Aust. J. Phys., 1976, **29**, 389–94

Geometry Factors for TE_{0mn} Resonance Cavities for Diagnostics of Large-diameter Plasmas

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Abstract

Mode-suppressing resonators oscillating in TE_{0mn} modes only are suggested here for the diagnostics of large-diameter plasma columns. Circular cylindrical cavities composed of electrically insulated parallel rings have been found to give the best Q values. Geometry factors are presented for TE_{0mn} resonance mode cavities ($m = 1, ..., 7$; arbitrary n) for different density profiles and filling ratios, allowing absolute electron density determinations.

Introduction

Microwave cavity resonators are widely used for the determination of electron densities (e.g. Biondi and Brown 1949; Brown and Rose 1952; Eckhardt *et al.* 1954; Blevin and Reynolds 1969; Janzen 1971*a*, 1971*b*; Janzen *et al.* 1974) and electron collision frequencies in plasmas (e.g. Janzen *et al.* 1975). The fundamentals of this diagnostic method are briefly as follows. When a plasma with an average electron density $\bar{n}_e$ is introduced into a right circular cylindrical microwave cavity which oscillates in a TE_{lmn} or TM_{lmn} mode at a resonance frequency f_0, it causes a frequency shift Δf in f_0 of the form

$$\Delta f/f_0 = \phi\,\bar{n}_e/n_c\,, \qquad \bar{n}_e \ll n_c\,, \tag{1}$$

where ϕ is the geometry factor and n_c is the cutoff density which is given by

$$n_c = 1\cdot 24\times 10^{10} f_0^2 \quad \mathrm{cm}^{-3}, \tag{2}$$

with f_0 expressed in GHz. The collision frequency ν can be determined by the change in the Q factor of the resonance line.

For the measurement of the plasma parameters, the plasma either fills the cavity resonator entirely or, more commonly, is introduced in a vessel through concentric holes in the top and bottom lid of the cylindrical cavity. Usually the diameter D and length L of a microwave cavity resonator are nearly equal to the wavelength λ_0, thus limiting the diagnostics of plasma columns to diameters of approximately half a wavelength. (The diameter of the holes in the end plates of a resonator should not exceed half the diameter of the resonator in order to avoid perturbing the fields too much.) Larger cavity resonators that allow large plasma columns to be introduced show too many resonance lines lying close together, thus making it very difficult to identify modes.

If the cylindrical wall of a cavity is slotted perpendicularly to the axis or, in other words, if the wall is composed of electrically separated rings or of a helix, then no RF currents can flow parallel to the axis of the cylinder. The only class of resonance modes which does not require wall currents in this direction is the TE_{0mn} type. Mode-suppressing resonators of this kind can then be built with much larger diameters and still allow identification of resonance lines. The possible numbers N_{max} of resonance lines for a normal type cavity resonator and a mode-suppressing ring or helix resonator for four cavity diameters D are set out below (only the first longitudinal mode is considered here, that is, $n = 0$ for TM modes, $n = 1$ for TE modes; the length L is taken to be equal to D).

	$D/\lambda_0=1$	3	10	15
N_{max}(TE and TM)	2	24	254	568
$N_{max}(TE_{0mn})$	0	2	9	14

This way of suppressing modes is used in communication electronics for microwave waveguides (Unger 1961) and for high Q wavemeters (Barlow *et al.* 1960).

TE_{0mn} Resonance Cavity Partially Filled with Plasma

According to the linear perturbation theory for a cavity resonator (Mueller 1939; Bethe and Schwinger 1943; Slater 1946), a plasma partially filling the cavity causes a shift Δf in the resonance frequency f_0 given by equation (1). The geometry factor ϕ is a function of the resonance mode, the ratio R_p/R_r of the plasma radius to the resonator radius, and the radial and axial profiles $n_e(r)$ and $n_e(z)$ of the plasma electrons. Values of ϕ have been computed for a great number of modes, thus allowing absolute measurements of the electron density n_e (Janzen 1969, 1971*a*, 1971*c*).

