TIME DIRECTION OF INFORMATION PROPAGATION AND COSMOLOGY

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Abstract

In conventional electrodynamic theory, the advanced potential solution of Maxwell's equations is discarded on the *ad hoc* basis that information can be received from the past only and not from the future. This difficulty is overcome by the Wheeler–Feynman absorber theory, but unfortunately the existence of a completely retarded solution in this theory requires a steady-state universe. In the present paper conventional electrodynamics is used to obtain a condition which, if satisfied, allows information to be received from the past only, and ensures that the retarded potential is the only consistent solution. The condition is that a function U^{a} of the future structure of the universe is infinite, while the corresponding function U^{r} of the past structure is finite. Of the currently acceptable cosmological models, only the steady-state, the open big-bang, and the Eddington–Lemaître models satisfy this condition. In these models there is no need for an *ad hoc* reason for the preclusion of advanced potentials.

I. Introduction

The intriguing problem of the arrow of time has received renewed interest in the past decade. Basically, the problem is that intrinsically time-symmetric differential field equations, such as Maxwell's equations, seem in practice to produce time-asymmetric results, i.e. retarded but not advanced potentials. In general, fields can carry information into the future only and not into the past. If the asymmetry in time is not inherent in the differential equations, it is reasonable to assume that it originates somewhere in the imposed boundary conditions. For the arrow of time to have the same direction everywhere, universal time-asymmetric boundary conditions are needed. Gold (1962) has suggested that just such conditions may arise from the fact that the universe is expanding and not contracting.

An attempt to explain local time asymmetry in terms of cosmological boundary conditions has been made by Hogarth (1962) and Hoyle and Narlikar (1964). These authors used the classical electrodynamic absorber theory of Wheeler and Feynman (1945), in which the radiative reaction on an accelerated charge arises from the induced motion of the other charges in the universe. Unfortunately, the only current cosmology for which absorber theory gives the correct result is the steady-state model, and the position of this model is by no means secure. In the not unlikely event that the steady-state model is disproved, then absorber theory will be disproved also, and the problem of the arrow of time will be still unsolved.

It is the purpose of this paper to examine the problem in terms of the more conventional classical theory. Here radiative reaction and the associated loss of energy from an accelerated electron arise through the action of the electron on itself, as postulated originally by Lorentz (1909). It is assumed here that if the presence of

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the rest of the universe is ignored then the intrinsic time symmetry will allow information to be sent into the past as well as into the future. A search is then made for cosmological conditions which will preclude the transmission of information into the past.

II. Lorentz Theory

According to the Lorentz self-action theory of radiative reaction, one part A of an accelerated electron is acted on by the retarded potential due to the motion of another part B of the electron. This action removes energy from the electron and an equal amount of energy is carried off by the retarded potential field emitted by the electron. From the time symmetry of Maxwell's equations, the time reverse is also possible, where part A feels the advanced potential of part B. Here the electron is accelerated even more. For the gain in energy to be balanced by the energy carried away by the emitted field, this field must have an advanced potential. In terms of information transmission, in the first case A receives retarded information about the motion of B while in the second case A receives advanced information from B. In this model, therefore, whether retarded or advanced radiation is emitted will depend upon whether information can be received from the past or from the future respectively.

III. Reception of Information

Consider a source S transmitting information to a detector of bandwidth W at the origin and let P be the power received in this bandwidth from S. The rate of reception C of information in the band is (Bell 1956)

$$C = W\log_2(1+P/N)\,, \tag{1}$$

in units of bits per second. In equation (1) N is the total noise power which can be written as $N_{\mathrm{d}}+N_{\mathrm{u}}$, where N_{d} is due to the detector itself and N_{u} is due to other sources besides S which can also transmit in that band.

If we now assume that the detector can receive both retarded and advanced information from S, we can write

$$C^{\mathrm{r}} = W\log_2\{1+P^{\mathrm{r}}/(N_{\mathrm{d}}+N_{\mathrm{u}}^{\mathrm{r}}+N_{\mathrm{u}}^{\mathrm{a}})\} \tag{2a}$$

and

$$C^{\mathrm{a}} = W\log_2\{1+P^{\mathrm{a}}/(N_{\mathrm{d}}+N_{\mathrm{u}}^{\mathrm{r}}+N_{\mathrm{u}}^{\mathrm{a}})\}\,, \tag{2b}$$

where the superscripts r and a refer to information, power, and noise from the past (retarded) and from the future (advanced) respectively. N^{a} must be included because of the Copernican principle, which demands that S be not unique in its ability to send information into the past.

