ON THE RELATIONSHIP BETWEEN RECENT MEASUREMENTS OF COSMIC RAY ELECTRONS, NONTHERMAL RADIO EMISSION FROM THE GALAXY, AND THE SOLAR MODULATION OF COSMIC RAYS*

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Summary

Utilizing recent measurements of the cosmic ray electron spectrum at the Earth and the effects of solar modulation on this spectrum, possible limits on the local interstellar electron spectrum have been determined. Synchrotron emission from these interstellar electrons is then compared with the local (disk) volume emissivity of nonthermal radio emission as deduced from a study of radio intensity profiles along the galactic equator. The detailed spectrum and magnitude of radio emissivity can be reproduced from the electron spectrum only for very stringent conditions on the magnitude of the local interstellar magnetic field, and the amount of solar modulation of cosmic rays. Specifically it is found that $B_\perp \approx 7$ μG, and the residual modulation parameter $K_R \approx 0 \cdot 75$ GV. If solar modulation effects on the cosmic ray electron component are negligible then an implausibly high local field of ≈ 20 μG is required.

If the local interstellar electron spectrum that best reproduces the spectrum of local radio emissivity is compared with the electrons expected as secondaries from cosmic ray collisions in the Galaxy, it is found that most electrons $\lesssim 300$ MeV may originate via the secondary mechanism rather than be directly accelerated as are the the higher energy electrons.

Adaption of this local interstellar electron spectrum which is quite different from that at the Earth also greatly modifies the interpretation of the effects of interstellar absorption by ionized hydrogen on the low frequency end of the radio spectrum. Emission measures of $< 0 \cdot 5$, 20, and 10^3 cm^{-6} pc are found in the polar, anticentre, and galactic centre directions respectively. These values are substantially below earlier estimates and would seem to rule out the existence of a large HII region about the Sun, for example.

Finally it is noted that the interstellar intensity of cosmic ray nuclei above 30 MeV deduced using a residual modulation constant of $0 \cdot 75$ GV is inadequate by two orders of magnitude to produce the required heating of interstellar HII clouds. If this heating is produced by cosmic rays it must be caused by a low energy component with a very steep spectrum. It is argued that such a component might arise from cosmic ray emission from solar-type stars in the Galaxy.

Introduction

In this paper we propose to re-examine the familiar comparison between data on primary cosmic ray electrons and nonthermal radio emission from the Galaxy. This study is made in the light of recent measurements of the extraterrestrial electron intensity near the Earth in the energy range 15–200 MeV (Jokipii, L'Heureux, and Meyer 1967; Webber 1968), which indicate a much lower intensity than previously assumed. In addition, the first measurements of solar modulation effects on the electron component have recently been carried out (Webber 1967a; L'Heureux et al. 1968). These measurements enable useful limits on the electron spectrum in the local region of interstellar space to be deduced from the spectrum observed near the Earth.

The significance of this extrapolation lies in the fact that the interstellar electron spectrum can then be related via the synchrotron process to the nonthermal radio emissivity of nearby space. In this approach the present study differs importantly from most earlier attempts, which have compared the measured electron spectrum at Earth (usually without any consideration of solar modulation effects) with the radio emission that is assumed to emanate from the galactic halo, inferring a characteristic halo magnetic field in the process. It is clear that a comparison with the local radio emissivity is much more relevant and this comparison is greatly facilitated by significant new measurements of the features of nonthermal radio emission from the Galaxy. In order to determine the local radio emissivity it is necessary to utilize both high and low resolution radio measurements to determine the relative importance of emission from the galactic disk, the halo, and from outside the Galaxy. Satellite and ground-based observations have now defined the polar radio spectrum in the 1–10 MHz range, and measurements of the Hobart group have defined the disk component for all but the lowest frequencies. At higher frequencies the work of the Cambridge group has been completely revised and extended.

The comparison between the primary electron spectrum and the nonthermal radio spectrum has important consequences with regard to the solar modulation of cosmic rays. It is generally accepted that even at the time of minimum solar activity there remains an appreciable residual solar modulation for the nucleonic components of the cosmic radiation. Even though there is much more accurate information on the solar modulation of nuclei than for electrons it is not possible at the present time to determine the magnitude of the residual modulation from these studies. As a result, differences of a factor of 1000 exist in the extrapolated intensities of cosmic ray nuclei (compare Durgaprasad, Fichtel, and Guss 1967 and Balasubrahmanyan et al. 1968).

On the other hand, the comparison between the primary electron spectrum and the nonthermal radio spectrum provides important constraints on the magnitude of the electron modulation. This, in turn, may be used to set limits on the modulation of nuclei. A comparison of the modulation experienced by electrons and by nuclei will allow one to distinguish between contributions due to rigidity- and velocity-dependent modulation and hence lead to definitive conclusions regarding the mechanism and magnitude of the solar modulation.

We now summarize the approach to be used in this paper.

(1) The electron spectrum measured at the Earth in 1966 is presented. This spectrum is now quite accurately known between about 5 MeV and 6 GeV energy.

(2) The effects of solar modulation on this spectrum are discussed. Although major uncertainties still exist in our knowledge of the magnitude and energy dependence of this modulation, sufficiently accurate limits on the interstellar electron spectrum can be set to enable a useful comparison with the nonthermal radio emission.

(3) The demodulated (interstellar) electron spectrum is then related to:
(a) The calculations of "secondary" electrons produced by cosmic ray nuclei moving in the Galaxy. An attempt will be made to separate the so-called "primary" and "secondary" components of electrons as a function of energy.
(b) The observations of nonthermal radio emission as deduced for the local region of the disk. Crucial to this comparison is the strength of the local galactic magnetic field. Certain limits as to the strength of this field are obtained.

Primary Electron Spectrum

The measurements on the primary electron spectrum appropriate to 1966 are summarized in Figure 1. Our measurements (Beedle and Webber 1968) and those of L'Heureux (1967) are seen to be in excellent agreement for the energy range 200–6 GeV, over which range the electron spectrum can be represented by

$$\frac{dj}{dE} = \frac{(10\pm2)\times10^2}{E^{1\cdot55\pm0\cdot1}} \qquad \left(\frac{electrons}{m^2\,sr\,sec\,MeV}\right).$$

Similarly our work and that of Cline, Ludwig, and McDonald (1964) in the energy range < 20 MeV are consistent. In this energy range there is considerably more uncertainty regarding the spectrum, however. A spectrum embracing all measurements can be represented by

$$\frac{dj}{dE} = \frac{(5\pm2)\times10^2}{E^{1\cdot8\pm0\cdot2}} \qquad \left(\frac{electrons}{m^2\,sr\,sec\,MeV}\right).$$

The derivation of the spectrum in the 20–200 MeV range has been discussed in two recent publications (Jokipii, L'Heureux, and Meyer 1967; Webber 1968). It is in this energy range that a significant kink in the electron spectrum occurs. This kink is apparent in both measurements. It plays a crucial role in the subsequent comparison of the electron spectrum with the nonthermal radio emission.

At least three measurements of the electron spectrum above 10 GeV presently exist (Daniel and Stephens 1967; Bleeker *et al.* 1968; Danjo *et al.* 1968). We regard the integral measurement of Daniel and Stephens above 16 GeV as the most reliable—since the energy is defined accurately by the well-known geomagnetic cutoff. Assuming a differential spectrum $j(E) = K_e/E^{2\cdot4}$ above 16 GeV allows us to plot the Daniel and Stephens point, as shown in Figure 1(b). The measurement of L'Heureux (1967)

does not give an integral flux above the highest differential energy interval; however, Beedle and Webber (1968) obtain a flux of $4\cdot5\pm0\cdot5$ electrons m^{-2} sr^{-1} sec^{-1} above 6 GeV. Comparing this integral value with the integral value obtained by Daniel and Stephens at the equator allows us to obtain the differential flux in the 6–16 GeV interval, as shown in Figure 1(b). It is evident that a simple extension of the spectrum measured below 6 GeV will not fit the data at higher energies. The dashed line in
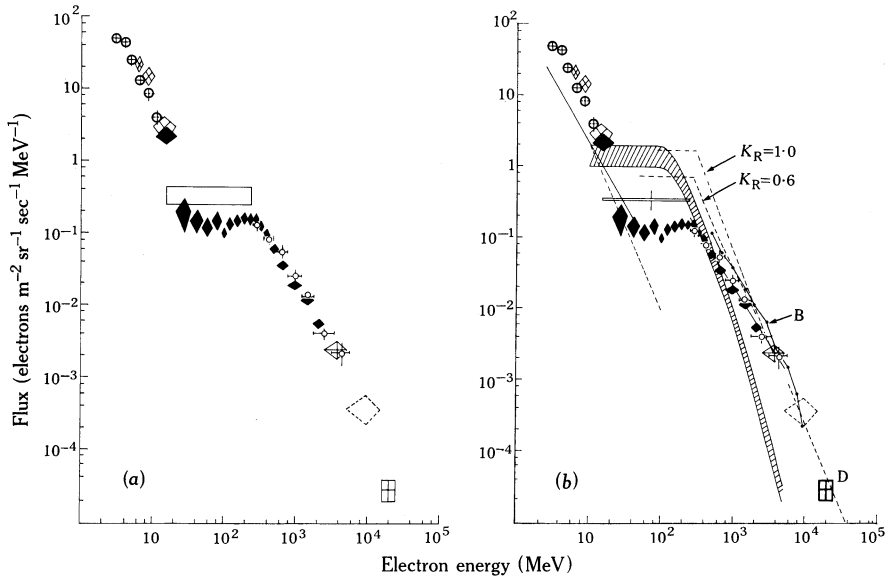

Fig. 1.—Extraterrestrial electron spectrum in 1966:

$\otimes$,	Cline, Ludwig, and McDonald (1964);
◆, ◇,	Beedle and Webber (1968) and Webber (1968);
□,	Jokipii, L'Heureux, and Meyer (1967);
○,	L'Heureux (1967).

