Aust. J. Phys., 1975, **28**, 15–20

The Application of the String Model of Hadrons to the Inelastic Scattering of Electrons

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Abstract

A simple string model of hadrons, which is in agreement with scaling, is applied to the deep inelastic scattering of electrons by the nucleon and is used to make predictions of the average charge and the types of particles produced. In particular, the total baryon number of the particles produced in the photon fragmentation region is shown to be 0 for $x \approx 0$ and 1 for $x \approx 1$, where x is the Bjorken scaling variable.

Introduction

The concept of the relativistic string (Nambu 1970; Susskind 1970*a*, 1970*b*) whose action integral is proportional to the area of the surface swept out by the string (Goto 1971; Minami 1972; Goddard *et al.* 1973) is leading to a better understanding of dual models of hadrons. However, most subsequent developments of the concept have been mathematical, rather than being aimed at testing the concept of the string against experiment. It seems worth while to look for simple consequences of the concept, which do not depend on complicated mathematical questions such as the number of dimensions of space–time needed for consistency, and which can be tested against experiment. In this paper, a simple string model of hadrons is applied to the deep inelastic scattering of electrons (or muons) by nucleons to yield results that may be tested against experiment.

String Model

Hadrons have been described (Tassie 1973*b*, 1974) as finite lengths of relativistic string in a treatment in which the relativistic string was taken to be a line of quantized magnetic flux. The existence of two types of quarks was assumed: those with magnetic charges of $+g$ and $-2g$, where $g = \frac{1}{4}\hbar c/e$ and e is the smallest unit of electric charge. A quark of magnetic charge $+g$ is an end of a string, and a quark of magnetic charge $-2g$ is a change of the sense of direction of the magnetic flux along the string. A string model depiction of a baryon is shown in Fig. 1, in which the arrowheads indicate the direction of magnetic flux along the string.

Alternatively, Nielsen and Olesen (1973) have considered the string, not as a line of quantized magnetic flux, but as a line of quantized flux associated with a Higg's vector field. The mathematics of the two cases is very similar and, in the present paper, we are not concerned whether the quantized flux associated with the string is magnetic or of some other kind. The essential features required from the previous treatment (Tassie 1973*a*, 1974) are that the string has a sense of direction and that there are two kinds of quarks. The magnetic charge of the previous treatment may be replaced by the 'charm' of the two-triplet model of Nambu (Bacry *et al.* 1964; Nambu 1965, 1966), so that magnetic charges of $+g$ and $-2g$ are here replaced by charm quantum numbers $C = +1$ and -2 respectively. As formerly all finite pieces of string had zero total magnetic charge, so here all hadrons have zero total charm. The zero-point energy of the vibrations of the string provides the rest mass of the nucleon. We assume throughout that the rest mass of the nucleon is spread uniformly along the length of the string.

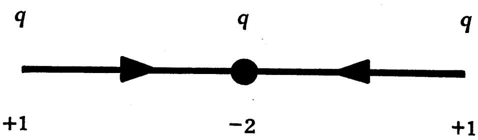

Fig. 1. String model depiction of a baryon, with the quarks q and their C quantum numbers indicated. The arrowheads show the direction of flux in the quantized flux lines.

Inelastic Electron Scattering

Scaling in the deep inelastic scattering of electrons by the nucleon is explained on the parton model (Bjorken and Paschos 1969; Feynman 1972) as the electron colliding with a parton, i.e. a point charge possessing a fraction x of the rest mass of the nucleon (Tassie 1973*b*), which then recoils as if free. The fraction x is called the Bjorken scaling variable and is given by

$$x = \tfrac{1}{2}q^2/M\nu\,,$$

where ν is the energy loss of the electron in the laboratory frame, q is the four-momentum transfer and M is the mass of the nucleon.

We assume throughout that quarks are the point charges with which the electron interacts, i.e. that the electric charge within the nucleon is concentrated in point charges at the ends and at the change in sense of direction of the string. We also assume that, in the interaction (or transfer of energy and momentum) between the electron and the nucleon, at most one break occurs in the string constituting the nucleon. After the interaction with the electron, the resulting pieces of string may break up further, but this does not affect the scattered electron. Multiple scattering of the electron by the string is negligible because of the weakness of the electromagnetic interaction.

