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Radiative Decay of the Low-lying States of ^{12}C

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Abstract

The B(E2) values for the two ^{12}C radiative transitions $2^+(4\cdot43\text{ MeV}) \to 0^+(\text{g.s.})$ and $0^+(7\cdot65\text{ MeV}) \to 2^+(4\cdot43\text{ MeV})$ are calculated, using a ground state wavefunction which is a mixture of quartet states. The two transitions can be adequately enhanced over the single-particle transition values. Indeed, the collectivity introduced by the quartet wavefunctions is not the dominant factor in providing the required enhancement; rather the use of a sufficiently large single-particle basis is necessary.

Introduction

The low-lying states of ^{12}C, and in particular the 4·43 MeV (2^+) state with its radiative transition to the ground state, have been the subjects of much discussion. A summary of the results of previous attempts to calculate the B(E2) value for this $2^+ \to 0^+$ transition, together with the experimental value, are set out below.

Adopted experimental value (Ajzenberg-Selove and Lauritsen 1970)	$8\cdot25 \pm 0\cdot44\ e^2\,\text{fm}^4$
Kurath (1957) theoretical result	$3\cdot4\ e^2\,\text{fm}^4$
Goswami and Pal (1963) theoretical result	$24\cdot6\ e^2\,\text{fm}^4$
Bouten *et al.* (1967) theoretical result	$11\cdot14\ e^2\,\text{fm}^4$

Kurath (1957) considered the wavefunctions for the 4·43 MeV state and the ground state to have only 1p-shell components. His tabulated result has been corrected with a more recent estimate (Bentz 1969) of $\langle r^2 \rangle$. The fact that his result is less than half of that given by experiment indicated the need to introduce collective effects into the description of a nucleus even as light as ^{12}C.

Goswami and Pal (1962, 1963) calculated B(E2) in the framework of the random phase approximation (RPA), and overestimated the experimental value by a factor of three. The validity of the RPA is doubtful in this case, since there is some evidence to show that in this framework the amount of shell-breaking in the ground states of doubly-closed-shell nuclei is overestimated. Agassi *et al.* (1969) and Ellis and Zamick (1969) have shown that the RPA overestimates shell-breaking in ^{12}C and ^{16}O respectively.

Bouten *et al.* (1967) carried out a projected Hartree–Fock calculation which also overestimated the value of B(E2). However, the ground state wavefunction obtained by these authors was not reasonable, since it contained 2s–1d and 2p–1f components which constituted over 25% in intensity of the wavefunction. Spicer (1973), from an analysis of the intensities of de-excitation γ-rays (Medicus *et al.* 1970) following the photodisintegration of ^{12}C, has estimated the total intensity of 2s–1d components in the ground state wavefunction of that nucleus to be approximately $1\frac{1}{4}\%$. There was no evidence for the presence of 2p–1f components in the ground state wavefunction, nor would any be expected considering the intensity deduced for the 2s–1d admixtures.

In a later paper Kurath (1960) demonstrated that the transition rate could be properly enhanced by using Nilsson wavefunctions. This showed that it is possible to account for the B(E2) value by introducing deformation effects.

The present paper reports a calculation which was aimed at discovering whether quartet model wavefunctions introduced sufficient collectivity to account for the γ-ray transition rates between the low-lying states of ^{12}C. The quartet model, the 'stretch scheme' and how they describe collective effects (as evidenced by the appearance of rotational bands) have been discussed by Gillet (1968). A quartet in the j shell is made up of two proton–neutron pairs, each coupled to angular momentum $2j$ and the two 'aligned' pairs coupled to zero angular momentum in the ground state. This configuration maximizes the effect of proton–neutron correlations.

Calculation

For the 2^+(4·43 MeV) $\rightarrow$ 0^+(g.s.) transition, the quantity to be calculated is

$$B(\mathrm{E2}) = \tfrac{1}{5}\sum_{M_i} |\langle 00|\, T_\kappa^{(2)}\, |2M_i\rangle|^2\,,$$

where $T_\kappa^{(2)} = (16\pi/5)^{\frac{1}{2}}\, Y_{2\kappa}(\theta,\phi)\, r^2$ is the electric quadrupole operator.