In (b) additional measurements of Bleeker *et al.* (1968) are shown as the (smoothed) beaded line (B), and Daniel and Stephens (1967) as the rectangle (D). The expected flux of secondary electrons arising from nuclear interactions of cosmic ray nuclei in the Galaxy is shown as the shaded area for passage of these nuclei through limits of 3 and 6 g cm^{-2} of hydrogen. The interstellar electron spectra obtained using solar demodulation constants of $0\cdot6$ and $1\cdot0$ GV are shown by the dashed lines.

Figure 1(b) is the best estimate of the spectrum between about 6 and 25 GeV. Thus it appears that there is a change of slope in the primary electron spectrum at ~ 6 GeV, and, between 6 and 25 GeV,

$$\frac{dj}{dE} = \frac{1\cdot6\times10^5}{E^{2\cdot4}} \qquad \left(\frac{\text{electrons}}{\text{m}^2 \text{ sr sec MeV}}\right).$$

The occurrence of such a break, arising as a result of the degradation of the high energy part of the electron spectrum through the interaction with the cosmic 3° black-body radiation has been the subject of much discussion since the original observations of Daniel and Stephens (1967). It is not the purpose here to add further

speculation to this question. As will be seen later the radio evidence for such a break is at least partially obscured by radio emission from this 3° radiation itself above 1000 MHz. A best estimate of the high energy part of the electron spectrum is presented so that the effects of solar modulation on the spectrum at low energies may be more fully appreciated.

SOLAR MODULATION OF ELECTRONS AND INTERSTELLAR ELECTRON SPECTRUM

In order to gain some insight into the problem of the solar modulation of electrons let us briefly summarize the current situation with regard to the modulation of cosmic ray protons. For these particles the generally recognized form for the modulation may be written (Nagashima, Duggal, and Pomerantz 1966)

$$n(r_0)/n(\infty) = \exp(-K_R/D),$$

where $n(r_0)$ and $n(\infty)$ are the densities of cosmic rays at the Earth and in interstellar space (outside the region of solar modulation) respectively. The quantity D is the diffusion coefficient describing the motion of the particles in the solar magnetic fields that permeate interplanetary space; K_R is a quantity related most directly to the bulk outward velocity of the solar plasma (the solar wind) and to the extent of the region of modulation about the Sun. D is dependent on the rigidity and species of particle in question but K_R is independent of these parameters. D can, and has been, accurately evaluated over a wide range of rigidities by studying the rigidity dependence of the proton and helium variations. The experimental results and theoretical predictions are in reasonable accord on most points (Webber 1967b). It has not yet been possible to evaluate the absolute value of K_R experimentally (and therefore the total modulation existing between the Earth and interstellar space); although various limits can be set on the basis of theoretical models (e.g. Quenby 1967). Generally the values of K_R obtained in this way are $\sim 0 \cdot 5$ GV. It is also possible to estimate K_R by making certain assumptions regarding the similarity of the demodulated (interstellar) proton and helium spectra. Values of K_R obtained in this way have ranged from $< 0 \cdot 5$ to as large as $2 \cdot 5$ GV (Balasubrahmanyan et al. 1968).

Consider particles with an "effective rigidity" of $0 \cdot 2$ GV $\equiv D$ (200 MeV electrons, 22 MeV protons). Then if we accept residual modulation parameters K_R as large as 2 GV, the intensity of particles of this particular rigidity in interstellar space is e^{10} or 2×10^4 times that at Earth. It is evident that uncertainties in the value taken for K_R lead to even greater uncertainties in the interstellar cosmic ray flux. Recently Gloeckler and Jokipii (1967) summarized all evidence and introduced arguments of their own to suggest that the best value for K_R is $0 \cdot 9$ GV, with it extremely unlikely that K_R is greater than $1 \cdot 2$ GV.

To actually perform the corresponding demodulation of the electron spectrum we require a knowledge of the rigidity dependence of the electron modulation itself, i.e. the effective diffusion coefficient for electrons. Two rather sharply divergent measurements of the electron modulation presently exist. Our measurements (Webber 1967a) covering the rigidity range $0 \cdot 3$–2 GV, give an electron modulation that is of the same order as that for protons at the same rigidity. Below $0 \cdot 3$ GV we have suggested that the effective electron modulation is independent of energy, in keeping

with a changeover to a purely velocity-dependent modulation, which has been observed for protons of these rigidities (Ormes and Webber 1968). L'Heureux *et al.* (1968) found no evidence for solar modulation effects on the electron component and as a result set an upper limit of $\sim 0 \cdot 2$ for the ratio of the electron–proton modulation at the same rigidity in the range $0 \cdot 3$–1 GV.

In Figure 1(*b*) we show the extrapolated interstellar electron intensities using our measurements of the solar modulation effects and values of $K_{\rm R} = 0 \cdot 6$ and $1 \cdot 0$ GV. If the modulation measurements of L'Heureux *et al.* (1968) are used then the interstellar electron intensity is virtually the same as that measured near the Earth in 1966, even for residual modulation parameters as large as 1 GV.

As a result, there exist two rather clear-cut limits on the possible interstellar electron spectrum, depending on which modulation is assumed for the electrons.

COMPARISON OF ELECTRON SPECTRA WITH CALCULATIONS OF "SECONDARY" ELECTRONS PRODUCED IN THE GALAXY

Two possible mechanisms have been proposed for the source of the energetic electrons that are observed near the Earth: (1) collisions of cosmic ray nuclei with interstellar material and the subsequent production of π mesons and decay muons and electrons, and (2) direct acceleration, presumably, although not necessarily, in the source regions that also accelerate the cosmic ray nuclei. The intensity of electrons from the first mechanism, known as secondary electrons, can and has been calculated using data on the cross sections and multiplicities for π-meson production and independent estimates of the amount of interstellar material that the cosmic ray nuclei have traversed. An analysis of this problem using contemporary estimates of the relevant quantities has been carried out by Ramaty and Lingenfelter (1966). Their estimates of this secondary electron spectrum for the limits that the energetic cosmic ray nuclei have passed through 3 and 6 g cm^{-2} of material are shown in Figure 1(*b*). A comparison of these calculations with the electron spectrum measured at Earth and that estimated to exist in interstellar space is illuminating. Firstly we observe that the commonly referenced situation wherein the observed electron intensity is much greater than the predicted secondary flux, thus suggesting another source for these particles, is certainly evident above 1 GeV. However, below 500 MeV the measured intensity of electrons at Earth is actually less than the secondary source. Between 30 and 150 MeV this deficiency is a factor of five.

If the calculated secondary intensities are now compared with those deduced for interstellar space we find that the two are comparable at energies < 200 MeV, if the electron modulation measured by Webber (1967*a*) is used.

If the electron modulation measurements of L'Heureux *et al.* (1968) are taken then the interstellar electron flux in the 30–200 MeV is inadequate by a factor of at least three to account for the expected secondary electron intensity. It would be necessary to assume that energetic (> 1 GeV nucleon^{-1}) cosmic ray protons have travelled through ~ 1 g cm^{-2} of interstellar material in order that the calculated secondary intensity agree with the extrapolated interstellar electron intensity. The best value for energetic (> 1 GeV nucleon^{-1}) heavier cosmic ray nuclei, obtained using measurements of the abundance of lithium, beryllium, and boron nuclei, is

$4 \cdot 5 \pm 1$ g cm^{-2} (Shapiro and Silberberg 1968). We believe that it is reasonable to assume that the amount of material traversed by protons and heavier nuclei is the same and that a substantial amount of solar modulation is affecting the low energy electrons observed at the Earth.

We will see that a comparison of the low energy interstellar electron intensity and the local low frequency radio emission also suggests that substantial modulation of the low energy electrons must be occurring.

RELATION BETWEEN ELECTRON SPECTRUM AND NONTHERMAL RADIO SPECTRUM

The synchrotron mechanism is generally considered to be responsible for most of the nonthermal radiation from our Galaxy. Several authors (Schwinger 1949; Oort and Walraven 1956) have discussed the theory of synchrotron radiation and have presented the necessary formulae. They are presented here only insofar as they are relevant to the present analysis.