(*a*) *Copernican Principle*

The Copernican principle forms the basis of modern cosmology (Bondi 1961). Briefly, it implies that there is no unique or specially privileged object in the universe. Applied to the present case, it means that if there is one such source as S which can transmit information into the past then there must be other such sources which can do likewise, spread fairly evenly throughout the universe. The principle allows us to write $N_{\mathrm{u}}^{\mathrm{a}}$ in the form

$$N_{\mathrm{u}}^{\mathrm{a}} = g^{\mathrm{a}}\,\sigma_0\,L^{\mathrm{a}}\,U^{\mathrm{a}}\,, \tag{3}$$

where L^{a} is the power emitted into the past by a typical source, σ_0 is the absorption cross section of the detector at the origin, U^{a} is a function of the structure of the universe, and g^{a} is a nonzero fraction that is dependent upon the distribution of noise power at the detector over all bandwidths. Similarly we have

$$N_{\mathrm{u}}^{\mathrm{r}} = g^{\mathrm{r}}\,\sigma_0\,L^{\mathrm{r}}\,U^{\mathrm{r}}\,. \tag{4}$$

The exact form of these equations is derived in Section IV.

Writing the power L in units of P, the power received by the detector from S, that is, putting $L = lP$, gives

$$N_{\mathrm{u}}^{\mathrm{a}} = g^{\mathrm{a}}\,\sigma_0\,l^{\mathrm{a}}\,P^{\mathrm{a}}\,U^{\mathrm{a}} = P^{\mathrm{a}}\,u^{\mathrm{a}} \tag{5a}$$

and

$$N_{\mathrm{u}}^{\mathrm{r}} = g^{\mathrm{r}}\,\sigma_0\,l^{\mathrm{r}}\,P^{\mathrm{r}}\,U^{\mathrm{r}} = P^{\mathrm{r}}\,u^{\mathrm{r}}\,, \tag{5b}$$

where

$$u = g\sigma_0\,lU\,. \tag{6}$$

The Copernican principle demands that l is nonzero.

Of course if *no* source can emit radiation into the past then no noise can be received from the future, irrespective of the size of U^{a} in equation (3). From Section II, this would be true if no advanced information could be transmitted across the electron, i.e. if C^{a} were zero for the receipt of information by A from B. This would follow if C^{a} were zero for the general case of detector and source. We can thus write

$$C^{\mathrm{a}} = 0 \quad \rightarrow \quad N_{\mathrm{u}}^{\mathrm{a}} = 0\,. \tag{7}$$

We do not need to know the exact functional relationship between C^{a} and $N_{\mathrm{u}}^{\mathrm{a}}$.

(*b*) *Condition for* $C^{\mathrm{a}} = 0$

Substituting equations (5a) and (5b) into (2a) and (2b) gives

$$C^{\mathrm{r}} = W \log_2\{1+P^{\mathrm{r}}/(N_{\mathrm{d}}+P^{\mathrm{r}}u^{\mathrm{r}}+N_{\mathrm{u}}^{\mathrm{a}})\} \tag{8}$$

and

$$C^{\mathrm{a}} = W \log_2\{1+(N_{\mathrm{d}}/P^{\mathrm{a}}+P^{\mathrm{r}}u^{\mathrm{r}}/P^{\mathrm{a}}+u^{\mathrm{a}})^{-1}\}\,. \tag{9}$$

We seek a condition which will ensure that $C^{\mathrm{a}} = 0$, $C^{\mathrm{r}} \neq 0$, in accord with the observational evidence that information can be received only from the past. From equation (9) we see that C^{a} is zero if

$$N_{\mathrm{d}}/P^{\mathrm{a}}+P^{\mathrm{r}}u^{\mathrm{r}}/P^{\mathrm{a}}+u^{\mathrm{a}} = \infty\,.$$

