greater cooling powers and continuous cooling capabilities by means of He^4/He^3 dilution refrigerators. The necessary large magnetic field may be an applied field from a superconducting solenoid, but much larger fields of order 100 T or higher may be provided by hyperfine interactions if the nuclei are incorporated in a ferromagnetic host metal. Techniques of dynamic polarization have the advantage of not requiring such low temperatures; they rely on the saturation of an appropriate electronic paramagnetic resonance, and have been summarized by Jeffries (1963, 1968) and Abragam and Borghini (1964). Very high degrees of orientation are often obtainable at liquid helium temperatures (~ 1–4 K).

The normalized angular distribution of the γ-ray radiation for a particular nuclear transition in axially oriented nuclei may be written (Blin-Stoyle and Grace 1957) as

$$W(\theta) = 1 + \sum_{\text{even } \nu} A_\nu \mathrm{P}_\nu(\cos\theta), \tag{11}$$

where the A_ν depend upon the parameters of the nuclear decay scheme and on the degree of orientation of the oriented parent nuclei. Here θ is the angle between the orientation axis and the direction of γ-ray emission. The A_ν may be either positive or negative, thus leading to a variety of radiation patterns. For highly oriented nuclei two common patterns are: a peaked emission along the axis with $W(0) \approx 2$ and an equatorial emission $W(\frac{1}{2}\pi) \approx \frac{1}{2}$; and a peaked equatorial emission with $W(\frac{1}{2}\pi) \approx 1{\cdot}25$ and $W(0) \approx 0$. While these maximum values of $W(\theta)$ do not imply a very large increase in the intensity of spontaneous emission in any direction over that for nonoriented nuclei, it should be possible to gain high degrees of directivity in grasers by the use of systems for which $W(0) \approx 0$ together with suitable sample geometries (Wilson 1977). This would involve either rod or disc shaped samples with the minor axis along the direction of minimum emission and with dimensions such that there is a large degree of graser amplification for photons emitted along the major axis.

The ability to rotate quickly the polarization axis (and hence the direction of maximum γ-ray emission) should be of very great importance in permitting the action of the graser to be switched on and off to achieve results analogous to laser Q switching. For example, if the active graser medium is in the form of a relatively thin slab or rod then, in the initial stages, the polarization direction should be such that the maximum emission is perpendicular to the slab. By switching the axis through 90°, controlled graser pulsing should be possible. One technique for achieving controlled rotations of nuclear polarization relies on the adiabatic theorem (see e.g. Abragam 1961). If the dominant orientation-dependent interaction of the nuclear spins is with a magnetic field, and this field is rotated with an angular velocity which is smaller than that for the Larmor precession, then the angle between the polarization and the field will remain constant. Using a sample of polarized ^{60}Co nuclei in iron at a temperature of $0 \cdot 02$ K, Chaplin *et al.* (1970) showed that the polarization direction may be magnetically switched in 1 s without any detectable change in the anisotropy of γ-ray emission. Another technique which permits much faster rotations is the precession of nuclei in the Larmor frame during pulsed n.m.r. (Abragam). Recently Foster *et al.* (1977) have demonstrated the rotation of γ-ray emission from ^{60}Co in iron using $0 \cdot 8$ kW resonant RF pulses and achieving switching times for 90° rotations as short as $0 \cdot 3$ μs. The two techniques described above for magnetic switching of the polarization direction should also be applicable to dynamically polarized nuclei.

As has been pointed out (e.g. by Baldwin and Khokhlov 1975) the development of a graser requires difficult improvements in techniques but does not appear to be in opposition to any limitations of a fundamental nature. Thus the first 'graser' will almost certainly involve the observation of a fractionally small degree of stimulated emission following the sudden application of a change in an external parameter. As mirrors cannot be used, a travelling wave system will be necessary. The main advantage of the system suggested here is that it involves a switching technique based on the exploitation of the anisotropic emission from oriented nuclei in conjunction with a suitably anisotropic sample geometry which would be required for any travelling wave system. In the decay of many isomeric nuclei, emission patterns for high nuclear polarizations will lead to extremely low emissions in some directions, making the switching quite feasible. However, the reduction in the required population densities will not be very great. In order to convincingly observe stimulated emission following such a switching it will be necessary to employ a switching time which is short in comparison with the lifetime of the excited nuclear states. The pulsed n.m.r. studies of Foster *et al.* (1977) on ^{60}Co nuclei have shown that extremely fast switching times are feasible. For systems employing long lived nuclear states (for example, $\tau \gtrsim 0 \cdot 1$ s), resonant line-narrowing techniques might be used. For these, nonresonant magnetic switching could be used.

Conclusions

The density of population inversion necessary for graser action has been derived as a function of photon energy. We have seen how the use of oriented nuclei with fast resonant switching techniques should facilitate the observation of graser action. Other advantages should result from an increase in the directional emission and a small reduction in the required population densities in a travelling wave system.

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