A relativistic electron gyrating in a magnetic field generates synchrotron radiation at a rate

$$P_{\text{tot}} = 6 \times 10^{-28} E^2 B_\perp^2 \qquad \text{erg sec}^{-1},$$

where E is in million electron-volts and $B_\perp$, the perpendicular component of the magnetic field, is in microgauss. The spectral distribution of this radiation is characterized by a frequency

$$\nu_c \text{ (MHz)} = 1 \cdot 6 \times 10^{-5} \{E \text{ (MeV)}\}^2 B_\perp \text{ (μG)}.$$

The actual spectral distribution of power emitted by a single electron is given by

$$dP/d\nu = 2 \cdot 3 \times 10^{-29} F(\alpha) B_\perp \qquad \text{erg sec}^{-1} \text{MHz}^{-1},$$

with $\alpha = \nu/\nu_c$. $F(\alpha)$ has been tabulated (Westfold 1959) and is found to have a maximum at $\alpha_m \sim 0 \cdot 5$ decreasing as $(\nu/\nu_c)^{\frac{1}{3}}$ for $(\nu/\nu_c) \ll 1$, and exponentially for $\nu/\nu_c \gg 1$. In actual fact the frequency at which maximum power is emitted ν_m is about $\frac{1}{2}\nu_c$.

Suppose now there exists a differential spectrum of electrons given by

$$j(E) \, dE = K_e/E^m,$$

then the volume emissivity of synchrotron emission per unit frequency interval is given by

$$\epsilon(\nu) = \frac{dP_\nu}{d\nu \, d\tau} = \int d\Omega \int_E^\infty \frac{dP}{d\nu} \, n(E) \, dE,$$

where $n(E) \, dE$ is the density of electrons in the energy interval E to $E + dE$. The intensity of synchrotron emission along a particular line of sight is

$$I(\nu) = \int_0^R \epsilon(\nu) \, dr,$$

where the extent of the radiating region is given by R.

Usually to obtain $I(\nu)$ a number of assumptions are made: (1) the electron distribution is isotropic and $n(E)$ is constant over the region of integration; (2) the

magnetic field is disordered or chaotic. It is also frequently assumed that all the emission takes place at the characteristic frequency ν_c. In this instance the spectral form of the emission takes the particularly simple form

$$I(\nu) \sim \nu^\gamma ,$$

where γ is related to the electron spectral exponent by $\gamma = \frac{1}{2}(1-m)$ (for $\nu > \nu_c$). The intensity of emission at a particular frequency is related to the magnetic field strength $B_\perp$ through the electron spectrum by
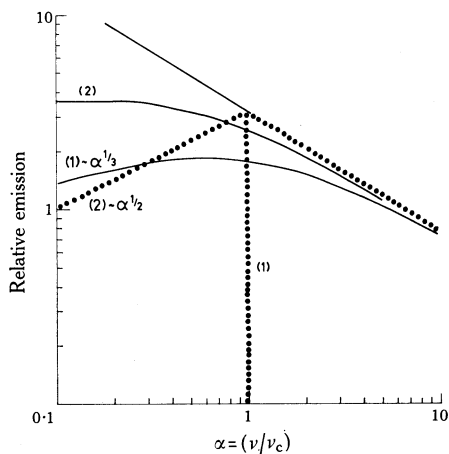
$$I(\nu) \sim B_\perp^{\frac{1}{2}(m+1)} .$$

Fig. 2.—Radio synchrotron spectra as a function of $\alpha = (\nu/\nu_c)$ obtained for an electron spectrum given by $j(E)\,dE = K_e/E^{2\cdot 2}$ for $E > E_1 \equiv \nu_{c1}$ and
(1) $j(E)\,dE = 0$, $E < E_1$;
(2) $j(E)\,dE = $ constant, $E < E_1$.
The δ-function approximation is shown by the dotted lines.

This so-called δ-function approximation is particularly useful for relating an electron spectrum of constant spectral index to the spectrum of radio emission. If the electron spectral index is changing with energy or has a discontinuity then the δ-function approximation is inadequate, particularly for obtaining the low frequency part of the radio emission spectrum.

Consider the following examples. The electron spectrum is given by $j(E)\,dE = K_e/E^{2\cdot 2}$ above some energy E_1 corresponding to the characteristic frequency ν_{c1}. Below this energy the electron spectrum is given by (1) $j(E)\,dE = 0$; and (2) $j(E)\,dE = $ constant. Figure 2 shows the relative radio emission as a function of frequency calculated for these electron spectra using the δ-function approximation and also by actually carrying out the required integration using the explicit values for the function $F(\alpha)$ as tabulated by Westfold (1959). The large difference in the two calculations at low frequencies is due mainly to the long low frequency tail on the function $F(\alpha)$. Even for an electron spectrum that becomes zero below some energy E_1 the synchrotron emission spectrum falls off no faster than $(\nu/\nu_c)^{\frac{1}{3}}$ at low frequencies. For a differential electron spectrum that becomes constant, the synchrotron spectrum becomes almost flat at low frequencies and shows a gradual flattening at $\sim 2\nu_c$ even though the change in the electron spectrum is abrupt (c.f. discussion of Turtle 1963).

In a plot such as Figure 2, the shape of the emission spectrum from different regions should be similar as long as we assume that the electron *spectrum* in these regions is also similar; however, this curve will be displaced along the $\log \nu$ axis by a constant amount depending on the ratio of the field strengths in the two regions (and also on any change in the absolute intensity of the electrons themselves).

Considerations regarding the actual distribution of field strength and direction along a line of sight and deviations of the electron distribution from isotropy may modify the above arguments slightly. We do not believe that the added complication introduced by considering these effects is justified at the present stage of analysis.

Nonthermal Radio Emission Profiles

The objective in this section is to derive the local interstellar volume emissivity of radio emission characterizing a region $\sim 0 \cdot 5$ kpc in diameter centred on the Sun. This emissivity will then be related to the interstellar electron spectrum. This approach differs from most of the previous approaches (e.g. Felton 1966) wherein the electron measurements near the Earth are related directly to radio emission from the galactic halo.

For the derivation of the local interstellar radio emission it shall be convenient to consider two regions of the radiofrequency spectrum.

(1) Frequencies $\gtrsim 30$ MHz where interstellar absorption by free electrons (HII regions) is not important.

(2) Frequencies < 30 MHz and extending down to ~ 1 MHz where interstellar absorption effects are becoming progressively more important, particularly in the direction of the galactic centre.

Eventually at the lowest frequencies the optical depth is ~ 1 at distances of less than 1 kpc in some directions and indeed we are seeing only the synchrotron emission from "local" electrons.

In both frequency ranges the radio emission as a function of frequency will be derived in four directions: the galactic centre, the anticentre, the north polar region, and the direction of minimum radio brightness (R.A. ~ 10 hr, Dec. $\sim 40°$). Then by a process of subtraction we shall derive the local radio emissivity spectrum.

In the case of the spectra in the directions of the galactic centre and anticentre it is necessary to use surveys with sufficiently narrow beamwidths (e.g. $\sim 1°$) to resolve the galactic disk. In some instances for measurements in the anticentre direction, we have utilized surveys of medium resolution ($\sim 10°$) at frequencies > 10 MHz to substantiate the narrow beam data when it is felt that they contribute a higher level of absolute accuracy (principally the recent measurements of the Cambridge group). For the present purposes the anticentre is defined as the region $b^{II} = \pm 2°$, $l^{II} = 175°$–$185°$; and the centre as $b^{II} = \pm 2°$, $l^{II} = 350°$–$10°$ (omitting the strong source Sgr A located at the origin of the new galactic coordinate system). The narrow and medium resolution surveys are synthesized to the anticentre region using the narrow beam survey of Blythe (1957) at 38 MHz.

In the cases of the spectra in the north polar region and the direction of minimum radio brightness both medium resolution and low resolution ($\sim 30°$)

studies have been used. It should be pointed out that the brightness in these directions does depend to some extent on the resolution of the instrument involved. Since at frequencies above 10 MHz the brightness is lowest in the polar direction, the better the resolution the lower the brightness (this can be clearly seen in the work of Bridle 1967). We have attempted to adjust all measurements in these two directions to a common aperture of $\sim 15°$ by $15°$ using the higher resolution measurements of Bridle (1967).