The necessity that C^{r} be nonzero precludes making N_{d} or $P^{\mathrm{r}}u^{\mathrm{r}}$ infinite. Thus we have the choice of conditions to impose: (i) set P^{a} equal to zero or (ii) make u^{a} infinite. The choice (i) corresponds to the conventional method of introducing a time asymmetry, i.e. the *ad hoc* rejection of the advanced potential solution of Maxwell's equations. Alternatively, this would follow from the adoption of absorber theory in a steady-state universe. Since we do not wish to use either of these methods, we are left with choice (ii) which will be determined by the particular model universe adopted.

If u^{a} is made infinite then C^{a} becomes zero, and from the relation (7) this means that equation (8) becomes the usual expression for the information received from the past, provided u^{r} is not infinite also. We note also that it follows from (7) and (5a) that an infinite u^{a} implies a zero P^{a}; this does not alter the result $C^{\mathrm{a}} = 0$ from (9) but reinforces it. A zero P^{a} means that a source cannot emit energy, and thus information, by means of the advanced potential. The result follows from an infinite u^{a} and does not need to be postulated separately.

The validity of the imposition of the time-asymmetric conditions $u^{\mathrm{a}} = \infty$, $u^{\mathrm{r}} \neq \infty$ will depend upon the particular time-asymmetric cosmological model eventually accepted as describing the actual universe. In some models this condition will be satisfied, in others it will not. In this paper we shall restrict ourselves to examining the general relativistic models and the steady-state model. It is more convenient to study the function U in place of u, and from (6) the conditions are then $U^{\mathrm{a}} = \infty$, $U^{\mathrm{r}} \neq \infty$.

IV. Calculation of U

To find the function U^{a} in equation (3), we must calculate the amount of noise in bandwidth W which the detector would receive *assuming* sources *could* radiate into the past. If the result of this calculation is infinite, then our condition is satisfied, and in reality sources would then *not* be able to radiate into the past.

To find the amount of radiant energy in the vicinity of the detector, we first note that there is a slight nonlinearity in the electromagnetic field brought about by photon–photon interaction (Karplus and Neuman 1950). The interaction cross section is very small but nevertheless finite, and at the extremely high radiation densities considered here collisions and energy exchange among photons will be important. The effect will be to spread the energies of the photons over some equilibrium distribution. The actual form of this distribution is not critical to our argument, provided it is reasonably well behaved. For example, if we assume a Planckian distribution, we would have the energy density in bandwidth $\mathrm{d}\nu$ given by

$$E_{\nu}^{\mathrm{a}}\,\mathrm{d}\nu \propto \nu^{3}\,\mathrm{d}\nu\{\exp(h\nu/kT_{E})-1\}^{-1},$$

where T_E is the characteristic distribution temperature which is dependent upon the total energy density. For example, a Stefan–Boltzmann type law would imply that T_E is proportional to the fourth root of the energy density. It follows that the noise in any bandwidth will be infinite if the total energy density is infinite.

The other condition that U^{r} is finite, i.e. that the retarded radiation from the rest of the universe is finite, is satisfied for all models which obey the Olbers condition of a dark sky. This is so for all acceptable models.

(a) General Relativistic Models

A formula for the total retarded radiant power received by the detector at the present epoch T for general relativistic models with the standard Robertson–Walker line element has been given by Pegg (1971) as

$$\sigma_0 I^{\mathrm{r}} = \sigma_0 cn(T)\, R^{-1}(T) \int_{T_\mathrm{i}}^{T} L^{\mathrm{r}}(t_1)\, R(t_1) \times \exp\left(-cn(T)\, R^{3}(T) \int_{t_1}^{T} \sigma(t)\, R^{-3}(t)\, \mathrm{d}t\right) \mathrm{d}t_1\,, \qquad (10)$$

where $n(T)$ is the present number density of sources, $L^{\mathrm{r}}(t_1)$ the retarded power emitted by a source, R the scale factor, and σ the cross section of a typical absorber. The limit T_i represents the earliest time the universe was in existence with $T_\mathrm{i} \leqslant t_1 \leqslant t \leqslant T$. The corresponding formula for the advanced power is easily found from a time inversion to be