Collisions, in which the string does not break, correspond to elastic scattering or to inelastic scattering with the excitation of a nucleonic resonance. Interactions with the electron, in which the string breaks, correspond to inelastic scattering, and x is the fraction of the string which was struck by the electron. It should be noted that, in this treatment, quarks are not particles. They are ends or changes in direction of flux of the string, and this is why free quarks do not occur. The electric charges and SU(3) quantum numbers are assigned to these ends and changes in direction.

At the break of the string, two new ends are created with $C = +1$ and -1, corresponding to the creation of a quark and an antiquark. The string breaks into two pieces, one piece recoiling with momentum approximately equal to the momentum lost by the scattered electron, and the other remaining (or spectator) piece having only a small velocity in the laboratory frame. Both the recoiling and spectator pieces may break up further if sufficient energy is available. In the terminology of inclusive reactions, the hadrons coming from the recoiling piece are in the photon fragmentation region, and those from the spectator piece are in the target fragmentation region (Brasse 1974; Dakin 1973).

Baryon Number of Hadrons in Final State

Let us assume initially for simplicity that the quark with $C = -2$ is exactly half-way along the string. Then, for $x < \frac{1}{2}$, the recoiling piece does not contain the central quark and so is a meson with baryon number 0. Although it may break into several pieces, including possible baryon and antibaryon pairs, the total baryon number of all the hadrons coming from the recoiling piece (i.e. in the photon fragmentation region) must be 0. However, for $x > \frac{1}{2}$, the recoiling piece contains the central $C = -2$ quark, and so is a baryon with baryon number 1. If the recoiling piece breaks up, the total baryon number of the pieces must be 1.

While, by symmetry, the average position of the $C = -2$ quark will be half-way along the string, in general it is expected that the position of the $C = -2$ quark will be determined by a wave function of its position along the string. This wave function must vanish for $x = 0$ and 1. Thus $x < \frac{1}{2}$ and $x > \frac{1}{2}$ in the above predictions should be replaced by $x \approx 0$ and $x \approx 1$ respectively. Also, the same considerations apply to the scattering of any leptons by the nucleon.

Thus, in the deep inelastic scattering of leptons by nucleons, for x small ($x \approx 0$), the recoiling piece of the nucleon will be a hadron or hadrons with total baryon number $B = 0$; while, for x large ($x \approx 1$), the recoiling piece will be a hadron or hadrons with total baryon number $B = 1$. The experimental data on inclusive reactions, which are available for $x \approx 0$, show that nucleons have only a small probability of coming out in the photon fragmentation region (Brasse 1974), so that part of the above result is confirmed by experiment.

Ratio of Neutron to Proton Scattering

For $x \approx 0$, only the end quarks with $C = +1$ contribute to inelastic electron scattering. However, for $x \approx 1$, all quarks contribute, although the central quark may not be equally as effective as the end quarks, since the probability of the string breaking is not necessarily the same when a central quark is struck as when an end quark is struck. The ratios of the inelastic scattering from neutrons to that from protons are

$$\left(\frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{p}}}\right)_{x\approx 0} = \sum_i{}^1 \mathscr{P}(\mathrm{N},i)\,Q_i^2 \Big/ \Big(\sum_i{}^1 \mathscr{P}(\mathrm{P},i)\,Q_i^2\Big),$$

$$\left(\frac{\sigma_{\mathrm{n}}}{\sigma_{\mathrm{p}}}\right)_{x\approx 1} = \frac{\sum_i{}^1 \mathscr{P}(\mathrm{N},i)\,Q_i^2 + \alpha\sum_i{}^{-2} \mathscr{P}(\mathrm{N},i)\,Q_i^2}{\sum_i{}^1 \mathscr{P}(\mathrm{P},i)\,Q_i^2 + \alpha\sum_i{}^{-2} \mathscr{P}(\mathrm{P},i)\,Q_i^2},$$

where $\sum^1$ and $\sum^{-2}$ indicate summations subject to the conditions $C_i = 1$ and -2 respectively, $\mathscr{P}(\mathrm{N},i)$ and $\mathscr{P}(\mathrm{P},i)$ are the probabilities of a quark of type i occurring in the neutron and proton respectively, Q_i is the electric charge of a quark of type i, and α is the ratio of the effectiveness of a central quark to that of an end quark.