For TE resonance modes the influence of the radial and axial electron density profiles on the geometry factor ϕ can be separated as

$$\phi(r,z) = F(r)\,.\,G(z),$$

with $G(z) = 1$ for a homogeneous axial density distribution. The radial electron density profile is assumed to have the form

$$n_e(r) = n_0\{1-s(r/R_p)^2\}, \quad r \leqslant R_p, \tag{3}$$

where n_0 is the density on the axis and the profile factor s varies between 0 (rectangular profile) and 1 (parabolic profile). Values of the factor $F(r)$ for TE_{0mn} modes ($m = 1,...,7$; arbitrary n) have been calculated here and are presented in Table 1. Fig. 1 shows $F(r)$ as a function of R_p/R_r for the modes TE_{01n}, TE_{02n}, TE_{04n} and TE_{07n} for rectangular radial density profiles ($s = 0$).

The geometry factor ϕ, as a result of linear perturbation theory, is applicable to plasmas with $\bar{n}_e \ll n_c$. The tolerable upper limit for $\bar{n}_e$ depends on the resonance mode, the plasma diameter and the radial profile, and can be determined by an 'exact' theory (Shohet 1966; Blevin and Reynolds 1969; Shohet and Hatch 1970; Raeuchle 1972; Janzen 1972). However, the application of the exact theory is difficult when the plasma is not homogeneous. The holes in the end plates of the resonator also influence the value of ϕ (Thomassen 1965). Computations for TM_{010} mode resonators show that the electron density will be measured $<10\%$ too low for plasma columns with diameters of half the resonator length ($R_p = \frac{1}{4}L$) and

Table 1. Geometry factors $F(r)$ for TE_{0mn} modes

The values of $F(r)$ are given as a function of the filling ratio R_p/R_r and the profile parameter s for the TE_{0mn} modes ($m = 1,...,7$; arbitrary n). The number in parentheses for each entry is the power of the 10 multiplier