$$\sigma_0 I^{\mathrm{a}} = \sigma_0 cn(T)\, R^{-1}(T) \int_{T}^{T_\mathrm{f}} L^{\mathrm{a}}(t_1)\, R(t_1) \times \exp\left(-cn(T)\, R^{3}(T) \int_{T}^{t_1} \sigma(t)\, R^{-3}(t)\, \mathrm{d}t\right) \mathrm{d}t_1\,, \qquad (11)$$

where now $T \leqslant t \leqslant t_1 \leqslant T_\mathrm{f}$, T_f being the most distant future time the universe is in existence.

We note that these calculations have ignored photon–photon scattering. This, however, does not affect the result, as we are interested in the total energy (momentum) of the photons which reach the detector and not in the particular photons that do so. Conservation of momentum in collisions ensures the results.

In equation (11), $L^{\mathrm{a}}(t_1)$ will represent the power emitted by a typical source into the past, assuming such a source could emit power into the past. This power will change with the time t_1 and we can write

$$L^{\mathrm{a}}(t_1) = L^{\mathrm{a}}(T)\, F(t_1)\,, \qquad (12)$$

where $F(t_1)$ is a function of time only. Writing g^{a} as the fraction of the total power in the bandwidth W, we obtain from equations (11) and (12) a formula for the noise $N_{\mathrm{u}}^{\mathrm{a}}$ in the form of (3) with

$$U^{\mathrm{a}} = cn(T)\, R^{-1}(T) \int_{T}^{T_\mathrm{f}} F(t_1)\, R(t_1) \times \exp\left(-cn(T)\, R^{3}(T) \int_{T}^{t_1} \sigma(t)\, R^{-3}(t)\, \mathrm{d}t\right) \mathrm{d}t_1\,. \qquad (13)$$

When an accelerated electron emits energy into the past, the electron gains energy and so increases its ability to emit into the past. If we allow both the retarded and advanced solutions, then on the average the electron will neither gain nor lose energy. Therefore, if we are testing for the consistency of the assumption that a source can emit only advanced radiation, we must put $F(t_1) > 1$. If we are testing for the possibility that a source can emit both advanced and retarded radiation we put $F(t_1) = 1$. Thus in the expression for U^{a} we have $F(t_1) \geqslant 1$.

(i) *Ever-expanding Models*

For models which expand forever it seems likely that the cross section $\sigma(t)$ would have an upper bound as t increases, and even if this were not so it is extremely unlikely that $\sigma(t)$ would increase as quickly as $R^2(t)$. Thus the exponent in equation (13) will always be finite for all t_1, and the exponential factor will be greater than some nonzero constant. Thus with $F(t_1) \geqslant 1$, we have

$$U^{\mathrm{a}} \geqslant K \int_T^{\infty} R \,\mathrm{d}t_1 = K A_{\mathrm{f}} ,$$

where K is nonzero and A_{f} is the future area under the $R(t)$ versus t curve for the model. This is indeed infinite for such models, and the present condition is satisfied.

(ii) *Static Model*

In the static model $R(t)$ is constant and there is no time-asymmetry in the model. We can therefore write $U^{\mathrm{a}} = U^{\mathrm{r}}$, and the result of the Olbers paradox argument that U^{r} is finite, although high, also ensures that U^{a} is finite. Models for which $R(t)$ asymptotically becomes constant also have a finite U^{a}.

(iii) *Closed Model*

The exponential factor in equation (13) is always less than unity, and in the closed model T_{f} is finite. Thus $F(t_1)$ will have an upper bound and so

$$U^{\mathrm{a}} \leqslant \text{const.}\ A_{\mathrm{f}} .$$

Here A_{f} is finite and so U^{a} is finite.

(iv) *Condition for* $U^{\mathrm{r}} \neq \infty$

As mentioned above, the condition that U^{r} is finite is just the normal Olbers paradox condition. A time-reversal of the above arguments shows that, assuming the stars have been shining for a sizeable fraction of the life of the universe, U^{r} is finite for models with a finite past area under the R versus t curve. For models in which R eventually becomes static and nonzero in the infinite past, U^{r} is finite also.