We consider the three sets of triplet structures listed in Table 1. These are defined by the values of the charges Q_p, Q_n and Q_λ assigned to the p, n and λ quarks respectively for either value of the charm quantum number C. Briefly the models are: (A) Gell-Mann–Zweig quarks (Gell-Mann 1964; Zweig 1965) for which we assume that both the $C = 1$ and -2 types of quarks have the same electric charges; (B) the two-triplet model of Nambu (1965, 1966) and Bacry *et al.* (1964) for which all quarks have integral charges; and (C) an additional two-triplet model in which all $C = 1$ quarks have charges of the same magnitude.

Table 1. Assignment of quark quantum numbers

Quark model	Quantum numbers C	Q_p	Q_n	Q_λ
Gell-Mann–Zweig quarks		$+2/3$	$-1/3$	$-1/3$
Nambu two-triplet model	$+1$	$+1$	0	0
	-2	0	-1	-1
Two-triplet model	$+1$	$+1/2$	$-1/2$	$-1/2$
	-2	$+1$	0	0

Table 2. Comparison of predicted and experimental (Bloom 1973) ratios of electron inelastic scattering cross sections for neutron and proton targets

Results	$(\sigma_n/\sigma_p)_{x\approx 0}$	$(\sigma_n/\sigma_p)_{x\approx 1}$	
Model A	2/3	2/3	
Model B	1/2	$1/2 \leqslant (2+2\alpha)/(4+\alpha) \leqslant 2$	[4/5]*
Model C	1	$1/2 \leqslant (3+2\alpha)/(3+4\alpha) \leqslant 1$	[5/7]*
Bloom (1973)	1	$\sim 0 \cdot 4$	

* Value of expression for $\alpha = 1$

The results for σ_n/σ_p for these models are listed in Table 2. In the case of $x \approx 1$, the actual expression, its range and the value for $\alpha = 1$ are given for models B and C. The comparison with the experimental results of Bloom (1973) shows that only model C is not in contradiction with experiment.

Properties of Hadrons in Final State

On the assumption that the break in the string has equal probability of occurring as $p\bar{p}$, $n\bar{n}$, or $\lambda\bar{\lambda}$, the average charge of the recoiling piece can be related to the charges assigned to the quarks for $x \approx 0$. However, for $x \approx 1$, an additional assumption is needed about the relative effectiveness of $C = 1$ and -2 quarks.

For $x \approx 0$, the average charge of the recoiling piece of the hadronic string is given by

$$\langle Q \rangle = \langle Q \rangle_s + \langle Q \rangle_b ,$$

where $\langle Q \rangle_s$ is the average charge of the end quark struck by the electron, i.e.

$$\langle Q \rangle_s = \sum_i^1 \mathscr{P}(i)\, Q_i^3 \Big/ \Big(\sum_i^1 \mathscr{P}(i)\, Q_i^2 \Big) ,$$

while $\langle Q \rangle_b$ is the average charge of $\bar{p}$, $\bar{n}$ and $\bar{\lambda}$ antiquarks with $C = -1$, which constitute the broken end of the recoiling piece. The values of $\langle Q \rangle$ obtained from the three models for proton and neutron targets are:

	Model A	Model B	Model C
Proton target	5/9	2/3	1/3
Neutron target	1/3	2/3	0

These values should be compared with the average of the total charge of all hadrons eventuating from the recoiling piece, and they should be identical with the total forward charge defined by Brasse (1974). Measurements of the total forward charge have been made (see Ballam *et al.* 1974 and Dakin *et al.* 1974, both cited after Brasse 1974) but these results do not allow definite conclusions to be reached at this stage.

Table 3. Ratios R_j of types of mesons produced for $x \approx 0$ and $x_F = 1$

Model	Proton target R_{π^+}	R_{π^0}	R_{π^-}	R_{K^+}	R_{K^0}	Neutron target R_{π^+}	R_{π^0}	R_{π^-}	R_{K^+}	R_{K^0}
A	8	9/2	1	8	1	4	3	2	4	2
B	2	1	0	2	0	1	1/2	0	1	0
C	2	3/2	1	2	1	1	3/2	2	1	2

