Aust. J. Phys., 1975, **28**, 77–83

Electron Distribution Moments and Precipitation Dependence on Anisotropic Magnetospheric Content

J. R. Catchpoole

Mathematics Department, Mitchell College of Advanced Education,
Bathurst, N.S.W. 2795.

Abstract

Precipitation of spiralling electrons from the magnetosphere is investigated, under acceleration by field line convergence and by superimposed electric field. Instead of an isotropic content, packets of electrons of fairly narrow pitch range are considered. These are treated as initially normally distributed about those angles which contribute most to the precipitation. Calculations indicate the effects on the pitch distribution and on the total magnitude of precipitation. The importance of these effects lies in the critical dependence of subsequent collision atmosphere phenomena on such details of distribution. A comparison of total precipitation is made with the isotropic case for various parameters. As measures of the acceleration and trapping modifications, the third and fourth moments of the electron distributions are indicated.

Introduction

Important atmospheric effects are determined by electrons which are precipitated from spiralling orbits in the magnetosphere into a denser lower atmosphere. The extent of this precipitation depends, amongst other factors, on various magnetospheric acceleration mechanisms. One of these is the line convergence of the Earth's magnetic field which produces a Lorentz force component retarding the precipitating motion. This is a function of the colatitude of precipitation considered. A second significant acceleration results from the force experienced by the spiralling charged particle being in a longitudinal electric field. Such an electric field, parallel to the guiding magnetic field line, will be small in magnitude. Nevertheless, estimated values (Block 1967) are sufficient to cause important variations in precipitation into the atmosphere. The parallel field may arise from dissimilarities in the angular velocity distributions of spiralling particles of opposite charges (Alfven and Falthammar 1963). Sufficient field strength will also result, independently, from ionospheric potential differences at conjugate points (Catchpoole 1971).

Variations in precipitation into the ionizing atmosphere caused by the above accelerations have been indicated previously (Catchpoole 1966), for different colatitudes θ_0 and for various parallel fields $E_{\|}$. Consequences at levels below that at which the charged particle may be supposed to have escaped its trapped orbit can similarly be predicted. One such consequence is the variation in typical height profiles of auroral ultraviolet radiation (Catchpoole 1973). The factor most relevant to what follows in this work, however, is that the extent of such secondary effects has been shown to be dependent to a marked degree on the pitch angle distribution of the ionizing agency on precipitation. It is in terms of this that the accelerating field controls both the precipitation itself and also its subsequent effectiveness.

Total Ionospheric Precipitation

Previously, an isotropic distribution has been assumed in the equatorial plane, which is taken as a reference plane for the magnetosphere. With incomplete data for a more realistic pitch angle distribution, some such assumption must be adopted (Gustafsson 1973). The first objective of the present work, however, has been to make some initial estimates of the effects of an equatorial magnetospheric content which is not isotropic. Still considering the same two acceleration mechanisms, the effect of this change in assumed initial distribution is investigated. An electron pitch distribution which is peaked at angles other than 90° is now assumed (the measure still being at the reference plane). As before, α_0 is used to denote pitch angles in this plane. Following acceleration, some of this equatorial content is injected into the ionosphere at an incident pitch angle denoted by α_p.

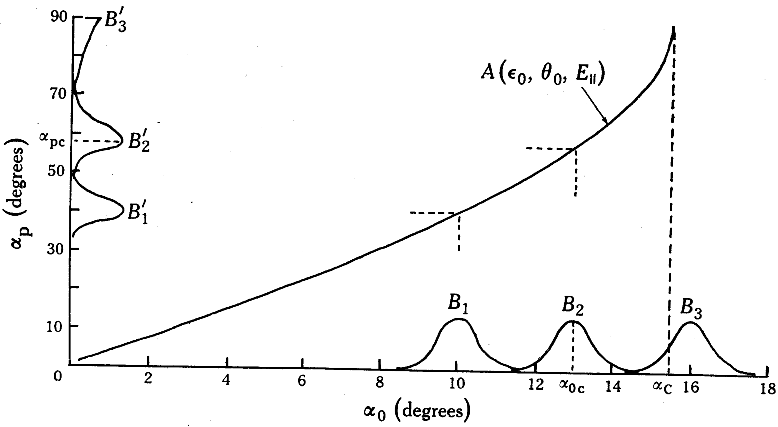


Fig. 1. Asymmetries of the precipitated pitch angle α_p, resulting from normal equatorial distributions, as a function of the initial central location α_{0c}, the initial energy ε_0, the colatitude of the spiralling line θ_0, and the accelerating parallel electric field $E_\parallel$.

