ESTIMATION OF THE PROPERTIES OF QUARKS

II.* SCALAR QUARK–QUARK INTERACTION

By G. B. Smith†

[*Manuscript received June* 11, 1970]

Abstract

Describing the hadrons by a relativistic independent quark model, using the Dirac equation with a scalar potential for the effective interaction, the range of the quark–quark interaction is found to lie between 0·3 and 0·4 fm. The hadronic spectrum is shown to be independent of the quark mass and the form of the quark–quark interaction.

I. Introduction

In Part I (Smith and Tassie 1970; present issue, pp. 615–25) we estimated the quark mass and the root-mean-square separation distance of quarks in the hadronic core when the quark–quark interaction was assumed to be a vector-type interaction. It has been suggested (Lipkin and Tavkhelidze 1965) that magnetic moment considerations imply a scalar-type quark–quark interaction, rather than vector. However, this is not certain (Kokkedee 1969) as these considerations assume that the quark magnetic moment is a function of the quark binding rather than of the meson cloud surrounding it.

In the present paper we assume that the quark–quark interaction is a scalar-type interaction and use the Dirac equation for the dynamical description of the hadrons.

II. Dynamical Description

As in Part I, we use an independent quark model of both mesons and baryons and consider only states which differ from the ground state by a change in the quantum numbers of one quark. The quark wavefunction ψ_{nlj} is an eigenfunction of the Dirac equation

$$(c\boldsymbol{\alpha}\cdot\boldsymbol{p}+\beta mc^2+\beta V)\psi_{nlj} = E_{nlj}\psi_{nlj}, \tag{1}$$

where m is the reduced mass of the quark and the spherically symmetric static scalar potential V is taken to have the same form for both baryons and mesons. As for the static vector potential the eigenvalues of (1) are bounded as

$$-mc^2 < E_{nlj} < +mc^2.$$

However, unlike the vector case, as the potential strength is increased an eigenvalue whose energy is positive for a weak potential strength has positive energy for any

* Part I, *Aust. J. Phys.*, 1970, **23**, 615–25.

† Research School of Physical Sciences, Australian National University, P.O. Box 4, Canberra, A.C.T. 2600; present address: Canberra College of Advanced Education, P.O. Box 381, Canberra City, A.C.T. 2601.

potential strength.* We interpret the state with the lowest possible eigenvalue $E_{nlj} = 0$ as a hadron of zero rest mass. Then the hadron mass M_{nlj} corresponding to a bound state at energy E_{nlj} will be

$$M_{nlj}\,c^2 = E_{nlj}\,.$$

As for the vector case (Rose 1961), the solution of equation (1) in a spherically symmetric scalar potential is

$$\psi_{nlj} = \begin{pmatrix} g(r) & \chi^{\mu}_{+k} \\ \mathrm{i}f(r) & \chi^{\mu}_{-k} \end{pmatrix},$$

where $\chi^{\mu}_{\pm k}$ are two component spinors and f and g are the solutions of

$$\frac{\mathrm{d}\mu_1}{\mathrm{d}x} + \frac{K\mu_1}{x} - (\epsilon+v+1)\mu_2 = 0\,, \qquad \frac{\mathrm{d}\mu_2}{\mathrm{d}x} - \frac{K\mu_2}{x} + (\epsilon-v-1)\mu_1 = 0\,, \tag{2}$$

where $\mu_1 = rg$, $\mu_2 = rf$, $x = rmc/\hbar$, $\epsilon = E/mc^2$, and $v = V/mc^2$.

Eigenvalues of equations (2) were found numerically (for V as a square well, a cut-off harmonic oscillator, a Woods–Saxon potential, a Gaussian potential, and an exponential potential) in an analogous manner to the eigenvalues of equations (12) in Part I.

III. Bound States and Bound State Wavefunctions

Bound states of the Dirac equation for a scalar interaction are quite dissimilar to their vector counterparts. Bound states of equations (2) are symmetric in ϵ. If μ_1^+ and μ_2^+ are the bound state wavefunctions for $\epsilon = \epsilon_0 > 0$ then μ_1^- and μ_2^- are bound state wavefunctions at $\epsilon = -\epsilon_0$ where

$$\mu_1^- = (\epsilon_0 - v - 1)\mu_1^+\,, \qquad \mu_2^- = -(\epsilon_0 + v + 1)\mu_2^+\,. \tag{3}$$