When the recoiling piece does not break up, predictions can be made of the ratios of the various types of meson that the recoiling piece may be, for $x \approx 0$ and for proton and neutron targets. The predicted ratios R_j, where j designates one of π^+, π^0, π^-, K^+ or K^0, are listed in Table 3. A particle possessing the whole of the momentum lost by the electron will be the sole recoiling piece. For it we have $x_F = 1$, where x_F is the Feynman scaling variable defined by

$$x_F = p_{\parallel}^{*}/p_{\max}^{*}.$$

In this equation, $p_{\parallel}^{*}$ is the momentum of the observed particle in the direction parallel to that of the momentum loss of the electron, and $p_{\max}^{*}$ is the maximum possible value of $p_{\parallel}^{*}$, both quantities being measured in the centre-of-mass frame of the final state hadrons. We note that the predicted ratios given in Table 3 for model A are identical with those obtained by Pantin (1972) for a dual parton model. Furthermore, it should be noted that, independently of the assignment of charges to the quarks, we have

$$R_{K^0}/R_{\pi^-} = R_{K^+}/R_{\pi^+} = 1$$

for both neutron and proton targets.

Discussion

A simple string model, has been used here to calculate various quantities which can be compared with experiment. The validity of the assumptions of the model can ultimately only be tested by experiment. The phenomenon of scaling in electron scattering requires that the charge in the nucleon is concentrated in points or at least in small regions. The assumption of only a single break in the string during the interaction with the electron is partly confirmed by the lack of nucleons (Brasse 1974) in the photon fragmentation region for $x \approx 0$, and yields the prediction that nucleons should appear in the photon fragmentation region for $x \approx 1$.

The assumption that the break in the string has equal probabilities of occurring as $p\bar{p}$, $n\bar{n}$, or $\lambda\bar{\lambda}$ can only be an approximation, and should only be valid at energies large compared with the difference between the kaon and pion masses. However, SU(3) breaking should not affect the equality

$$R_{\pi^+}/R_{\pi^-} = R_{K^+}/R_{K^0}$$

for both neutron and proton targets. Finally, we note that the predictions derived here for electron scattering apply also to muon scattering.

Acknowledgment

The author is very grateful to Dr B. G. Kenny for stimulating discussions and many comments.

References

Bacry, H., Nuyts, J., and van Hove, L. (1964). *Phys. Lett.* **9**, 279.
Bjorken, J. D., and Paschos, E. A. (1969). *Phys. Rev.* **185**, 1975.
Bloom, E. D. (1973). A.I.P. Conf. Proc. No. 12, p. 277.
Brasse, F. W. (1974). Int. Symp. on Electron and Photon Interactions at High Energies, Bonn, 1973 (North–Holland: Amsterdam).
Dakin, J. (1973). A.I.P. Conf. Proc. No. 12, p. 346.
Feynman, R. P. (1972). 'Photon–Hadron Interactions' (Benjamin: Reading, Mass.).
Gell-Mann, M. (1964). *Phys. Lett.* **8**, 214.
Goddard, P., Goldstone, J., Rebbi, C., and Thorn, C. B. (1973). *Nucl. Phys.* B **56**, 109.
Goto, T. (1971). *Progr. Theor. Phys.* **46**, 1560.
Minami, M. (1972). *Progr. Theor. Phys.* **48**, 1308.
Nambu, Y. (1965). Proc. 2nd Coral Gables Conf. on Symmetry Principles at High Energy (Freeman: San Francisco).
Nambu, Y. (1966). 'Preludes in Theoretical Physics' (Ed. A. de-Shalit, H. Feshbach and L. van Hove) p. 133 (North–Holland: Amsterdam).
Nambu, Y. (1970). Proc. Int. Conf. on Symmetries and Quark Models, Wayne State Univ., 1969, p. 296 (Gordon and Breach: New York).
Nielsen, H. B., and Olesen, P. (1973). *Nucl. Phys.* B **61**, 45.
Pantin, C. F. A. (1972). *Nucl. Phys.* B **46**, 205.
Susskind, L. (1970*a*). *Phys. Rev.* D **4**, 1182.
Susskind, L. (1970*b*). *Nuovo Cimento* A **69**, 457.
Tassie, L. J. (1973*a*). *Phys. Lett.* B **46**, 397.
Tassie, L. J. (1973*b*). 'The Physics of Elementary Particles' (Longmans: London).
Tassie, L. J. (1974). *Int. J. Theor. Phys.* **9**, 167.
Zweig, G. (1965). 'Symmetries in Elementary Particle Physics. 1964 International School of Physics *Ettore Majorana*' (Academic Press: New York).

Manuscript received 5 August 1974