In the present calculation and comparison, consideration is given to packets of electrons which initially are normally distributed with pitch angle about some central reference α_{0c} (peak and mean). The calculations include packets with various dispersions, but in all cases these are kept relatively narrow in order to enable the pitch dependence of present interest to be followed. It is therefore meaningful to compare the present normal results with the previous isotropic results.

Fig. 1 illustrates schematically the procedure adopted in the present investigation. Curve A is representative of any of a family of curves showing the relationship between the precipitated pitch angle α_p and the corresponding equatorial plane pitch angle α_0. Parameters of these curves are the colatitude of precipitation θ_0, the magnitude of the parallel electric field $E_\parallel$ and the initial electron energy ε_0 (equatorial plane). The derivation and features of these relations have been noted previously (Catchpoole 1969). Their details are inherent in the following calculations.

The procedure begins with the narrow packets of spiralling electrons, equatorially normally distributed, as in the curves B of Fig. 1. The group B_1, for instance, is centred about a pitch angle $\alpha_{0c} = 10°$ as it crosses the equatorial plane. Next, these distributions are 'reflected' in various curves such as A, which, from previous results, determine pitch angles on precipitation; that is, on entering a denser (600 km) atmosphere from which it is supposed that re-entry to the magnetosphere by mirroring is impossible. Finally, the curves B' indicate the 'images', or group angular distribution on precipitation.

An indication is also given in Fig. 1 of the asymmetry of the precipitation, together with magnitude and distribution effects of the cutoff pitch angle α_C. Both of these characteristics of the ionizing agency at precipitation have significant effects on subsequent processes within the collision atmosphere. Dissimilar effects will result, for example, from distributions B_1, B_2 and B_3. Comparisons throughout are made by supposing the same initial total magnetospheric content in all cases of different angular composition.

Angular Distribution of Precipitation

Measures have been made of the peakedness and asymmetry of the precipitated distributions, still, of course, as a function of the parameters of the curves A. These measures are based on the normal initial curves of various standard deviation σ and of different central location α_{0c}. For calculation purposes, each half of the input distribution was divided into $k = 20$ class intervals with relative frequencies f_i from the normal distribution. Corresponding class boundaries at precipitation were found from curve A, as were the locations of the precipitation peaks α_{pc}.

The third and fourth moments of the precipitated magnetospheric electron distribution were also calculated. These are the central moments about α_{pc}. When α_{0c} is trapped ($\alpha_{0c} > \alpha_C$), moments are calculated about $\alpha_p = 90°$ (distribution comparisons thus becoming less valid further into the trapping region). The moments were found from the expressions

$$\mu_3 = n^{-1}\sum_{i=1}^{k} f_i(\alpha_{pi}-\alpha_{pc})^3, \qquad \mu_4 = n^{-1}\sum_{i=1}^{k} f_i(\alpha_{pi}-\alpha_{pc})^4,$$