As the strength of the scalar interaction increases the bound states do not necessarily become more tightly bound. This can be demonstrated by writing equations (2) as a second-order differential equation in μ_1:

$$\frac{\mathrm{d}^2\mu_1}{\mathrm{d}x^2} - \frac{\mathrm{d}v/\mathrm{d}x}{\epsilon+v+1}\frac{\mathrm{d}\mu_1}{\mathrm{d}x} - \frac{K(K+1)}{x^2}\mu_1 + \{\epsilon^2 - (1+v)^2\}\mu_1 - \frac{K}{x}\frac{\mathrm{d}v/\mathrm{d}x}{\epsilon+v+1}\mu_1 = 0\,. \tag{4}$$

For a square well potential $\mathrm{d}v/\mathrm{d}x = 0$, and the only term of equation (4) containing the potential is $\{\epsilon^2 - (1+v)^2\}\mu_1$ which is symmetric about $v = -1$. The two potentials $v_1 = -1+\delta$ and $v_2 = -1-\delta$, $0 < \delta < 1$, act the same in equation (4). In essence the part of the potential deeper than $v = -1$ acts as a repulsion in this term.

* Adjusting the strength of v in equations (3), Section III, such that $\epsilon_0 \to 0$ we see that $\mu_1^- \nleftrightarrow \mu_1^+$ and $\mu_2^- \nleftrightarrow \mu_2^+$ for an arbitrary form of v. Therefore an eigenfunction at $\epsilon > 0$ cannot become an eigenfunction at $\epsilon < 0$ and vice versa.

It should be noted that even though v_1 and v_2 act the same in equation (4) this does not mean that their eigenvalues are the same. Eigenvalues are determined by matching μ_1 and μ_2 at the edge of the square well potential and

$$\mu_2 = \frac{1}{\epsilon+v+1}\left(\frac{d\mu_1}{dx}+\frac{K\mu_1}{x}\right)$$

is not the same for v_1 and v_2. In general for slowly varying potentials the part of the potential $v < -1$ acts as a repulsion and, for a potential like a Gaussian for example,

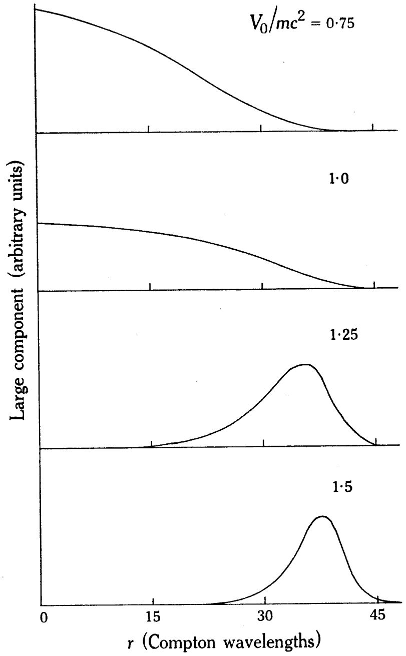

Fig. 1.—Large component of the s-state Dirac scalar wavefunction for a Woods–Saxon potential (equation (5)) at four potential strengths V_0.

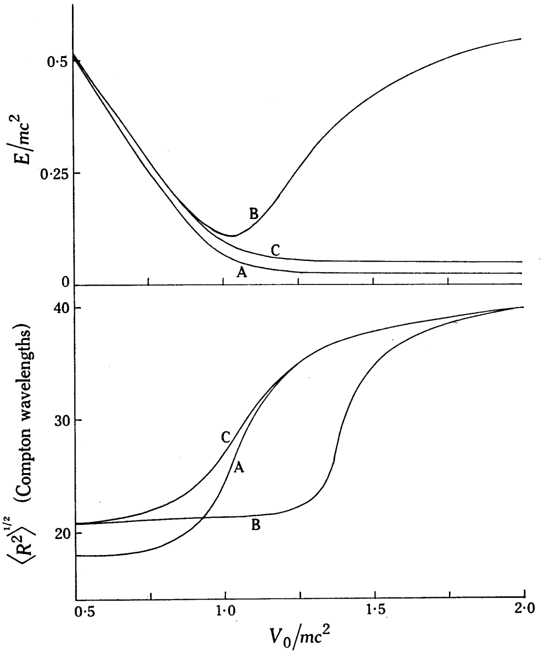

Fig. 2.—Bound state energy E and root-mean-square radius $\langle R^2\rangle^{\frac{1}{2}}$ of a Woods–Saxon wavefunction plotted against potential strength V_0: A, $l = 0$, $j = 1/2$; B, $l = 1$, $j = 1/2$; C, $l = 1$, $j = 3/2$.

acts as a repulsive core. This can be seen in Figure 1, where the large component of the wavefunction $g = \mu_1/r$ for a Woods–Saxon potential

$$V = -V_0/[1+\exp\{-13{\cdot}2(1-x/40)\}] \tag{5}$$

is plotted for four values of V_0. The effect also shows up in the bound state energy and the root-mean-square radius of the wavefunction. For a potential (5) these are shown in Figure 2. For $j = l-\frac{1}{2}$, $K = l$ (Rose 1961) and we see that bound states are not necessarily more tightly bound as the potential strength increases.

