LOW FREQUENCY ABSORPTION IN ISOTROPIC WORLD MODELS

By A. D. Payne*

[*Manuscript received April* 12, 1967]

Summary

The present paper is concerned with the free-free absorption by intergalactic ionized hydrogen of the radiation from discrete radio sources. The absorption for different world models has been calculated as a function of redshift, electron density, and temperature.

I. Introduction

It is well known that discrete radio sources can have low frequency turnovers in their spectra owing to a combination of thermal absorption, synchrotron self absorption, and/or absorption in the HII region that surrounds the galaxy. There is a further possible cause of low frequency turnover and this is free-free absorption in intergalactic space, which, according to some cosmological theories, is filled with ionized hydrogen of electron density in the range 10^{-4}–10^{-5} cm^{-3}. The aim of the present paper is to show the effects of intergalactic free-free absorption on the spectra of radio sources as a function of their redshift.

Several authors (Field and Henry 1964; Gould and Ramsay 1966) have determined limits for the temperature of the intergalactic medium, and it seems that the most probable equilibrium temperature lies in the range $1–5\times 10^4$ °K. Sciama (1964), in a brief review of the problem of the temperature of intergalactic matter, concluded that the most likely temperature is about 10^5 °K maintained by cosmic ray heating, which balances losses due to bremsstrahlung and recombination radiation in the plasma. Gold and Hoyle (1959) proposed a hot intergalactic medium at 10^9 °K but it has been shown (Gould and Burbidge 1963) that this model must be incorrect. For the purposes of the present paper it will be assumed that the temperature lies in the range $1–5\times 10^4$ °K.

II. Absorption in Expanding World Models

Uniform world-model universes can be represented by space–times with metrics of the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \frac{R^2(t)}{c^2}\left(\frac{\mathrm{d}r^2+r^2\,\mathrm{d}\theta^2+r^2\sin^2\theta\,\mathrm{d}\phi^2}{(1+\frac{1}{4}kr^2)^2}\right). \tag{1}$$

This equation describes a spherically symmetric space–time in which the Earth is considered to be located at the origin of the co-moving coordinates (r,θ,ϕ). $R(t)$ is the expansion factor, which is a function of the cosmic time t, k is the curvature constant, and c is the local velocity of light.

* Physics Department, University of Tasmania, Hobart.

We consider a photon of radiation emitted at time t_e and received by the observer at time t_0. The motion of the photon is represented diagrammatically as follows.

$$\underset{\mathrm{O}}{\underline{(0,t_0)}} \qquad \underset{\mathrm{P'}}{\underline{(r-\delta r, t+\delta t)}} \qquad \underset{\mathrm{P}}{\underline{(r,t)}} \qquad \underset{\mathrm{E}}{\underline{(r_e,t_e)}}$$

The coordinate velocity $\mathrm{d}r/\mathrm{d}t$ of the radiation travelling from E to O is

$$\mathrm{d}r/\mathrm{d}t = -c(1+kr^2)/R(t) \tag{2}$$

and integrating we obtain the basic equation for the motion of the radiation

$$-\int_r^0 \frac{\mathrm{d}r}{1+\frac{1}{4}kr^2} = c\int_t^{t_0} \frac{\mathrm{d}t}{R(t)}. \tag{3}$$

The luminosity distance D can be written

$$D = R_0^2 r/R(1+\tfrac{1}{4}kr^2). \tag{4}$$

Suppose a radio source at (r_e, t_e) has an emission spectrum $P(\lambda_e, t_e)$. The rate of emission of energy in the waveband $(\lambda_e, \lambda_e+\mathrm{d}\lambda_e)$ will be $P(\lambda_e, t_e)\,\mathrm{d}\lambda_e$ watts. An observer at the origin $r = 0$ will receive this energy at time t_0 in the frequency range $(\lambda_0, \lambda_0+\mathrm{d}\lambda_0)$ at a flux density $S(\lambda_0)$ watts per square metre given by

$$S(\lambda_0)\,\mathrm{d}\lambda_0 = \{P(\lambda_e, t_e)/4\pi D^2\}\,\mathrm{d}\lambda_e. \tag{5}$$

Now we put

$$\omega(t) = R(t)/R(t_0), \tag{6}$$

so that

$$\nu(t) = \nu_0/\omega(t) \tag{7}$$

and

$$\mathrm{d}\lambda_e = \omega(t_e)\,\mathrm{d}\lambda_0, \tag{8}$$

$$\mathrm{d}t_e = \omega(t_e)\,\mathrm{d}t_0. \tag{9}$$

If absorption of the radiation is neglected, the total flux S_0 received at O will be, in virtue of equations (5), (6), and (8),

$$S_0\,\mathrm{d}\lambda_0 = \frac{P\{\omega(t_e)\lambda_e\}\,\omega(t_e)}{4\pi D_e^2}\,\mathrm{d}\lambda_0. \tag{10}$$

When free-free absorption by ionized hydrogen occurs in the intergalactic medium, the received flux S_0' can be written

$$S_0'(\nu_0) = S_0(\nu_0)\exp\{-\tau(\nu_0)\}, \tag{11}$$

where $\tau(\nu_0)$ is the optical depth for the radiation received at frequency ν_0. We require to calculate the absorption factor $\exp\{-\tau(\nu_0)\}$ as a function of redshift for various world models.