In no instance has the published brightness been modified by more than 30% by these adjustments; however, we believe that truer representations of the actual spectra are obtained.
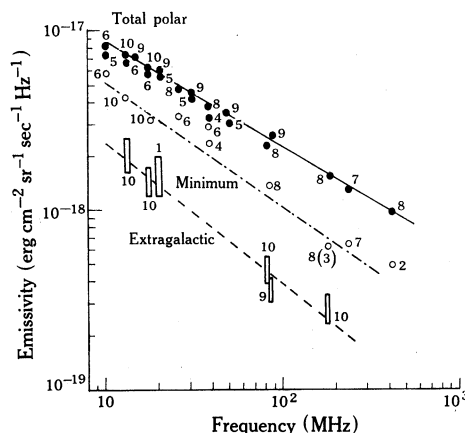

Fig. 3.—Radio brightness spectra in the direction of the north galactic pole and the direction of minimum brightness. The extragalactic component deduced by a number of observers is also shown:
 1, Shain, Komesaroff, and Higgins (1961)
 2, Pauliny-Toth and Shakeshaft (1962)
 3, Turtle and Baldwin (1962)
 4, Kenderdine (1963)
 5, Parthasarathy and Lerfeld (1965)
 6, Andrew (1966)
 7, Large, Quigley, and Haslam (1966)
 8, Purton (1966)
 9, Yates and Wielebinski (1966)
10, Bridle (1967)

The spectra above 10 MHz for the polar direction and the direction of the minimum brightness are shown in Figure 3. (The spectra below 10 MHz for the polar region are presented in Figure 9.) The numbers beside each point indicate the authors responsible for the measurement. Measurements of the total polar emission lean heavily on the most recent work of the Cambridge group (e.g. Andrew 1966; Purton 1966; Bridle 1967), which has updated and extended the earlier work of Turtle *et al.* (1962), Turtle and Baldwin (1962), Turtle (1963), and others. It should be pointed out that the emissivity in the southern polar region, as illustrated by the measurements of Yates and Wielebinski (1966), is identical to within 20% with that in the northern polar region. The solid line in Figure 3 represents a simple spectrum of the form

$$I(\nu) = 3\cdot4 \times 10^{-16}\nu^{-0\cdot60} \qquad \text{erg cm}^{-2}\text{sr}^{-1}\text{sec}^{-1}\text{MHz}^{-1}.$$

There is some evidence that the spectrum is becoming flatter at the lower frequencies although it is difficult to see how a spectral exponent much greater than $0\cdot7$ can be taken at frequencies > 50 MHz. It should be noted that Anand, Daniel, and Stephens (1968), using much the same data, have drawn a smoothly varying curve through the data. Their curve, which gives a spectral exponent $\sim 0\cdot4$ at the lowest frequencies, is certainly an alternative fit to our data points. Individual authors have also tried to fit their own data points and arrived at essentially the same

conclusion. For example, Andrew (1966), Purton (1966), and Bridle (1967) all found that an exponent $\sim 0 \cdot 4$ was most suitable in the range 10–100 MHz, whereas above this frequency they supported an exponent $\sim 0 \cdot 9$. Yates and Wielebinski (1966) found an exponent $\sim 0 \cdot 5$ at 85 MHz slowly decreasing to $0 \cdot 3$ at the low frequency end of their range. Above 85 MHz they favoured an exponent $\sim 0 \cdot 6$.

The emission in the direction of minimum brightness is about 50% of that in the polar region. The spectrum in this direction is not as well defined but appears to be very similar to that in the polar direction.
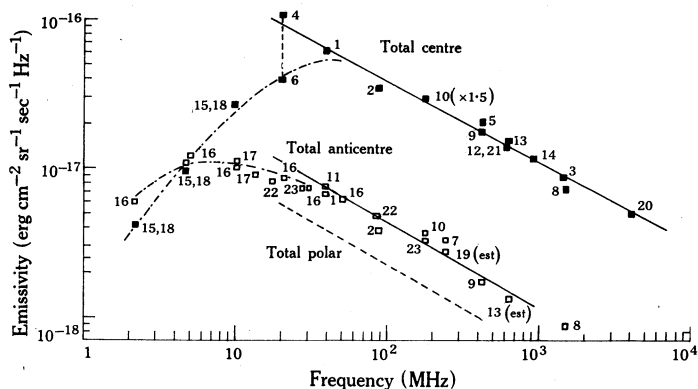

Fig. 4.—Radio brightness spectra in the direction of the galactic centre and anticentre (as defined in the text). The total polar spectrum is shown as a reference: 1, Blythe (1957); 2, Hill, Slee, and Mills (1958); 3, Westerhout (1958); 4, Komesaroff (1961); 5, Large, Mathewson, and Haslam (1961); 6, Shain, Komesaroff, and Higgins (1961); 7, Davies and Hazard (1962); 8, Mathewson, Healey, and Rome (1962); 9, Pauliny-Toth and Shakeshaft (1962); 10, Turtle and Baldwin (1962); 11, Kenderdine (1963); 12, Turtle (1963); 13, Moran (1964); 14, Davies (1965); 15, Ellis (1965); 16, Parthasarathy and Lerfeld (1965); 17, Andrew (1966); 18, Ellis and Hamilton (1966); 19, Large, Quigley, and Haslam (1966); 20, Penzias and Wilson (1966); 21, Purton (1966); 22, Bridle (1967); 23, Parthasarathy (1967).

From observations of the variation of brightness and spectral index across the sky, a number of observers have attempted to determine the percentage of the radio emission that could be extragalactic. It is assumed that this extragalactic emission is isotropic and has a different (steeper) spectral index than that coming from the various regions of the Galaxy. The estimates of this extragalactic component are shown in Figure 3. They seem to define a spectrum of slope $\sim 0 \cdot 8$ and of magnitude $\sim 30\%$ of the total polar emission at ~ 10 MHz. If indeed the extragalactic component has such a steep spectrum and extends to lower frequencies then it may dominate the flattening total polar spectrum at frequencies of 1–2 MHz. This interesting possibility has been discussed in some detail by Smith (1965).

The situation in the galactic centre and anticentre directions is shown in Figure 4. At frequencies below ~ 38 MHz there is a lack of high resolution data for the anticentre region. For this reason we have used medium and low resolution measurements, synthesizing the emissivities found in these lower resolution studies to the standard anticentre direction using the high resolution measurements of Blythe

(1957) at 38 MHz. These adjustments amount to multiplying the given low resolution radio intensities by factors of from $1 \cdot 1$ to $1 \cdot 3$.

The dramatic decrease in the spectra below 20 MHz in the centre and anti-centre directions due to absorption by ionized hydrogen is clearly evident. Above about 30 MHz, however, both spectra follow very closely the solid lines that are drawn for spectral indices of $-0 \cdot 6$. The magnitude of the anticentre emission is about twice that in the polar direction whereas the emission in the direction of the galactic centre is about 10 times that in the anticentre direction. The characteristics of the spectrum above 10 MHz in the anticentre region are in fact not noticeably different to the spectrum in the polar direction, a point that is in agreement with the conclusions of Andrew (1966), Purton (1966), and Bridle (1967). The index of $-0 \cdot 6$ in the direction of the galactic centre is identical with that found by Komesaroff (1961).

Local Disk Emissivity

To derive from these measurements a value for the local disk emissivity we must first consider a simple geometrical model for the galactic disk and halo. This would be a spherical halo of radius 15 kpc, and a flat disk also of radius 15 kpc and of semi-thickness $0 \cdot 4$ kpc. In this model the Sun is at a distance of 10 kpc from the centre, approximately on the galactic equator (see Fig. 6(a)).

If the emissivity were uniform throughout the disk the ratio of intensities in the centre–anticentre direction would be $5 : 1$. The observed ratio of about $10 : 1$ indicates that the average emissivity must increase as one moves towards the centre of the Galaxy. To examine this behaviour more closely we have utilized the results of six surveys of nonthermal radio emission with sufficiently narrow beamwidths to resolve the galactic disk. For $b^{\mathrm{II}} = 0$ (galactic equator) the longitudinal variation of the nonthermal component of radio emission is plotted in Figure 5. The data are normalized in the anticentre direction using a $\nu^{-0 \cdot 6}$ dependence for the emission.

The galactic profile is very similar from each of these studies and shows an increasing wealth of detail with increasing resolution (related to spiral arm structure etc). The intensity profile to be expected if the emissivity is uniform throughout the disk is shown as curve A. The fact that the observed intensity profiles follow this curve for all directions except within $50°$ of the galactic centre indicates that the emissivity must be almost independent of radius at distances $\gtrsim 10$ kpc from the galactic centre. This leads us to consider a very simple form for emissivity as a function of distance out to 8 kpc from the galactic centre,

$$\epsilon(r) = \epsilon_{\mathrm{S}}(5 \cdot 8 - 0 \cdot 6r),$$

where r is the distance (kpc) and ϵ_{S} is the emissivity near the Sun. Beyond 8 kpc the emissivity remains constant out to the boundary of the disk at 15 kpc. The corresponding galactic disk intensity profile is given by curve B in Figure 5. Approximately as good a fit to the observed profile would be obtained if