IV. Quark Parameters

In Part I, independently of the dynamical description, we obtained the result that the root-mean-square separation distance $\langle R^2\rangle^{\frac{1}{2}}$ of the quarks in the core is

$$0\cdot 1\ \mathrm{fm} \lesssim \langle R^2\rangle^{\frac{1}{2}} \lesssim 1\cdot 0\ \mathrm{fm}. \tag{6}$$

Further, we estimated from the hadronic spectra the mass ranges in which the bound state energies of the Dirac equation must lie, namely

$$600\ \mathrm{MeV}/c^2 \lesssim M(1\mathrm{s}_{\frac{1}{2}}) \lesssim 1320\ \mathrm{MeV}/c^2, \tag{7a}$$

$$400\ \mathrm{MeV}/c^2 \lesssim M(1\mathrm{p}_{\frac{1}{2}})-M(1\mathrm{s}_{\frac{1}{2}}) \lesssim 610\ \mathrm{MeV}/c^2, \tag{7b}$$

$$470\ \mathrm{MeV}/c^2 \lesssim M(1\mathrm{p}_{3/2})-M(1\mathrm{s}_{\frac{1}{2}}) \lesssim 680\ \mathrm{MeV}/c^2. \tag{7c}$$

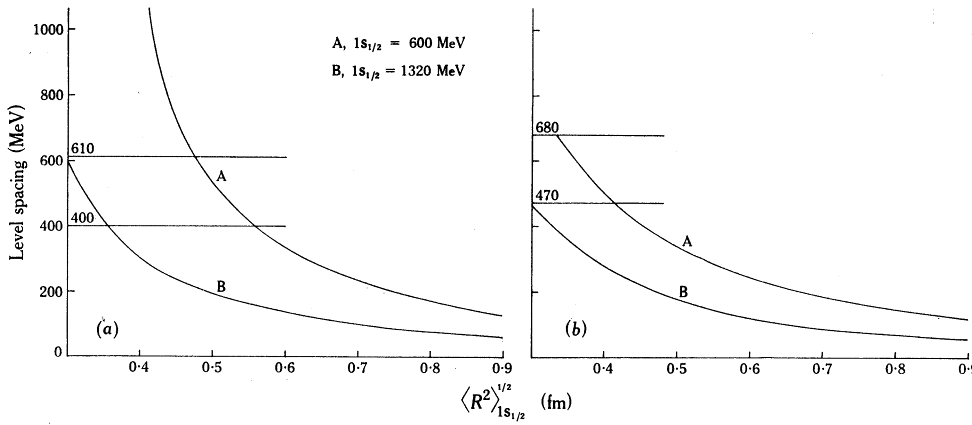

Fig. 3.—Variation of the (*a*) $1\mathrm{p}_{1/2}$–$1\mathrm{s}_{1/2}$ and (*b*) $1\mathrm{p}_{3/2}$–$1\mathrm{s}_{1/2}$ energy spacings of Dirac scalar bound states as a function of the root-mean-square radius of the s-state wavefunction.

Adjusting the strength of the scalar potential to fit the $1\mathrm{s}_{\frac{1}{2}}$ bound state of equations (3) to the value of $M(1\mathrm{s}_{\frac{1}{2}})$ given in (7a), we find that over the ranges (6) and (7) and for all potentials considered $E(1\mathrm{p}_{3/2})-E(1\mathrm{s}_{\frac{1}{2}})$ and $E(1\mathrm{p}_{\frac{1}{2}})-E(1\mathrm{s}_{\frac{1}{2}})$ are independent of the quark mass. Further, we find for a given $\langle R^2\rangle^{\frac{1}{2}}$ that $E(1\mathrm{p}_{3/2})-E(1\mathrm{s}_{\frac{1}{2}})$ and $E(1\mathrm{p}_{\frac{1}{2}})-E(1\mathrm{s}_{\frac{1}{2}})$ are independent of the potential shape. Thus the p-state, s-state energy spacing depends only on $\langle R^2\rangle^{\frac{1}{2}}$.