Now it is assumed that $\tau(\nu_0)$ is given by the ordinary equation for free-free absorption

$$\delta\tau(\nu_0) = (\zeta N^2/\nu^2 T^{3/2})\delta D, \tag{12}$$

where N is the electron density and T the absolute temperature at the point (r, t).

ζ is a slowly varying function of N and T. For the conditions appropriate to this problem ζ is about $0 \cdot 4$ in CGS units.

The electron density can be represented in terms of the local value from

$$\left.\begin{aligned} N(t) &= N_0 && \text{(steady-state model),} \\ N &= N_0\,\omega^{-3} && \text{(evolving models).} \end{aligned}\right\} \tag{13}$$

The expansion of evolving models is assumed to be adiabatic and unaffected by radiation pressure. T can therefore be written in the form

$$\left.\begin{aligned} T &= T_0\{N(t)/N_0\}^{\gamma-1} = T_0 && \text{(steady-state model),} \\ T &= T_0\,\omega^{3(1-\gamma)} = T_0\,\omega^{-2} && \text{(evolving models),} \end{aligned}\right\} \tag{14}$$

where $\gamma = 5/3$.

Combining equations (7), (12), (13), and (14) we find

$$\tau(\nu_0) = -\frac{\zeta N_0^2}{T_0^{3/2}\nu_0^2}\int_{D_e}^{0} \omega^2\,\mathrm{d}D \tag{15}$$

for the steady-state model, and

$$\tau(\nu_0) = -\frac{\zeta N_0^2}{T^{3/2}\nu_0^2}\int_{D_e}^{0} \omega^{-1}\,\mathrm{d}D \tag{16}$$

for evolutionary models. It is the integrals in equations (15) and (16) that must be evaluated for different world models. The redshift δ is related to ω by the equation

$$1+\delta = \omega^{-1}\,. \tag{17}$$

III. Absorption in Steady-state Universe

The steady-state model universe can be specified by

$$R = R_0\exp\{H(t-t_0)\}\,, \tag{18}$$

$$k = 0\,, \tag{19}$$

where H is the Hubble parameter.

Integrating equation (3) with $k = 0$, we obtain

$$r = (c/H)\{\exp(-Ht)-\exp(-Ht_0)\}\,. \tag{20}$$

Using equations (4) and (6) and substituting $t_0 = 0$, we find

$$D = (c/H)(\omega^{-2}-\omega^{-1}) \tag{21}$$

and

$$\mathrm{d}D/\mathrm{d}\omega = -(c/H)(2\omega^{-3}-\omega^{-2})\,. \tag{22}$$

Hence

$$\begin{aligned} \int_{D_e}^{0} \omega^2\,\mathrm{d}D &= (c/H)\int_{\omega_e}^{1}(2\omega^{-1}-1)\,\mathrm{d}\omega \\ &= 2(c/H)(\ln\omega_e^{-1}+\omega_e-1)\,. \end{aligned} \tag{23}$$

So that, finally

$$\tau(\nu_0) = \frac{\zeta N_0^2}{T^{3/2}\nu_0^2}\frac{c}{H}\left(2\ln(1+\delta)+(1+\delta)^{-1}-1\right). \tag{24}$$

The equation (24) is plotted in Figure 1 as a function of frequency for various values of the redshift δ. The temperature of the intergalactic medium is taken to be 2×10^4 °K, the electron density 5×10^{-5} cm^{-3}, and the Hubble constant is assumed to have the value 100 km sec^{-1} Mpc^{-1}. The absorption factor will be further decreased if, as some authors suggest, the Hubble constant is as low as 75 km sec^{-1} Mpc^{-1}. It is obvious that free-free absorption for the low frequency radiation from discrete sources in a steady-state universe is not a strong function of the redshift.