The ground state wavefunction used was

$$\begin{aligned}\psi_{\mathrm{g.s.}} = {} & 0\cdot781\,(1\mathrm{s}_{1/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0} + 0\cdot625\,(1\mathrm{s}_{1/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0}(1\mathrm{p}_{1/2})^4_{J=0}\\ & + 0\cdot071\,(1\mathrm{s}_{1/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0}(2\mathrm{s}_{1/2})^4_{J=0} + 0\cdot071\,(1\mathrm{s}_{1/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0}(1\mathrm{d}_{5/2})^4_{J=0}\\ & + 0\cdot05\,(1\mathrm{s}_{5/2})^4_{J=0}(1\mathrm{p}_{3/2})^4_{J=0}(1\mathrm{d}_{3/2})^4_{J=0}\,. \qquad (1)\end{aligned}$$

The first component is the simplest 'shell model' state, and the rest are formed from it by the promotion of a whole quartet from the $1\mathrm{p}_{3/2}$ single-particle state to the $1\mathrm{p}_{1/2}$ state or the 2s–1d shell. The amplitudes of the components were deduced from two sources: First, the calculation of Rowe and Wong (1970), which neglected all the 2s–1d components, gave fractional occupation probabilities for the $1\mathrm{p}_{3/2}$ and $1\mathrm{p}_{1/2}$ subshells. If the form of wavefunction given in equation (1) is assumed, then the coefficients of its first two terms can be deduced from the fractional occupation probabilities. Second, the analysis of de-excitation γ-rays following the photodisintegration of ^{12}C, noted earlier (Spicer 1973), indicates the intensities of the last three components of the wavefunction as 0·005, 0·005 and 0·0025 respectively. The coefficients of the first two terms were reduced marginally to allow the presence of these small contributions in a normalized wavefunction.

The wavefunction of the 2^+ state at 4·43 MeV was obtained by allowing components formed by all possible E2 single-particle transitions and all quartets to recouple to $J = 2$, consistent with the Pauli principle, for each component of the ground state wavefunction. Where a single component of the ground state wavefunction gives rise to a number of components in the wavefunction of the 2^+ state, these latter components were weighted according to the magnitude of the E2 matrix element producing them. For example, the second component of the ground state wavefunction (equation 1) gives rise to nine components in the wavefunction of the 2^+ state. It turned out that only the first two terms of the ground state wavefunction, and those components of the 2^+ state derived from them, contributed significantly in the calculation of B(E2) for the $2^+ \rightarrow 0^+$ (g.s.) transition, and so those components of the ground state wavefunction involving configurations in the 2s–1d shell are neglected hereafter.

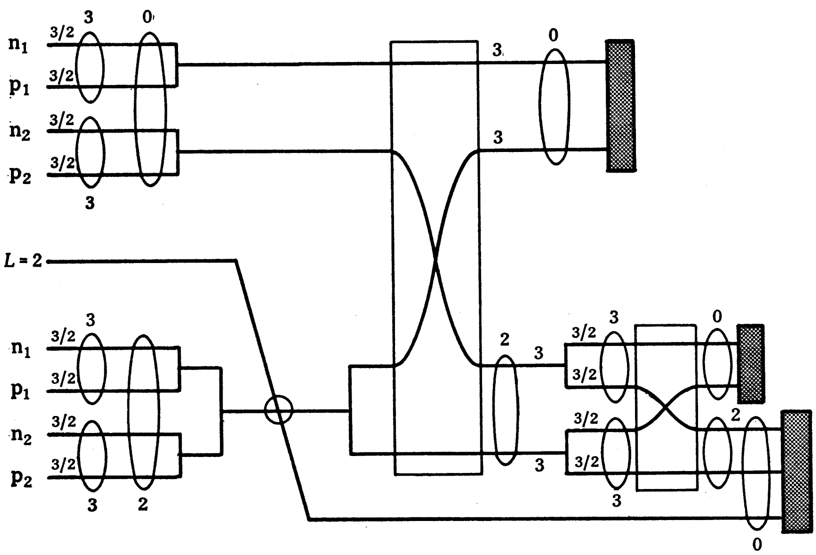

Fig. 1. Angular momentum recoupling diagram enabling the expression of a quartet–quartet transition projection integral in terms of a single-particle transition projection integral.