Index	Ratio	Geometry factor $F(r)$					
m	R_p/R_r	$s = 0$	0·2	0·4	0·6	0·8	1·0
1	0·05	3·51 (−5)	3·38 (−5)	3·22 (−5)	3·01 (−5)	2·74 (−5)	2·35 (−5)
	0·10	5·52 (−4)	5·32 (−4)	5·07 (−4)	4·74 (−4)	4·31 (−4)	3·70 (−4)
	0·20	8·21 (−3)	7·92 (−3)	7·56 (−3)	7·09 (−3)	6·47 (−3)	5·61 (−3)
	0·30	3·67 (−2)	3·55 (−2)	3·40 (−2)	3·21 (−2)	2·95 (−2)	2·59 (−2)
	0·40	9·75 (−2)	9·46 (−2)	9·11 (−2)	8·66 (−2)	8·05 (−2)	7·20 (−2)
	0·50	1·90 (−1)	1·85 (−1)	1·80 (−1)	1·72 (−1)	1·63 (−1)	1·49 (−1)
	0·60	2·98 (−1)	2·93 (−1)	2·87 (−1)	2·79 (−1)	2·68 (−1)	2·53 (−1)
	0·70	3·97 (−1)	3·94 (−1)	3·91 (−1)	3·86 (−1)	3·80 (−1)	3·72 (−1)
	0·80	4·65 (−1)	4·68 (−1)	4·71 (−1)	4·75 (−1)	4·81 (−1)	4·89 (−1)
	0·90	4·95 (−1)	5·06 (−1)	5·19 (−1)	5·35 (−1)	5·58 (−1)	5·89 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
2	0·05	2·09 (−4)	2·02 (−4)	1·92 (−4)	1·80 (−4)	1·63 (−4)	1·40 (−4)
	0·10	3·15 (−3)	3·03 (−3)	2·90 (−3)	2·72 (−3)	2·48 (−3)	2·14 (−3)
	0·20	3·92 (−2)	3·80 (−2)	3·66 (−2)	3·46 (−2)	3·21 (−2)	2·85 (−2)
	0·30	1·30 (−1)	1·28 (−1)	1·24 (−1)	1·20 (−1)	1·14 (−1)	1·06 (−1)
	0·40	2·27 (−1)	2·27 (−1)	2·26 (−1)	2·25 (−1)	2·23 (−1)	2·21 (−1)
	0·50	2·67 (−1)	2·73 (−1)	2·81 (−1)	2·91 (−1)	3·05 (−1)	3·24 (−1)
	0·60	2·71 (−1)	2·84 (−1)	3·01 (−1)	3·22 (−1)	3·50 (−1)	3·89 (−1)
	0·70	3·16 (−1)	3·29 (−1)	3·46 (−1)	3·68 (−1)	3·98 (−1)	4·39 (−1)
	0·80	4·11 (−1)	4·22 (−1)	4·35 (−1)	4·52 (−1)	4·74 (−1)	5·06 (−1)
	0·90	4·85 (−1)	4·97 (−1)	5·11 (−1)	5·30 (−1)	5·55 (−1)	5·90 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
3	0·05	6·21 (−4)	5·99 (−4)	5·70 (−4)	5·34 (−4)	4·86 (−4)	4·19 (−4)
	0·10	8·72 (−3)	8·43 (−3)	8·06 (−3)	7·59 (−3)	6·96 (−3)	6·08 (−3)
	0·20	8·22 (−2)	8·04 (−2)	7·82 (−2)	7·53 (−2)	7·14 (−2)	6·61 (−2)
	0·30	1·71 (−1)	1·72 (−1)	1·73 (−1)	1·74 (−1)	1·76 (−1)	1·79 (−1)
	0·40	1·85 (−1)	1·93 (−1)	2·04 (−1)	2·17 (−1)	2·35 (−1)	2·60 (−1)
	0·50	2·31 (−1)	2·40 (−1)	2·51 (−1)	2·66 (−1)	2·85 (−1)	3·11 (−1)
	0·60	3·23 (−1)	3·30 (−1)	3·39 (−1)	3·51 (−1)	3·66 (−1)	3·88 (−1)
	0·70	3·44 (−1)	3·57 (−1)	3·74 (−1)	3·95 (−1)	4·24 (−1)	4·65 (−1)
	0·80	3·80 (−1)	3·95 (−1)	4·15 (−1)	4·41 (−1)	4·75 (−1)	5·22 (−1)
	0·90	4·72 (−1)	4·85 (−1)	5·02 (−1)	5·23 (−1)	5·52 (−1)	5·92 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
4	0·05	1·35 (−3)	1·30 (−3)	1·24 (−3)	1·17 (−3)	1·06 (−3)	9·17 (−4)
	0·10	1·73 (−2)	1·67 (−2)	1·61 (−2)	1·52 (−2)	1·40 (−2)	1·24 (−2)
	0·20	1·11 (−1)	1·10 (−1)	1·09 (−1)	1·07 (−1)	1·06 (−1)	1·03 (−1)
	0·30	1·41 (−1)	1·47 (−1)	1·54 (−1)	1·64 (−1)	1·77 (−1)	1·95 (−1)
	0·40	1·94 (−1)	2·00 (−1)	2·08 (−1)	2·18 (−1)	2·31 (−1)	2·49 (−1)
	0·50	2·61 (−1)	2·69 (−1)	2·78 (−1)	2·91 (−1)	3·07 (−1)	3·30 (−1)
	0·60	2·81 (−1)	2·93 (−1)	3·09 (−1)	3·29 (−1)	3·55 (−1)	3·92 (−1)
	0·70	3·68 (−1)	3·78 (−1)	3·91 (−1)	4·08 (−1)	4·30 (−1)	4·61 (−1)
	0·80	3·84 (−1)	4·00 (−1)	4·21 (−1)	4·47 (−1)	4·82 (−1)	5·31 (−1)
	0·90	4·59 (−1)	4·74 (−1)	4·92 (−1)	5·17 (−1)	5·49 (−1)	5·94 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
5	0·05	2·45 (−3)	2·37 (−3)	2·26 (−3)	2·12 (−3)	1·94 (−3)	1·68 (−3)
	0·10	2·78 (−2)	2·70 (−2)	2·61 (−2)	2·48 (−2)	2·32 (−2)	2·09 (−2)
	0·20	1·11 (−1)	1·12 (−1)	1·15 (−1)	1·17 (−1)	1·21 (−1)	1·25 (−1)