$$\epsilon(r) = \epsilon_{\mathrm{S}}(-1 \cdot 6r/r_{\mathrm{S}})$$

(c.f., for example, Okuda and Tanaka 1968), the biggest difference being in the

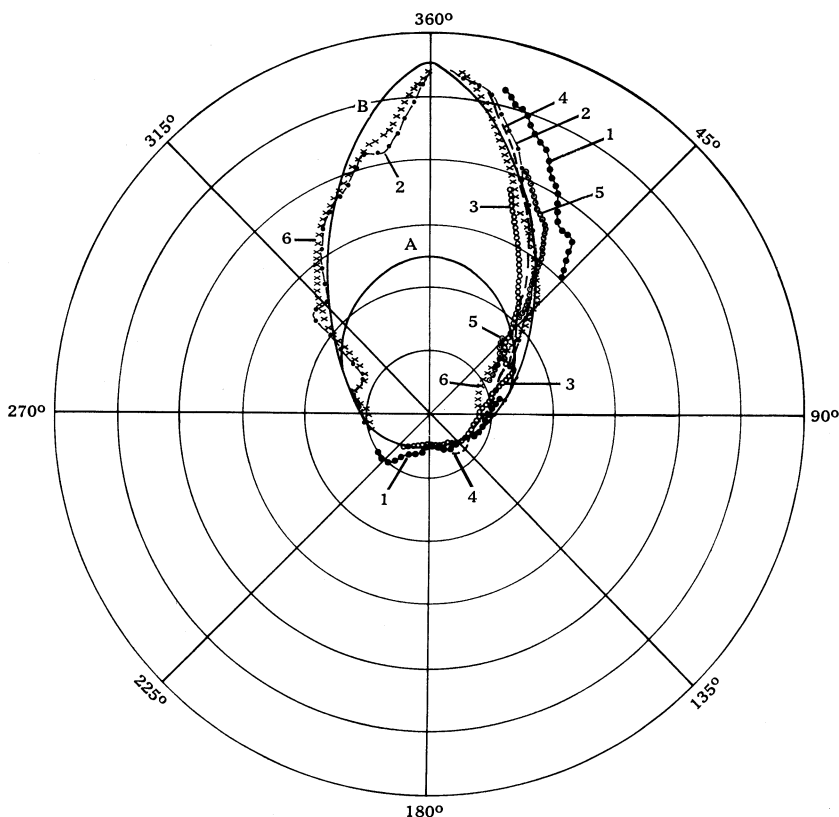continued decrease in emissivity beyond 10 kpc, which is not evident in the galactic disk intensity profiles.

Fig. 5.—Polar diagram of radio emission measured along the galactic plane with high resolution surveys. Data at six frequencies are shown (normalized in the anticentre direction): 1, 38 MHz (Blythe 1957); 2, 85 MHz (Hill, Slee, and Mills 1958); 3, 178 MHz (Turtle and Baldwin 1962); 4, 404 MHz (Pauliny-Toth and Shakeshaft 1962); 5, 610 MHz (Moran 1964); 6, 1440 MHz (Mathewson, Healey, and Rome 1962). Curve A represents the polar diagram to be expected if the radio emission is uniform throughout the disk. Curve B is obtained for the radial profile of emissivity given in Figure 6(b).

To obtain the emissivity per unit volume in the disk near the Sun it is only necessary to divide the intensity of radio emission in the anticentre direction by $(4\pi) \times 5$ kpc, the assumed distance over which this emission is coming. This procedure neglects the extragalactic component, which is $\sim 10\%$ of the total emission in the anticentre direction. It also neglects the fact that the emission is probably not uniformly distributed over the 5 kpc distance to the boundary but is concentrated in the spiral arms. Since the Sun is located in an arm (at the edge of the Orion arm), a consideration of this nonuniformity would tend to enhance the values for the local emissivity.

If one were to take the exponential decrease of emissivity illustrated in Figure 6, the local emissivity would need to be ∼1·4 times greater.
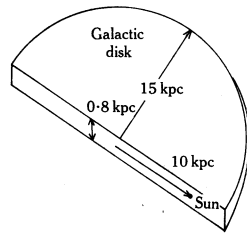

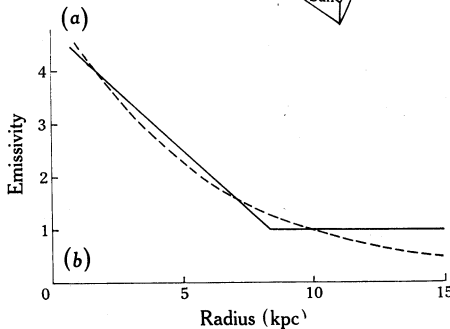Fig. 6(a).—Schematic representation of galactic disk and halo.

Fig. 6(b).—Radial dependence of emissivity (arbitrary units) in galactic disk required to produce profile B in Figure 5.

In Figure 7 we show the local emissivity deduced from the model in which the emissivity is uniform beyond 8 kpc. This local emissivity can be represented by a form

$$\epsilon(\nu) = 5 \times 10^{-38}\, \nu^{-0.6} \qquad \mathrm{erg\, cm^{-3}\, sec^{-1}\, Hz^{-1}}$$

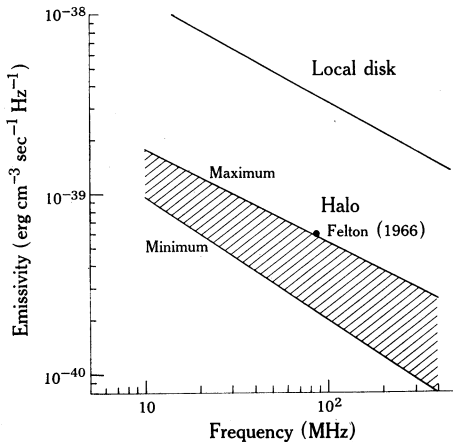above 20 MHz, flattening appreciably at lower frequencies.

Fig. 7.—Local spectrum of radio emissivity from the galactic disk. The maximum and minimum allowable emissivity for a uniform halo are also shown, as is the halo emissivity at 81 MHz deduced by Felton (1966).

Before comparing this directly with the electron spectrum, let us attempt, using the above spectrum for the local emissivity, to derive a characteristic emissivity spectrum for the halo. Now as one looks out in the polar direction and the direction of minimum nonthermal radio emission, contributions will occur from radio emission

in the disk and the halo as well as the extragalactic component. The spectrum from the extragalactic component has already been derived, and using the above local emissivity and assuming a disk semithickness of 400 pc (Baldwin 1966) we can estimate that part of the emission spectrum from the disk. This turns out to be $\approx 20\%$ of the total polar emission, or comparable to the extragalactic component.

If the remaining emission is to be ascribed to a spherical halo, then calling this remainder in the polar direction the maximum halo, and in the direction of minimum radio emission the minimum halo, we have the halo emissivities per unit volume given in Figure 7. (Note that the recent estimate of halo emissivity at 81 MHz by Felton (1966) lies almost on top of our maximum halo spectrum.)
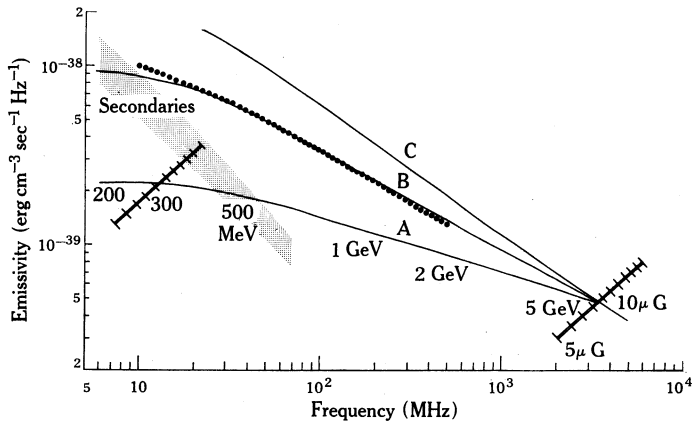

Fig. 8.—Comparison of local disk emissivity (●) with the emissivity to be expected from secondary electrons and (A) electron spectrum measured at Earth in 1966, (B) interstellar electron spectrum obtained with a residual modulation parameter of $0 \cdot 6$ GV, and (C) interstellar electron spectrum with a residual modulation parameter of $1 \cdot 0$ GV. The manner in which the curves A, B, and C must be displaced for different galactic disk magnetic field strengths is shown, as are the equivalent electron energies.

The characteristic halo emissivity is an order of magnitude less than the local disk emissivity and if one takes the minimum halo emissivity then the halo is a very weak radio emitter indeed and it becomes reasonable to ask whether there is a halo at all. Of course more sophisticated models of the halo and disk distributions can be taken (e.g. Mills 1959) but it seems that the central problem concerns the magnitude and uniformity of the disk component. If there is considerable structure to the disk, in the form of loops and spurs in addition to a more regular component of semithickness ~ 400 pc then the minimum halo emissivity that we have derived is probably the most realistic one.

Turning now to a comparison of the previously derived spectrum of local emissivity with that to be anticipated from the interstellar electron spectrum, the situation is summarized in Figure 8. The manner in which this emissivity spectrum varies with the magnetic field strength is given in nomogram fashion in the figure. The emissivities deduced from the electron spectra are illustrated for an interstellar magnetic field $B_\perp = 8 \ \mu$G. This magnetic field strength provides an excellent fit for

the interstellar electron spectrum obtained with a residual modulation parameter of 0·6 GV. Even the low frequency flattening of the radio spectrum is reproduced as a result of the flattening of the electron spectrum below 300 MeV.

If the emissivity from the interstellar electron spectrum, derived using a residual modulation parameter of 1·0 GV, is compared with that deduced from the radio measurements, a weaker magnetic field ($\sim 5 \, \mu$G) is required to produce an approximate agreement. In this instance, however, the emissivity obtained from the electron spectrum has a notably steeper spectrum than that deduced from the radio measurements.

If, in turn, the interstellar electron spectrum is essentially that measured at the Earth in 1966, then the local interstellar magnetic field must be at least 18 μG to even approximately reproduce the deduced radio emissivity. The emissivity obtained from this electron spectrum also has a much flatter spectrum than any reasonable limitation on the measured emissivity.