Then, considering the wavefunctions of both the 0^+ and the 2^+ states as a sum of components, we write

$$B(\mathrm{E2}) = \tfrac{1}{5}\Big|\sum_n \sum_i C_n C_n^{(i)} [0^+ \,|\, T^{[2]} \,|\, 2^+]_n^i\Big|^2, \qquad (2)$$

where $C_n^{(i)}$ is the coefficient of the component of the 2^+-state wavefunction and C_n is the coefficient of the corresponding component in the ground state wavefunction. The quantity $[0^+ \,|\, T^{[2]} \,|\, 2^+]$ is a projection integral (using the notation of Danos 1971), which is related to a reduced matrix element by the relation

$$\langle \psi^{[j_1]} \,||\, T^{[k]} \,||\, \psi^{[j_2]} \rangle = (-)^{2k}(-)^{j_1+k+j_2} [\psi^{[j_1]} \,|\, T^{[k]} \,|\, \psi^{[j_2]}]. \qquad (3)$$

Harmonic oscillator radial wavefunctions (Mayer and Jensen 1955) were used throughout; this has the consequence that all projection integrals contain a factor α^2, where $\alpha = (m\omega/\hbar)^{\frac{1}{2}}$ has the dimensions of an inverse length, m being the nucleon reduced mass and $\hbar\omega$ the characteristic oscillator energy. The value of α^2 was calculated (Mayer and Jensen) from the root mean square radius of the ^{12}C nucleus as given by elastic electron scattering (Bentz 1969).

To calculate the quartet projection integrals, use was made of the Danos (1971) angular momentum recoupling diagrams, thus enabling them to be expressed in terms of projection integrals with single-particle wavefunctions in the initial and final states. For example, for the $(1p_{3/2})^4$ quartet, Fig. 1 gives

$$[0^+ \,|\, T^{[2]} \,|\, 2^+] = \begin{bmatrix} 0 & 2 & 2 \\ 2 & 0 & 2 \\ 2 & 2 & 0 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 3 & 3 & 2 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} \frac{3}{2} & \frac{3}{2} & 3 \\ \frac{3}{2} & \frac{3}{2} & 3 \\ 0 & 2 & 2 \end{bmatrix} [3\,|\,3][\tfrac{3}{2}\,|\,\tfrac{3}{2}\,|\,T^{[2]}][\tfrac{3}{2}\,|\,\tfrac{3}{2}]$$

$$= -(\sqrt{3/5})[j = \tfrac{3}{2} \,|\, j = \tfrac{3}{2} \,|\, T^{[2]}]\,. \qquad (4)$$

Use of a similar diagram to separate the space and spin parts of the projection integral leads to

$$[(1p_{3/2})^4_{J=0} \,|\, T^{[2]} \,|(1p_{3/2})^4_{J=0}] = -(8\sqrt{3}/5\sqrt{5})\langle r^2\rangle_{1p}\,.$$

This result includes the factor of two arising from the presence of two protons in Fig. 1 which could have been singled out for the final projection integral.

The process of constructing the wavefunction of the 2^+ state, as described earlier, gives no information regarding the relative phases of the components. In order to limit the possible number of phase combinations, an attempt has been made to satisfy two other conditions simultaneously. These are that the B(E2) value for the 0^+(7·65 MeV) → 2^+(4·43 MeV) transition be satisfactorily given, and that the wavefunction for the 2^+ state lead to the expectation of only a small f-wave contribution in the ^{11}B(d, n)^{12}C*(4·43 MeV) and ^{11}B(^{3}He, d)^{12}C* reactions, as indicated by experiment (Almond and Risser 1965; Crosby and Legg 1967). In each case, the DWBA fit for $l = 1$ proton transfer provides a good fit to the differential cross section in the angular region where the $l = 3$ contribution is expected to be a maximum.

Since we have approximated the wavefunction for the ground state by

$$0\cdot781\,(1s_{1/2})^4_{J=0}(1p_{3/2})^4_{J=0}(1p_{3/2})^4_{J=0} + 0\cdot625\,(1s_{1/2})^4_{J=0}(1p_{3/2})^4_{J=0}(1p_{1/2})^4_{J=0}\,,$$

and since the 7·65 MeV state is the only 0^+ state close to the ground state, it follows that a good approximation to the wavefunction of the 7·65 MeV state, within the framework specified in this paper, is

$$(7\cdot65\text{ MeV}, 0^+) = 0\cdot625\,(1s_{1/2})^4(1p_{3/2})^4(1p_{3/2})^4 - 0\cdot781\,(1s_{1/2})^4(1p_{3/2})^4(1p_{1/2})^4\,.$$

The ft values of the β-decay branches of both ^{12}B and ^{12}N to the ground state and 7·65 MeV state of ^{12}C tend to support this assignment (Ajzenberg-Selove and Lauritsen 1970).