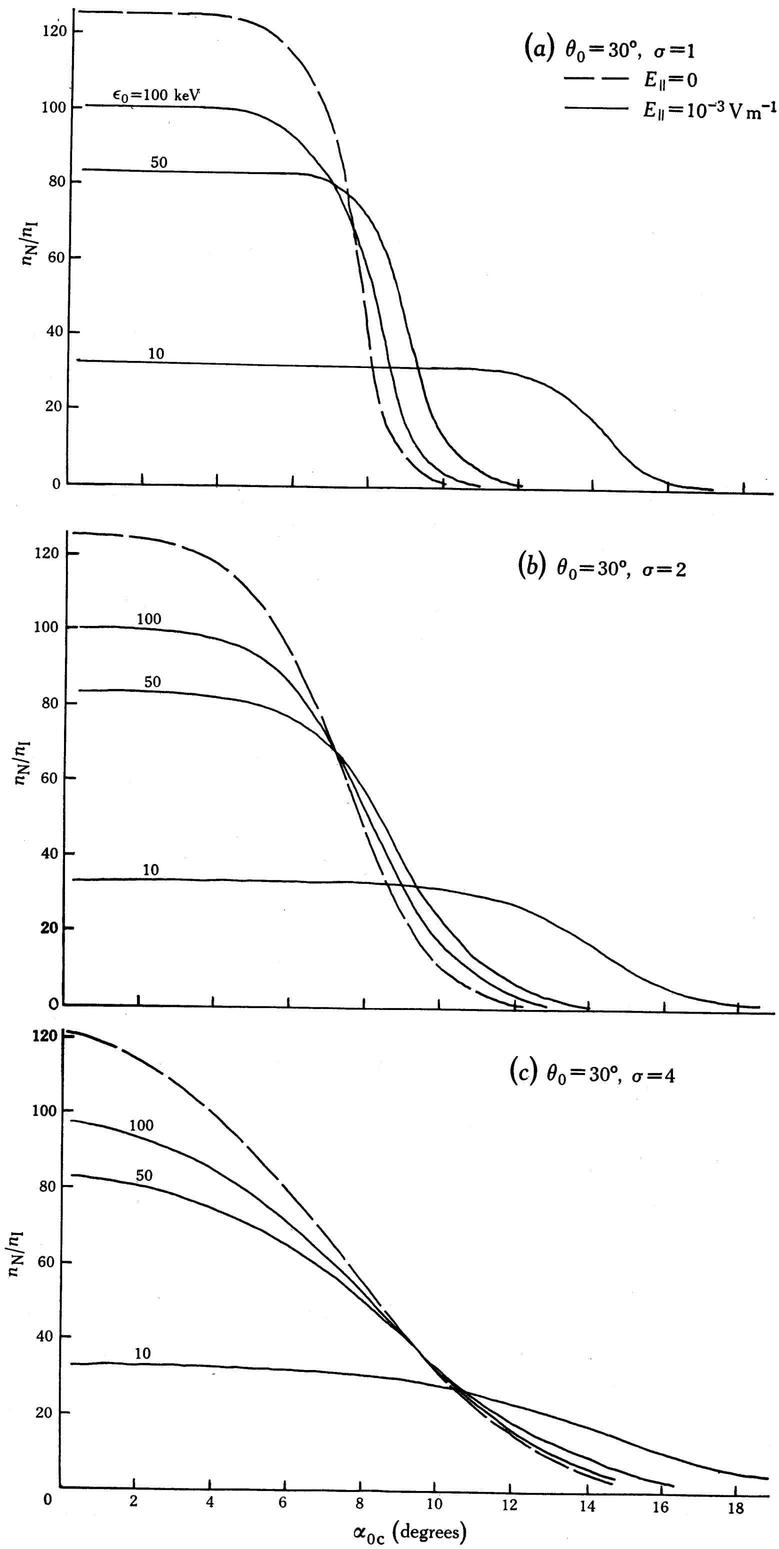
where n is the (arbitrary) total electron content and α_{pi} is the precipitated angle class mark corresponding to the initial classes noted above.