An interstellar field of this magnitude seems much too large in view of all of the other observational evidence (Davies 1965). This difficulty with the magnitude of the interstellar field is enhanced when we recall that the emissivity deduced from the radio measurements probably tends to be slightly underestimated for the reasons discussed earlier. We therefore believe that this comparison supports the idea of a large modulation for electrons in the solar environment. Indeed, the agreement between emissivities when an interstellar electron spectrum obtained with a residual modulation parameter of 0·6 GV is used gives strong support to the argument that the energy dependence of the solar modulation is reasonably given by the form measured by Webber (1967a).

A further observation concerns the comparison of the emissivity to be expected from the spectrum of secondary electrons only and the emissivity deduced from the radio measurements. The limits on the emissivity from the secondary spectra calculated by Ramaty and Lingenfelter (1966) for passage of cosmic ray nuclei through 3 and 6 g cm^{-2} of material are shown in Figure 8. If the radio emission from these secondary electrons were to exceed the measured emission, this would be suggestive that one of the arguments relating to the comparison was incorrect (e.g. the interstellar magnetic field $> 8 \, \mu$G, or the path length for cosmic ray nuclei < 3 g cm^{-2}).

However, the situation is such that the radio emission from secondary electrons alone does not exceed the measured emission, although it is becoming an increasingly greater fraction of it as one goes to lower frequencies.

RADIO SPECTRUM BELOW 10 MHz AND INTERSTELLAR ELECTRON SPECTRUM AT LOW ENERGIES

The interpretation of the galactic radio spectrum below 10 MHz is treated separately from the high frequency part of the spectrum for two reasons. Firstly, the uncertainties in the measured radio emission are much larger at these frequencies, particularly in the polar direction. Secondly, the effects of absorption by ionized hydrogen in the disk of the Galaxy become important at these frequencies and tend to influence the interpretation of the results.

The experimental situation below 10 MHz in the centre, anticentre, and polar directions is summarized in Figure 9. The intensity versus frequency profiles for the centre and anticentre directions are taken from Figure 4. The data below 10 MHz in these directions are almost entirely due to Ellis and co-workers at Hobart.

The situation in the polar directions is unfortunately not decisive from the point of view of trying to determine a radio spectrum. The obvious differences in the measurements do not seem to be clearly related to whether the measurements are made from the ground, where ionospheric absorption could play an important role, or from satellites, where calibration difficulties are encountered. For example, the
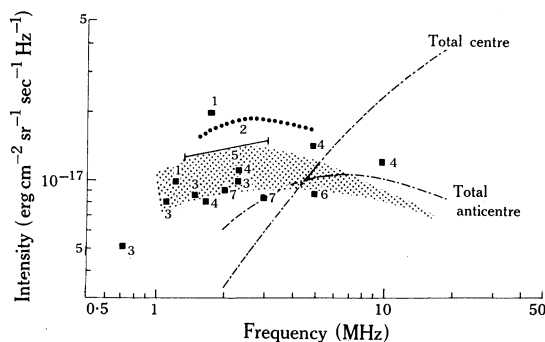

Fig. 9.—Measurements of radio brightness below 10 MHz in the direction of the galactic pole. The total brightness in the centre and anticentre directions is also shown.
1, Walsh, Haddock, and Schulte (1963)
2, Hartz (1964)
3, Benediktov et al. (1965)
4, Ellis (1965)
5, Hugill and Smith (1965)
6, Parthasarathy and Lerfeld (1965)
7, Parthasarathy (1967)

polar intensities measured from the ground by Ellis (1965) and Parthasarathy (1967) differ by a factor of more than two at 5 and 10 MHz and have quite a different slope at the lower frequencies. There is some evidence from satellite observations (Hartz 1964) that emission from the south polar regions is greater than from the north polar region at low frequencies. This might account for some of the difference between the two ground-based observations, although it should be recalled that no difference between south polar and north polar radio intensities is noticed above 10 MHz. The reader is referred to a more thorough discussion of the possible north–south differences by Andrew (1966).

The situation regarding the agreement between the individual satellite measurements below 5 MHz is equally uncertain. It is not the present purpose to attempt to resolve these differences but mainly to try and determine an applicable spectrum of radio emission in the polar direction. To be realistic such a spectrum must encompass the shaded region in Figure 9, and is well determined above 10 MHz by the data already presented in Figure 3.

The polar spectrum that we shall adapt is a smooth curve drawn through the centre of the shaded region in Figure 9. This polar spectrum is shown again in Figure 10 along with the spectra in the directions of the galactic centre and anticentre. It should be pointed out that the uncertainty in the polar spectrum is reflected in the subsequent comparison of the spectra as presented in Figure 10.

It is obvious nevertheless that the spectra in the centre and anticentre directions are turning over at low frequencies as a result of absorption in interstellar ionized

hydrogen. The same effect may also be occurring in the spectrum in the polar direction but it is much less evident. In fact, as has been emphasized earlier, this flattening could be directly related to the flattening of the low energy electron spectrum.

It is convenient at this point to introduce the concept of a "projected" radio intensity or brightness. This intensity is defined as that to be expected in a particular direction in the absence of absorption by ionized hydrogen. As a result it is directly related to true local emissivity in that direction. Two possibilities for the spectrum of projected intensity (emissivity) will be considered.

(1) For the first model (I) we shall utilize the fact that the spectra in both the centre and anticentre directions are $\sim \nu^{-0.6}$ at higher frequencies where absorption effects are negligible and write for the projected intensity

$$\mathscr{I}(\nu) = (\nu/\nu_0)^{-0.6}\, I(\nu_0),$$
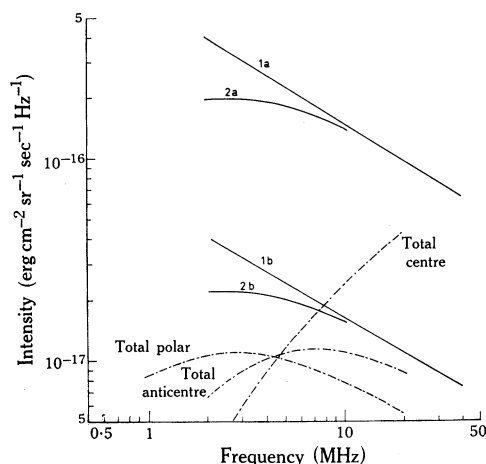

Fig. 10.—Comparison of radio brightness at low frequencies in the polar, anticentre, and centre directions. Curves 1a and 1b represent the projected brightness in the centre and anticentre directions under the circumstances of no absorption by ionized hydrogen and a volume emissivity $\epsilon(\nu) \sim \nu^{-0.6}$. Curves 2a and 2b are the same except that the volume emissivities are taken to have the same spectrum as the total polar brightness.

where ν_0 is a frequency where absorption effects are negligible. Projected intensity spectra according to this relation are shown in Figure 10, in the centre and anticentre directions. Komesaroff (1961) has introduced a similar concept to examine the effects of absorption in the direction of the galactic centre, and has used an identical spectral index for the projected intensity.

(2) For the second model (II) we shall assume that above 2 MHz absorption effects in the polar direction are in fact negligible and let the measured spectrum above this frequency be the projected intensity spectrum as well. In other words, we shall make an important departure from earlier work and allow the intrinsic emissivity spectrum itself to flatten at low frequencies. The justification for this is, of course, the indication that the electron spectrum may also flatten at low energies. The corresponding projected spectra in the centre and anticentre directions are shown as curves 2a and 2b in Figure 10.

Let us now consider in some detail the effects of absorption by ionized hydrogen. The coefficient of interstellar absorption by the free–free process in the radio region

is (Ginzburg 1961)

$$K_\nu = \frac{1 \cdot 38 \times 10^{-14}}{T_e^{3/2}} \left(\frac{N_e^2}{\nu^2} \right) g \,,$$

where ν is in megahertz, N_e is the electron density in CGS units, and g is a quantity given by

$$g = \{17 \cdot 7 + \ln(T^{3/2}/\nu)\} \,.$$

For frequencies ≈ 1 MHz and effective temperatures between 10^3 and $10^4 \,^\circ$K, g is ≈ 18. The electron temperature associated with the ionized hydrogen is usually taken to be $10^4 \,^\circ$K, so that

$$K_\nu = 2 \cdot 5 \times 10^{-19} (N_e^2 / \nu^2) \,.$$

The optical depth is

$$\tau = \int K_\nu \, dr \,.$$

For the usual case where K_ν is taken not to vary with distance and integrating over a distance of 1 pc (with L in pc),

$$\tau = K_\nu r = 0 \cdot 75 (N_e^2 / \nu^2) L \,.$$

Defining a quantity called the emission measure $E = N_e^2 L$,

$$\tau = 0 \cdot 75 \, E / \nu^2 \qquad \text{cm}^{-6} \, \text{pc} \,.$$

For examining the effects of absorption we may consider two simplified galactic models. In the first instance it is assumed that all of the HII lies between the observer and the nonthermal region. In this case

$$I(\nu) = \mathscr{I}(\nu) \exp(-0 \cdot 75 \, E / \nu^2) \,.$$

Figure 11 shows the ratios of $I(\nu)/\mathscr{I}(\nu)$ in the centre and anticentre directions deduced from the measured values of $I(\nu)$ and for the two assumptions regarding the spectrum of the projected emission $\mathscr{I}(\nu)$. The values for $I(\nu)/\mathscr{I}(\nu)$ expected on the basis of model I are also shown in the figure, normalized at values of $\tau = 1$. Model I gives a very poor fit to the data, predicting a much more rapid cutoff of $I(\nu)$ than is actually observed.