From the previously found projection integrals and the wavefunction of the 4·43 MeV (2^+) state as determined above, the B(E2) values of the two radiative transitions 0^+(7·65 MeV) → 2^+(4·43 MeV) and 2^+(4·43 MeV) → 0^+(g.s.) were calculated, this time allowing the relative phases in the wavefunction of the 2^+ state to be modified in order to fit as well as possible the constraints noted above. The wavefunction for the 2^+ state arrived at by this process is given in the Appendix. The results of the calculation gave the B(E2) values for the two radiative transitions respectively as 8·43 and 8·37 e^2 fm^4. These are to be compared with the experimental values of $10\cdot2 \pm 3\cdot7$ and $8\cdot25 \pm 0\cdot44$ e^2 fm^4.

Conclusions

It has been demonstrated that, under the assumption of a quartet model wavefunction for the ground state of ^{12}C, it is possible to calculate, within experimental errors, the B(E2) values for the radiative transitions between the 7·65 MeV (0^+) and 4·43 MeV (2^+) states and between the 4·43 MeV and ground (0^+) states. At the same time it is possible to minimize the estimated net f-wave contribution in the wavefunction of the 4·43 MeV state. This contribution would be sampled by the ^{11}B(d, n)^{12}C* reaction, and experiment gives no evidence for any significant f-wave contribution.

The above results indicate that it is possible to obtain the appropriate enhancements of the transition probabilities for the two radiative transitions. However, the enhancements are not due to the quartet nature assumed for the ground and 7·65 MeV states, but rather to the inclusion in the calculation of a sufficiently large single-particle basis.

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Appendix

The wavefunction used for the 4·43 MeV (2^+) state of ^{12}C was

$$
\begin{aligned}
&+0\cdot285\,(1s_{1/2})^4(1p_{3/2})^7(1p_{1/2}) - 0\cdot221\,(1s_{1/2})^4(1p_{3/2})^7(1f_{5/2})\\
&+0\cdot541\,(1s_{1/2})^4(1p_{3/2})^7(1f_{7/2}) - 0\cdot180\,(1s_{1/2})^4(1p_{3/2})^7(2p_{1/2})\\
&+0\cdot180\,(1s_{1/2})^4(1p_{3/2})^7(2p_{3/2}) + 0\cdot271\,(1s_{1/2})^3(1p_{3/2})^8(1d_{5/2})\\
&+0\cdot221\,(1s_{1/2})^3(1p_{3/2})^8(1d_{3/2})\\
&+0\cdot191\,(1s_{1/2})^4(1p_{3/2})^4_{J=2}(1p_{1/2})^4 - 0\cdot151(1s_{1/2})^4(1p_{3/2})^3(1p_{1/2})^4(1f_{5/2})
\end{aligned}
$$

$$+0\cdot369\,(1s_{1/2})^4\,(1p_{3/2})^3\,(1p_{1/2})^4\,(1f_{7/2})+0\cdot123\,(1s_{1/2})^4\,(1p_{3/2})^3\,(1p_{1/2})^4\,(2p_{1/2})$$
$$+0\cdot123\,(1s_{1/2})^4(1p_{3/2})^3\,(1p_{1/2})^4\,(2p_{3/2})+0\cdot185\,(1s_{1/2})^3\,(1p_{3/2})^4\,(1p_{1/2})^4\,(1d_{5/2})$$
$$-0\cdot151\,(1s_{1/2})^3\,(1p_{3/2})^4\,(1p_{1/2})^4\,(1d_{3/2})-0\cdot282\,(1s_{1/2})^4\,(1p_{3/2})^4\,(1p_{1/2})^3\,(1f_{5/2})$$
$$+0\cdot123\,(1s_{1/2})^4\,(1p_{3/2})^4\,(1p_{1/2})^3\,(2p_{3/2})\,.$$

Where the coupling of four particles is not specified, the total angular momentum is zero. The coupling of three particles, each of total angular momentum j, is taken to give a total angular momentum j.

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