(b) *Steady-state Model*

Equation (10) was derived by Pegg (1971) on the assumption that the proper number density of sources is given by

$$n(t) = n(T)\, R^{-3}(t)\, R^{3}(T) .$$

In the steady-state model, however, because of continual creation,

$$n(t) = n(T) .$$

This condition modifies both integrals in equation (13) to give

$$U^{\mathrm{a}} = cn(T)\,R^{-4}(T)\int_{T}^{T_{\mathrm{f}}} R^{4}(t_1) \times \exp\left(-cn(T)\,\sigma(T)\int_{T}^{t_1} \mathrm{d}t\right)\mathrm{d}t_1\,,$$

since $L^{\mathrm{a}}(t)$ and $\sigma(t)$ are now constant by the nature of the model.

Substituting for $R(t) = \exp Ht$ (Bondi 1961), where H is Hubble's constant, we find that U^{a} is infinite if and only if

$$c \leqslant 4H\sigma^{-1}n^{-1}(T)\,. \tag{14}$$

Taking the typical sources to be galaxies, and using Bonnor's (1964) estimate for the absorption of radiation passing through a galaxy, we find

$$\sigma n(T) \sim H/c\,, \tag{15}$$

and thus the relation (14) is just satisfied.

We have written (14) such that all the properties of the structure of the universe are on one side of the equation and the maximum velocity at which information can propagate is on the other. Read in this way, the relations (14) and (15) indicate that, given the structure of the steady-state universe, information seems to travel at close to the maximum allowable speed when propagation into the past is prohibited. This interesting result is not considered further here, however.

Table 1

CONDITIONS SATISFIED BY CLASSES OF COSMOLOGICAL MODELS

Models which satisfy both conditions allow information to be sent into the future only

Model class	Condition $U^{\mathrm{a}} = \infty$	Condition $U^{\mathrm{r}} \neq \infty$	Both conditions
I	Not satisfied	Satisfied	Not satisfied
II	Satisfied	Satisfied	Satisfied
III	Satisfied	Satisfied	Satisfied
IV	Not satisfied	Satisfied	Not satisfied
V	Not satisfied	Satisfied	Not satisfied
VI	Satisfied	Not satisfied	Not satisfied
S-S	Satisfied	Satisfied	Satisfied

V. Summary of Results

If the two conditions $U^{\mathrm{a}} = \infty$ and $U^{\mathrm{r}} \neq \infty$ are satisfied by a model, then information can only be transmitted into the future, and an electron can only transmit energy through the retarded potential. To summarize the results, we use the classification of general relativistic models given by Bondi (1961). The results are shown in Table 1. The classes represent the following models: I, Einstein (static); II, expanding from $R = 0$ to ∞ (big-bang open model); III, Eddington–Lemaître;

IV, expanding from $R = 0$ to constant; V, expanding and recontracting (big-bang closed model); VI, contracting and re-expanding infinitely; S-S, steady-state.

Since the equations of general relativity are time symmetric as well, there are also seven possible contracting models which are the time inverses of those listed. If the first condition $U^{\mathrm{a}} = \infty$ is satisfied for the expanding model then U^{r} is infinite for the corresponding contracting model, which does not satisfy the second condition. Similarly, if the second condition $U^{\mathrm{r}} \neq \infty$ is satisfied for the expanding model, the first condition is not satisfied for the contracting model. In all the expanding models listed, at least one condition is satisfied and therefore no contracting models can satisfy both conditions. We find therefore that of the 14 possible classes of general relativistic and steady-state models only **3** satisfy the present requirements. In these models information can be propagated into the future only.

VI. Conclusions

The structures of the ever-expanding big-bang model, the steady-state model, and the Eddington–Lemaître model are such as to prohibit the propagation of information into the past. In these models energy and information can only be carried by fields with a retarded potential. These results follow from the conventional, but time-symmetrized, Lorentz theory of self-action as the cause of radiative reaction. The results may be contrasted with those based on the Wheeler–Feynman absorber theory, which allows retarded radiation in only the steady-state model.

VII. References

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