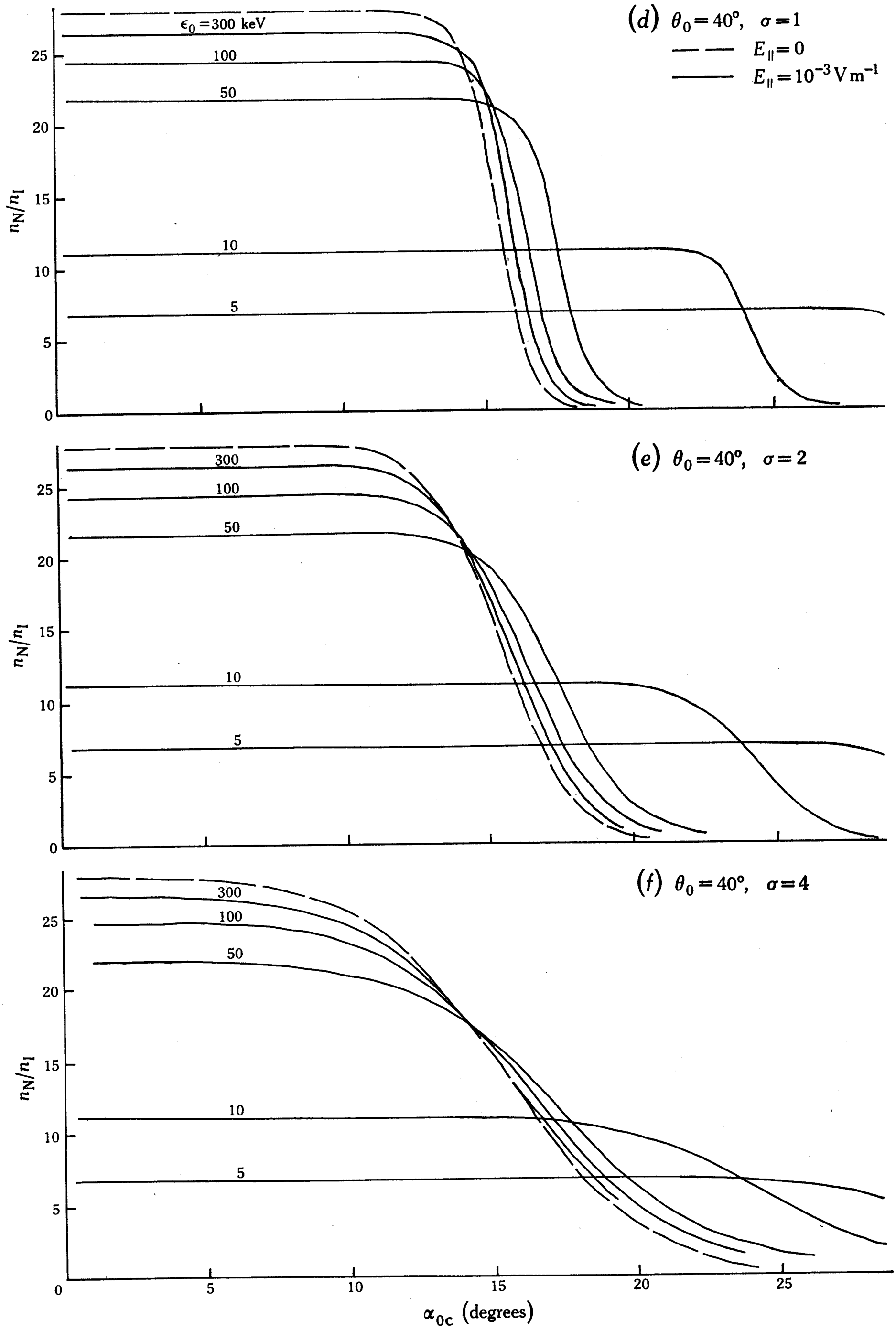
Final results indicating the distribution of electrons precipitated into the atmosphere are given in terms of the corresponding coefficients. Asymmetry is measured by the moment coefficient of skewness a_3 and peakedness by the moment coefficient of kurtosis a_4, where

$$a_3 = \mu_3/\mu_2^{3/2} = \mu_3/\sigma_p^3, \qquad a_4 = \mu_4/\mu_2^2 = \mu_4/\sigma_p^4.$$

Results

For various parameters, Figs 2*a*–2*f* show some of the results from the first part of the calculations. In each case, the curves indicate the relation between the ratio n_N/n_I and α_{0c}; that is, the ratio of total electron precipitation from the magnetosphere as a function of the central pitch angle assumed by the spiralling packet of electrons in the equatorial plane. The ratio is that of the number n_N precipitated in the present model of a narrow normal distribution about α_{0c} to the number n_I precipitated from an isotropic distribution of the same total content, the latter being determined from the previous work referred to above. The different graphs are for


(a) $\theta_0=30°$, $\sigma=1$
$E_{\parallel}=0$
$E_{\parallel}=10^{-3}\,\mathrm{V\,m^{-1}}$
$\epsilon_0=100$ keV
50
10
(b) $\theta_0=30°$, $\sigma=2$
100
50
10
(c) $\theta_0=30°$, $\sigma=4$
100
50
10
n_N/n_I
0
20
40
60
80
100
120
2
4
6
8
10
12
14
16
18
α_{0c} (degrees)



Figs 2*a*–2*f*. Ratios of the total magnetospheric electron precipitation (preferred pitch angle to isotropic distribution) for the indicated values of the parameters θ_0, the colatitude of precipitation, $E_\parallel$, the superimposed parallel electric field, and σ, the standard deviation about the central pitch angle α_{0c}.