These results are related to using a scalar potential for the quark–quark interaction, as this behaviour is not exhibited by vector potentials. From equation (1) it can be seen that the scalar interaction modifies the mass term so that in essence an effective mass $m^* = m+V$ is being used.

For the range of bound state masses given in (7) it is possible to estimate the root-mean-square radius of the quark–quark interaction from the hadronic spectra without reference to (6). This is shown in Figure 3. We obtain the result

$$0\cdot 3\ \mathrm{fm} \lesssim \langle R^2\rangle^{\frac{1}{2}} \lesssim 0\cdot 4\ \mathrm{fm}, \tag{8}$$

if both the $E(1p_{3/2})-E(1s_{\frac{1}{2}})$ and $E(1p_{\frac{1}{2}})-E(1s_{\frac{1}{2}})$ are fitted. This result is consistent with (6) which was obtained from consideration of the scattering sum rules and the nucleon form factors.

V. Rigid Rotor Model

For the vector interaction we reproduced the results of the Dirac equation with a simple rigid rotor model in which a spherically symmetric mass distribution rotated with a total angular momentum L given by

$$L^2 = l(l+1)\hbar^2 .$$

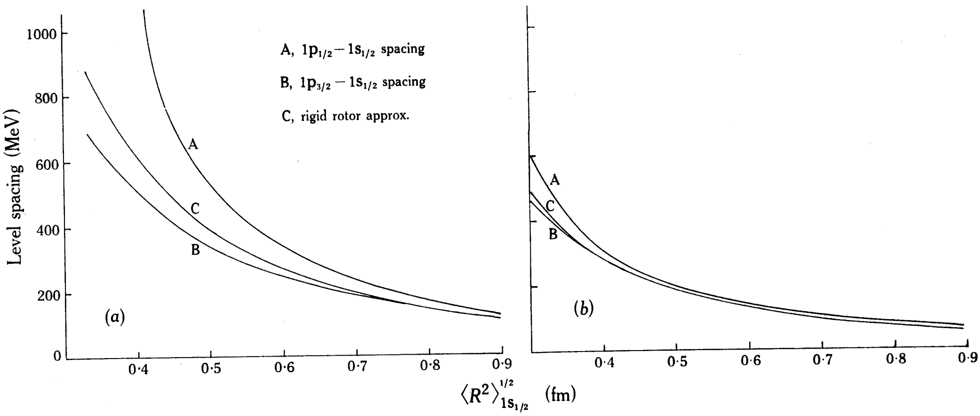

Fig. 4.—Rigid rotor approximation to the Dirac bound states for a scalar interaction and a $1s_{1/2}$ level energy of (a) 600 MeV and (b) 1320 MeV.

We do the same for the scalar interaction except that we use an effective mass, namely that of the s-wave bound state. The energy E of rotation of a rigid rotor is

$$E = L^2/2I = \hbar^2 l(l+1)/2I ,$$

where I is the moment of inertia of the rotor, which for a spherically symmetric mass distribution is

$$I = \tfrac{2}{3}m\langle R^2\rangle ,$$

where m is the total mass and $\langle R^2\rangle$ the mean-square radius of the mass distribution. Using an effective mass

$$m^* = E(1s)/c^2$$

we find the p-state, s-state energy spacing is

$$E(1p)-E(1s) = \tfrac{3}{2}\hbar^2c^2/\langle R^2\rangle_{1s}E(1s) ,$$

where $\langle R^2\rangle_{1s}$ is the mean-square radius of the s-state wavefunction. Figure 4 shows that this is a reasonable approximation to the behaviour of the Dirac bound states for a scalar interaction.

VI. Conclusions

For quarks whose interaction is described by a non-singular scalar interaction, the hadron spectrum implies a root-mean-square separation distance of the quarks in the core of

$$0\cdot 3\ \mathrm{fm} \lesssim \langle R^2\rangle^{\frac{1}{2}} \lesssim 0\cdot 4\ \mathrm{fm}\,.$$

For this interaction the hadron spectrum is independent of the quark mass and the shape of the interaction. The hadron spectrum is consistent with a rigid rotor model in which an effective quark mass is used.

VII. References

Kokkedee, J. J. J. (1969).—"The Quark Model." Ch. 11. (W. A. Benjamin: New York.)
Lipkin, H. J., and Tavkhelidze, A. (1965).—*Phys. Lett.* **17**, 331.
Rose, M. E. (1961).—"Relativistic Electron Theory." Ch. 5, pp. 27, 159. (Wiley: New York.)
Smith, G. B., and Tassie, L. J. (1970).—*Aust. J. Phys.* **23**, 615.