Table 1 (*Continued*)

Index m	Ratio R_p/R_r	Geometry factor $F(r)$ $s = 0$	0·2	0·4	0·6	0·8	1·0
5	0·30	1·35 (−1)	1·41 (−1)	1·48 (−1)	1·57 (−1)	1·70 (−1)	1·87 (−1)
	0·40	2·10 (−1)	2·16 (−1)	2·24 (−1)	2·33 (−1)	2·46 (−1)	2·63 (−1)
	0·50	2·39 (−1)	2·48 (−1)	2·61 (−1)	2·76 (−1)	2·97 (−1)	3·25 (−1)
	0·60	3·08 (−1)	3·18 (−1)	3·30 (−1)	3·47 (−1)	3·68 (−1)	3·99 (−1)
	0·70	3·43 (−1)	3·56 (−1)	3·73 (−1)	3·94 (−1)	4·22 (−1)	4·61 (−1)
	0·80	4·04 (−1)	4·18 (−1)	4·36 (−1)	4·59 (−1)	4·90 (−1)	5·33 (−1)
	0·90	4·48 (−1)	4·64 (−1)	4·85 (−1)	5·11 (−1)	5·46 (−1)	5·96 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
6	0·05	3·95 (−3)	3·81 (−3)	3·65 (−3)	3·43 (−3)	3·14 (−3)	2·74 (−3)
	0·10	3·86 (−2)	3·77 (−2)	3·66 (−2)	3·52 (−2)	3·33 (−2)	3·06 (−2)
	0·20	9·55 (−2)	9·93 (−2)	1·04 (−1)	1·10 (−1)	1·19 (−1)	1·30 (−1)
	0·30	1·59 (−1)	1·62 (−1)	1·67 (−1)	1·73 (−1)	1·80 (−1)	1·91 (−1)
	0·40	1·86 (−1)	1·95 (−1)	2·05 (−1)	2·18 (−1)	2·36 (−1)	2·62 (−1)
	0·50	2·58 (−1)	2·66 (−1)	2·76 (−1)	2·90 (−1)	3·07 (−1)	3·32 (−1)
	0·60	3·00 (−1)	3·11 (−1)	3·24 (−1)	3·41 (−1)	3·63 (−1)	3·95 (−1)
	0·70	3·40 (−1)	3·54 (−1)	3·72 (−1)	3·94 (−1)	4·24 (−1)	4·66 (−1)
	0·80	4·13 (−1)	4·26 (−1)	4·42 (−1)	4·64 (−1)	4·92 (−1)	5·32 (−1)
	0·90	4·41 (−1)	4·58 (−1)	4·80 (−1)	5·08 (−1)	5·45 (−1)	5·97 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)
7	0·05	5·83 (−3)	5·64 (−3)	5·40 (−3)	5·09 (−3)	4·68 (−3)	4·11 (−3)
	0·10	4·79 (−2)	4·71 (−2)	4·61 (−2)	4·48 (−2)	4·30 (−2)	4·06 (−2)
	0·20	8·71 (−2)	9·15 (−2)	9·69 (−2)	1·04 (−1)	1·13 (−1)	1·26 (−1)
	0·30	1·53 (−1)	1·58 (−1)	1·64 (−1)	1·73 (−1)	1·83 (−1)	1·98 (−1)
	0·40	2·09 (−1)	2·15 (−1)	2·22 (−1)	2·32 (−1)	2·44 (−1)	2·62 (−1)
	0·50	2·42 (−1)	2·52 (−1)	2·64 (−1)	2·79 (−1)	3·00 (−1)	3·29 (−1)
	0·60	2·93 (−1)	3·05 (−1)	3·19 (−1)	3·38 (−1)	3·64 (−1)	3·99 (−1)
	0·70	3·59 (−1)	3·71 (−1)	3·86 (−1)	4·05 (−1)	4·30 (−1)	4·66 (−1)
	0·80	4·03 (−1)	4·18 (−1)	4·35 (−1)	4·58 (−1)	4·88 (−1)	5·31 (−1)
	0·90	4·39 (−1)	4·57 (−1)	4·79 (−1)	5·07 (−1)	5·45 (−1)	5·99 (−1)
	1·00	5·00 (−1)	5·19 (−1)	5·42 (−1)	5·71 (−1)	6·11 (−1)	6·67 (−1)

$<5\%$ too low for $R_p \leqslant \frac{1}{8}L$. The influence of the discharge vessel on the value of ϕ can be neglected when the wall of the vessel is kept thin and placed preferably where the microwave field is zero (Janzen 1972).