Model I may be more reasonably applied to the data in the polar direction if it is assumed that most of the emission in this direction comes from beyond the disk. The crucial question is: what is the spectrum $\mathscr{I}(\nu)$ in this direction? If it is taken to be a simple extension of the spectrum $I(\nu) \sim \nu^{-0 \cdot 6}$ measured at higher energies, τ is comparatively large, being $\sim 1 \cdot 5$ at 1 MHz (e.g. Hoyle and Ellis 1963). The corresponding emission measure is then $\sim 2 \, \text{cm}^{-6} \, \text{pc}$.* The implications of this emission measure in the polar directions have been discussed by Ellis and Hamilton (1966), who attribute it to absorption in interstellar HII, and by Lencheck (1964)

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* From a detailed study of low frequency brightness profiles Ellis and Hamilton (1966) have derived an emission measure of 8 cm^{-6} pc, for $b^{II} = 60°$.

and Alexander and Stone (1965), who have attributed this absorption to the solar HII region.

The observed turnover of the total polar spectrum at low frequencies is totally unlike that to be expected on the basis of model I. In fact, down to ~ 2 MHz it follows exactly the form to be expected if the spectrum of electrons producing the emission is itself turning over. Only below ~ 2 MHz may the suggested fall-off of the total polar spectrum begin to indicate the effects of HII absorption between the emission and the source. If one assumes that this flattened spectrum does in fact resemble $\mathscr{I}(\nu)$, as we have earlier, then τ cannot be greater than about $0 \cdot 3$ at 1 MHz. The corresponding emission measure is $\sim 0 \cdot 4$ cm^{-6} pc, which is an order of magnitude less than previously assumed.
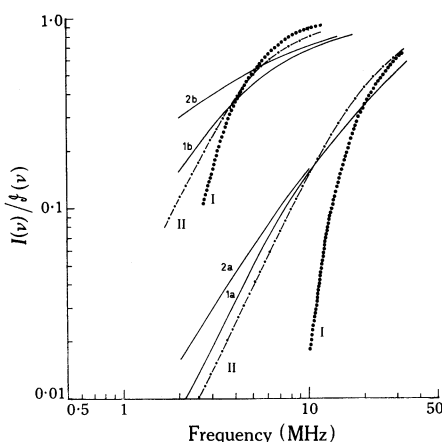

Fig. 11.—Ratio of measured brightness $I(\nu)$ to projected brightness $\mathscr{I}(\nu)$ as a function of frequency. The curves 1a, 1b, 2a, and 2b, have the same meaning as in Figure 10. The curves are also given for the ratios to be expected if models I and II for absorption and emission apply.

In model II we shall consider that nonthermal emission and absorption by interstellar HII occur continuously along the line of sight and that the ratio of these two quantities and the quantities themselves are constant. This more closely approximates conditions in the galactic disk, although it is still a very simplified picture. We know from our earlier discussion that $\epsilon(\nu)$ is certainly a function of distance at least within 10 kpc of the galactic centre. Further, it might be expected that regions of high absorption would be related to the regions of high emission (e.g. Fig. 2 of Smith 1965), although a strict constancy of the ratio of these quantities should not be expected. At any rate under the simplified assumptions of model II we have

$$I(\nu) = \mathscr{I}(\nu)\{\tau_\nu^{-1}(1-\mathrm{e}^{-\tau_\nu})\}.$$

Note that for $\tau_\nu \equiv K_\nu L \gg 1$, $I(\nu)/\mathscr{I}(\nu) \approx \nu^2$.

The calculated ratios $I(\nu)/\mathscr{I}(\nu)$ for $\tau = 600$ at 1 MHz in the direction of the galactic centre and $\tau = 40$ at 1 MHz in the direction of the anticentre are shown in Figure 11. These curves provide a much better fit to the data although there is evidence that the real $I(\nu)$ is decreasing somewhat less rapidly with frequency than expected on the basis of model II.

The calculated ratios correspond to emission measures 800 and 53 cm^{-6} pc respectively.* If the analysis in these two directions is to be consistent the ratio of emission measures should be approximately 10 : 1 (corresponding to the fact that the total path length L is five times longer in the direction of the galactic centre and the average emissivity in this direction is a factor of two larger, i.e. more simply the ratio I_ν (centre)/I_ν (anticentre) at high frequencies). The ratio of ~ 15 obtained above is indeed reasonable within the accuracy of the $I(\nu)/\mathscr{I}(\nu)$ curves, particularly since a close inspection of Figure 11 reveals that the calculated curves may be adjusted to give the expected ratio of 10 and still provide a reasonable fit to the measured data. The important point to note here is that a radial dependence of $\epsilon(\nu)$, which surely exists, will not affect the *shape* of the separate $I(\nu)/\mathscr{I}(\nu)$ curves but will only enter into the ratio of the emission measures calculated in the centre and anticentre directions as long as the ratio $\epsilon(\nu)/K_\nu$ remains constant. The only way the shape of the $I(\nu)/\mathscr{I}(\nu)$ curves themselves can be varied is to assume that $\epsilon(\nu)/K_\nu$ varies with distance. A comparison of the $I(\nu)/\mathscr{I}(\nu)$ curves calculated on the basis of model II and those deduced from the measurements reveals that $\epsilon(\nu)/K_\nu$ must vary in such a way that $\epsilon(\nu)/K_\nu$ becomes larger near the Sun. That is to say radio emission from electrons is relatively more important than absorption effects from interstellar HII in the local environment, as compared with the average along a line of sight in either the centre or anticentre directions. Obviously by choosing the proper variation of the ratio of $\epsilon(\nu)/K_\nu$ it is possible to reproduce either curves 1 or 2 in Figure 11. As a result this comparison is unable to provide a separate indication as to the actual form of $\epsilon(\nu)$ at low frequencies. To do this we must compare $\epsilon(\nu)$ with the various possible interstellar electron spectra, as will be done in the following section.

This approach does emphasize, however, how importantly our conclusions regarding the typical electron densities in the HII regions depend on the assumed shape of the projected brightness spectrum at low frequencies. For example, the values of emission measure in the centre and anticentre directions indicate an average electron density $\sim 0\cdot 10$ cm^{-3} in interstellar space near the Sun. The emission measure of 2 cm^{-6} pc obtained earlier in the polar direction when taken with this electron density gives a disk semithickness of ≈ 200 pc, whereas if the smaller polar emission measure of $0\cdot 5$ cm^{-6} pc is taken the disk semithickness is effectively only 40 pc (the absorption is assumed to be interstellar rather than from a solar HII region). From the point of view of radio emission, the characteristic semithickness of the disk is usually taken to be ~ 300–400 pc. Conversely, taking this semithickness as the region in which absorption occurs gives electron densities of $0\cdot 05$ and $0\cdot 01$ cm^{-3} respectively for the two values of emission measure. These two viewpoints can be interpreted in terms of a paucity of absorption relative to emission and may reflect the point we have deduced already from the $I(\nu)/\mathscr{I}(\nu)$ curves, namely that the Sun is in a region of relatively low radio absorption.

* Komesaroff (1961) has typically obtained values of $\tau \approx 10$ at 20 MHz corresponding to ~ 4000 at 1 MHz in the direction of the galactic centre, which is a factor of 10 larger than we obtain. However, our results represent an average over a band $\pm 2°$ on either side of the galactic equator. The value obtained by Komesaroff applies within $\pm 0\cdot 5°$ of the equator and he finds a decrease of an order of magnitude in the optical depth only $3°$–$4°$ off the equator. The two results are in fact in reasonable accord as is the value of 165 cm^{-6} pc for $b^{II} = 5°$ obtained by Ellis and Hamilton (1966).