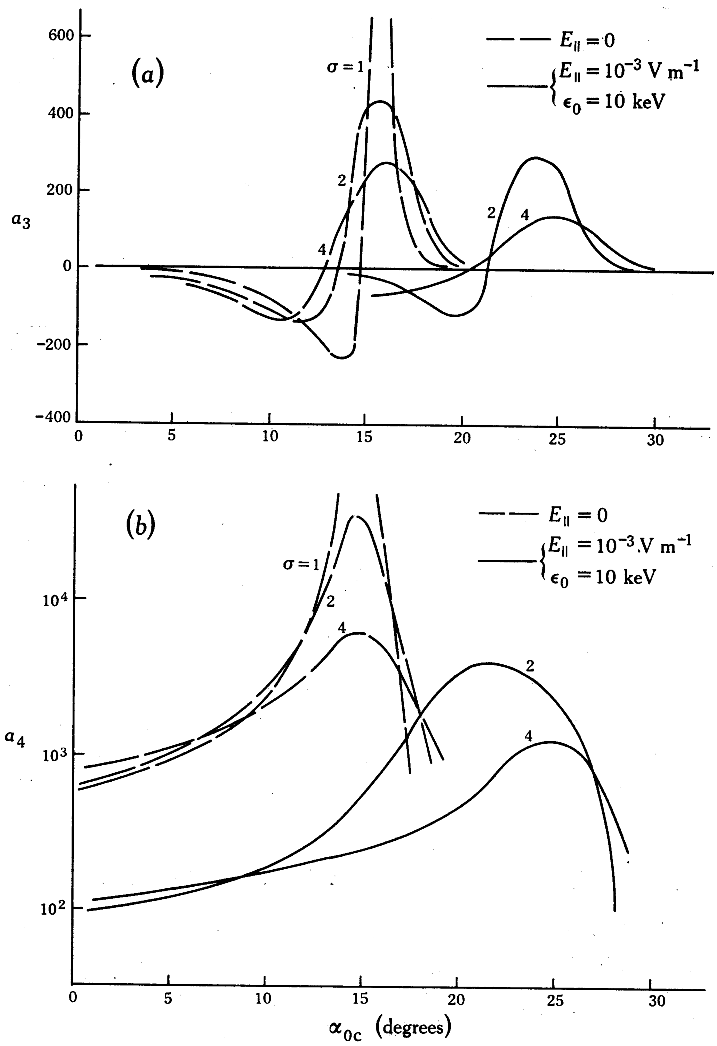


Fig. 3. Showing as a function of the central pitch angle α_{0c}, (*a*) the third moments a_3 and (*b*) the fourth moments a_4 of the precipitated magnetospheric electrons about the peak pitch angle α_{pc} (or about $\alpha_p = 90°$ when the peak is trapped), resulting from a normal distribution, with the indicated deviations σ, in an equatorial plane about α_{0c}. All curves are for a colatitude θ_0 of 40°.

two colatitudes of precipitation, $\theta_0 = 30°$ and 40°, and for standard deviations σ of 1, 2 and 4 about α_{0c}. The separate curves on each graph are for the further parameters involved: comparing the effects in the absence of a superimposed electric field ($E_{\parallel} = 0$; dashed curves) with those when this parallel field is equivalent to that of a uniform 10^{-3} V m^{-1} (full curves). As noted previously, this is the order of maximum likely magnitude, although lesser fields still produce significant effects on the degree and distribution of precipitation. Initial energies ε_0 before acceleration of from 5 to 300 keV are indicated as final parameters for individual curves.

Fig. 3*a* shows the third moments a_3 of the precipitated magnetospheric electrons about α_{pc}. Again this is a function of α_{0c}, with similar colatitude, acceleration and deviation parameters; resultant moments for an initial energy of 10 keV are shown. Finally, Fig. 3*b* indicates the fourth moments a_4 for the same range of α_{0c} and similar parameters.

The marked asymmetries and peakedness indicated for certain initial pitch angles and combinations of parameters is important in subsequent effects in the collision atmosphere. These effects can be very dependent on the pitch distribution of ionizing particles incident upon the atmosphere, on changes in the distributions brought about by various acceleration mechanisms, and especially on any anisotropy in magnetospheric content which augments the relative distribution at lower pitch angles.

Acknowledgments

This work is a development from studies undertaken at York University, Toronto, while the author held a Research Fellowship from the National Research Council of Canada. The present investigation now forms part of research being carried out within the Mathematics Department of the Mitchell College of Advanced Education.

References

Alfven, H., and Falthammar, C. G. (1963). 'Cosmical Electrodynamics' (Clarendon: Oxford).
Block, L. P. (1967). *Space Sci. Rev.* **7**, 198.
Catchpoole, J. R. (1966). *J. Atmos. Terr. Phys.* **28**, 75.
Catchpoole, J. R. (1969). *Aust. J. Phys.* **22**, 733.
Catchpoole, J. R. (1971). *Aust. J. Phys.* **24**, 881.
Catchpoole, J. R. (1973). *J. Atmos. Terr. Phys.* **35**, 861.
Gustafsson, G. (1973). *J. Geophys. Res.* **78**, 25.

Manuscript received 5 July 1974