Mode-suppressing Cavities

Four different types of cavities, resonators 1...4, with diameters of 100 and 240 mm each were built and tested. The characteristics of the resonators were: (1) continuous copper walls; (2) helically wound copper strips, 10 mm wide and spaced $<0\cdot5$ mm between turns; (3) helically wound enamelled copper wire, 0·8 mm diameter with close turns; (4) annular turns of copper strips, 10 mm wide, each soldered to form a ring and spaced $<0\cdot5$ mm between rings. The microwaves were fed through a coaxial cable and a loop (8 mm long, 4 mm wide) into the cavity. The loop, which lay in a cross-section plane at $z = \frac{1}{2}L$, preferred modes with odd n numbers. A second type of coupling launched the waves through a waveguide and a hole in one end plate at $r \approx 0\cdot7\,R_r$ into the cavity. The applied microwaves with frequencies between 3·2–6·5 GHz and 8·2–12·4 GHz yielded values of D/λ_0 in the range 1–8·3.

Resonator 1 showed the expected variety of resonance lines. Mode identification by comparison with computed resonance frequencies was not possible.

Resonators 2 and 3 showed resonance lines with such low Q factors that they could not be used for diagnostic purposes. The reason for the low Q in resonator 2 could lie in the large pitch of 0·1 of the helix. In resonator 3 the diameter D of the cavities was not well defined, since wire of circular cross section was used, and varied between 100–108 mm and 200–208 mm, which could result in an uncertainty in the resonance frequencies of up to 4–8%.

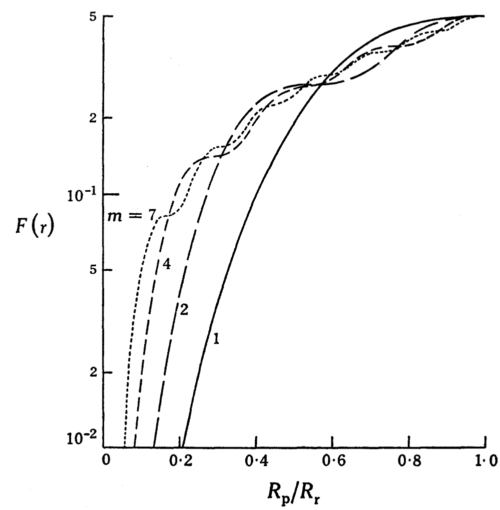

Fig. 1. Geometry factor $F(r)$ as a function of the filling ratio R_p/R_r for TE_{0mn} modes, $m = 1, 2, 4, 7$, with rectangular radial electron density profiles ($s = 0$).

Resonator 4 showed resonances with high Q values which could be identified by comparing measured and calculated frequencies. A final check on the mode was made by probing the field distribution of the resonance with a small dielectric sphere to determine the number of characteristic radial, axial or azimuthal maxima or minima present. The details obtained for the field distribution inside the TE_{0mn} cavity are given elsewhere (Janzen 1976).

Conclusions

It has been verified that mode-suppressing circular cylindrical microwave cavities oscillating in TE_{0mn} modes can be used for the diagnostics of large-diameter plasma columns. The present experiments have shown that ring-type resonators have the highest Q values of all mode-suppressing resonators tested. The geometry factors presented here for the TE_{0mn} resonators allow absolute determinations of electron densities.

Acknowledgments

The author thanks Miss B. Seybold for her kind assistance in the calculations. He would also like to express his gratitude to Professor H. K. Messerle for enabling him to work in the School of Electrical Engineering, University of Sydney, and to his colleagues in the Department. This work was supported by a Norman I. Price Research Fellowship.

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Manuscript received 27 January 1976