Let us now see what a comparison of the interstellar electron spectrum with the low frequency radio emissivity tells us. The low frequency radio emissivity is obtained in exactly the same manner as before and using the same dependence of $\epsilon(\nu)$ on r as at higher frequencies, except that we now have the possibility of using two curves for the projected intensity $\mathscr{I}(\nu)$ which is used in calculating $\epsilon(\nu)$. These are the curves 1 and 2 in Figure 10. The corresponding low frequency emissivity profiles are shown in Figure 12, and are simply an extension of the profile presented in Figure 8. The expected emissivity for various interstellar electron spectra is also shown in Figure 12 again for a local magnetic field of 8 μG. The manner in which this emissivity scales with $B_\perp$, and the corresponding electron energies, are also shown in
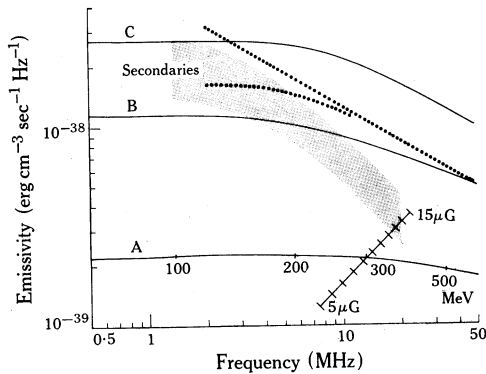

Fig. 12.—Comparison of local disk emissivity deduced at low frequencies (dotted curves) with the emissivity to be expected from:
A, electron spectrum measured at Earth in 1966,
B, interstellar electron spectrum obtained with a residual parameter of 0·6 GV, and
C, interstellar electron spectrum obtained with a residual parameter of 1·0 GV.
The manner in which the curves A, B, and C must be displaced for different galactic magnetic field strengths is also shown. The shaded area is the range of expected emission from secondary electrons.

the figure. It is seen that the expected emissivity from the low energy electron spectrum measured near the Earth in 1966 is almost an order of magnitude less than actually deduced. In order to provide sufficient emissivity from such a low intensity of electrons the local magnetic field is required to exceed 20 μG. Since a field as large as this is highly unlikely, solar modulation effects must be depressing the low energy electron spectrum near the Earth. Emissivities based on interstellar electron spectra obtained using demodulation constants of 0·6 and 1·0 GV are in much better accord with the deduced emissivities. The best agreement is obtained for $K_R = 0\cdot75$ GV and $B_\perp = 6$ μG. If $K_R = 0\cdot6$ GV then $B_\perp$ must be ~ 9 μG, whereas if $K_R = 1\cdot0$ GV then $B_\perp \sim 4$ μG.

The correspondence between the shapes of the emissivity spectra at low frequencies sets very severe restraints on the characteristics of the electron modulation at low energies. Using the energy dependence of the modulation given by Webber (1967a) the emission from the interstellar electron spectrum almost exactly reproduces the emission profile based on a projected intensity profile that flattens at low frequencies. This does not prove that such a profile is correct and the projected intensity profile based on an extension of the $\nu^{-0\cdot6}$ spectrum measured at higher frequencies will also be suitable, provided we assume a different electron modulation at lower

energies. The limits on the interstellar electron spectrum are rather clearly defined by the above comparison, however.

The question of the radio emission at these low frequencies from the secondary electrons is also relevant. As is evidenced in Figure 1(b) as well as Figure 13, if the calculations of the secondary electron intensity are correct, then most of the observed low energy electrons must be of secondary origin if the cosmic ray nuclei have passed through $\gtrsim 3 \ \mathrm{g\,cm^{-2}}$ of matter. The deduced emissivity at low frequencies sets a very positive upper limit of $< 6 \ \mathrm{g\,cm^{-2}}$ of material if *all* of the low energy electrons are secondaries.
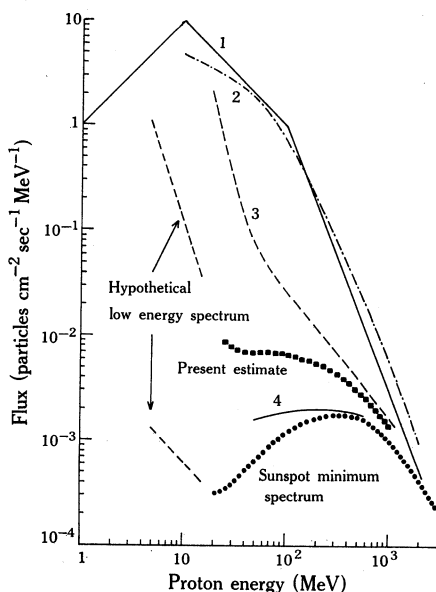

Fig. 13.—Interstellar cosmic ray proton spectrum obtained using a residual modulation parameter of $0 \cdot 75$ GV:
1, Hayakawa (1964)
2, Balasubrahmanyan *et al.* (1968)
3, Gloeckler and Jokipii (1967)
4, Durgaprasad, Fichtel, and Guss (1967)

INTERSTELLAR PROTON SPECTRUM

Utilizing the residual modulation constant of $0 \cdot 75$ GV derived from the electron data we may attempt to determine the interstellar proton spectrum. As noted earlier, the rigidity dependence of the proton modulation has been more completely measured (above $50 \ \mathrm{MeV} \equiv 0 \cdot 3$ GV rigidity) than for electrons; however, there is no *direct* method available to estimate the residual solar modulation of these particles from the data on protons alone—hence the huge differences in the estimates of the unmodulated (interstellar) proton spectrum. A summary of various estimates is given in Figure 13. Here the sunspot minimum spectrum is taken from the work of Gloeckler and Jokipii (1967). Curves 1, 2, 3, and 4 represent various estimates of the interstellar proton spectrum by Hayakawa (1964), Balasubrahmanyan *et al.* (1968), Gloeckler and Jokipii (1967), and Durgaprasad, Fichtel, and Guss (1967) respectively. Estimates 1 and 2 are based principally on the requirement that the rate of energy loss by ionization of these cosmic ray protons is sufficient to maintain the heating of interstellar HI clouds (see Balasubrahmanyan *et al.* for a discussion of this problem).

However, these spectra contain an energy density of cosmic rays $\sim$ 5–10 eV cm^{-3} as compared with an energy density $\sim 0\cdot5$ eV cm^{-3} for the sunspot minimum spectrum near the Earth. This interstellar energy density is equivalent to that contained in a magnetic field $\sim 20\ \mu$G, and according to Parker (1966) an energy density $\gtrsim 2$ eV cm^{-3} for cosmic rays leads to difficulties in holding together the combined magnetic-field–cosmic-ray system in the spiral arms by gravity.

The curve 3 is actually obtained using a residual modulation constant K_R of $0\cdot9$ GV and gives a more reasonable cosmic ray energy density ~ 1 eV cm^{-3} in interstellar space. This spectrum is also sufficient to produce the required heating of interstellar HI clouds, according to Balasubrahmanyan *et al.* (1968).

Our estimate differs from that of Gloeckler and Jokipii (curve 3) in that (1) we have used a slightly smaller demodulation constant as suggested by the data on electrons, and (2) we have used a modulation $\sim 1/\beta$ at low energies as indicated by the work of Ormes and Webber (1968) instead of a steeper function more like $1/\beta P$ used by Gloeckler and Jokipii (1967).

Below 20 MeV no reliable measurements are available on the proton modulation and there are very few data on the proton spectrum itself. Fan, Gloeckler, and Simpson (1965) give some evidence that the proton spectrum near the Earth begins to turn up at energies < 20 MeV, as represented by the dashed curve in Figure 13. It is not clear whether these protons are of solar origin or are merely a continuation of the higher energy part of the spectrum reaching us from the Galaxy. Suppose we take the latter point of view and suppose also we assume that the $1/\beta$ dependence of the modulation measured by Ormes and Webber (1968) for protons at intermediate energies extends to lower energies. This latter assumption is supported by the previously discussed measurements of a $1/\beta$ dependence for the electron modulation at equivalent rigidities (Webber 1967a). The low energy interstellar proton spectrum obtained by the resulting solar demodulation is shown as the upper dashed curve in Figure 13. This spectrum supplies a comparable amount of heating to interstellar HI regions as do spectra 1, 2, and 3, albeit from lower energy protons losing energy by ionization at a greater rate.

The demodulation effectively transforms a proton spectrum $\sim 1/E$ near the Earth to an interstellar spectrum $\sim 1/E^3$, one that is very similar to that actually observed for solar cosmic rays near the Earth. It is therefore tempting to ask whether such a low energy component of interstellar cosmic rays could be produced by solar type stars in the Galaxy. The answer, based on order-of-magnitude estimates of the number of particles emitted and the energy spectra involved, is yes. Consider the Sun. Estimates of the number of particles emitted during solar cosmic ray events can be made on a number of grounds (e.g. Webber 1963) and lead to an average rate of emission $\sim 10^{30}$–10^{31} particles sec^{-1} above a few million electron-volts averaged over the last 10 years. Now it is not clear what fraction of these actually escape into interstellar space; however, we may suppose that it is comparable to the number emitted. If the value of 10^{11} main sequence stars similar to the Sun is taken for our Galaxy, we have a total emission of low energy cosmic rays of $\sim 10^{41}$–10^{42} sec^{-1} from such sources. The lifetime of these cosmic rays is short ($\sim 3 \times 10^{13}$ sec for $n_H \sim 1$ cm^{-3}) as they rapidly lose energy by ionization loss. The total number in the

Galaxy at any one time is thus $\sim 3 \times 10^{54}$–3×10^{55} particles. Presumably these particles will not travel far from their source of origin, but will diffuse mainly in the disk of the Galaxy. The volume in which they reside is thus $\sim 10^{66}$–10^{67} cm³, depending on how closely they are confined to the spiral arms themselves. The density ρ that could be supplied by solar-type stars thus works out to be $\sim 3 \times 10^{-11}$–3×10^{-13} particles cm⁻³. The density required by the spectrum in Figure 13 is $\sim 3 \times 10^{-11}$ particles cm⁻³ above 5 MeV.

The near equality of these numbers suggests the plausibility of such a source for providing a prominent galactic spectrum of low energy particles.

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