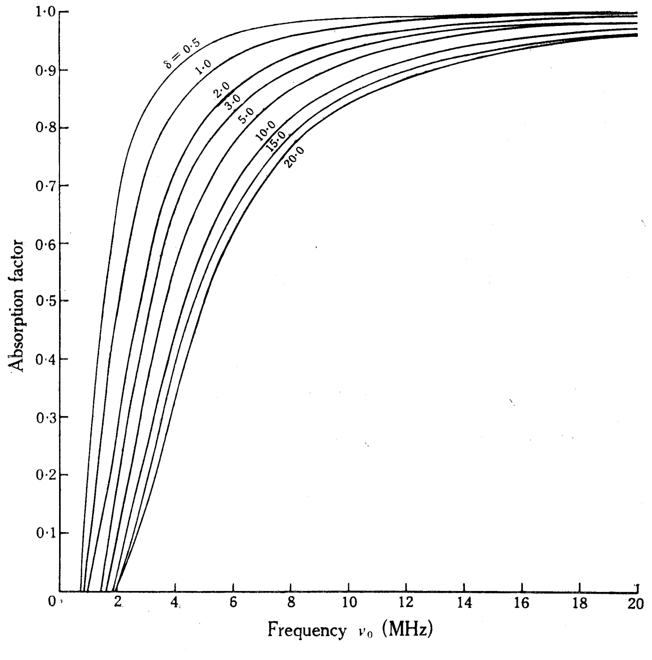

Fig. 1.—Steady-state universe; absorption factor for various values of the redshift δ at frequencies less than 20 MHz for $N_0 = 5\times10^{-5}$ cm^{-3}, $T_0 = 2\times10^4$ °K, $H = 100$ km sec^{-1} Mpc^{-1}.

IV. Absorption in Einstein–de Sitter Universe

For this model the parameters are defined by

$$R = R_0(t/t_0)^{2/3}, \qquad k = 0, \qquad H = 2/3t_0.$$

Equations (3), (4), and (6) give

$$D = (2c/H)\omega^{-1}(1-\omega^{\frac{1}{2}}). \tag{25}$$

Hence

$$\int_{D_e}^{0} \omega^{-1}\,\mathrm{d}D = (2c/H)\int_{\omega_e}^{1} (\tfrac{1}{2}\omega^{-5/2}-\omega^{-3})\,\mathrm{d}\omega$$
$$= (2c/H)(\tfrac{1}{3}\omega_e^{-3/2}-\tfrac{1}{2}\omega_e^{-2}+\tfrac{1}{6}),$$

so that

$$\tau(\nu_0) = \frac{\zeta N_0^2}{T_0^{3/2}\nu_0^2}\frac{2c}{H}\left(\tfrac{1}{2}(1+\delta)^2-\tfrac{1}{3}(1+\delta)-\tfrac{1}{6}\right). \tag{26}$$

The results of equation (26) are shown in Figure 2. Absorption in an Einstein–de Sitter universe is clearly quite strongly dependent on the redshift.

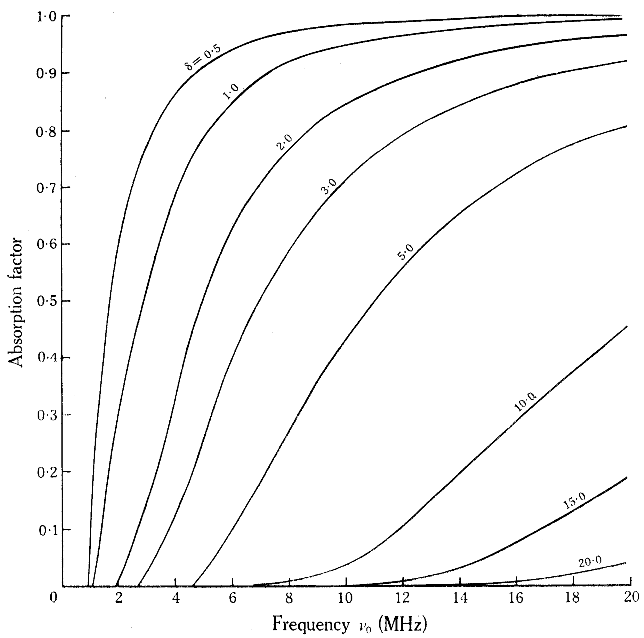

Fig. 2.—Einstein–de Sitter universe; absorption factor for various values of the redshift δ. $N_0 = 5\times10^{-5}\,\mathrm{cm}^{-3}$, $T_0 = 2\times10^4\,{}^\circ\mathrm{K}$, $H = 100\,\mathrm{km\,sec^{-1}\,Mpc^{-1}}$.

V. Absorption in Dirac Universe

The Dirac-model universe has the specifications

$$R = R_0(t/t_0)^{1/3}, \qquad k = 0, \qquad H = 1/3t_0. \tag{27}$$

Again with the aid of equations (3), (4), (6), and (27) we have

$$D = (c/2H)\{\omega^{-1}(1-\omega^2)\}$$

and

$$\int_{D_e}^{0} \omega^{-1}\,\mathrm{d}D = -(c/2H)\int_{\omega_e}^{1} (\omega^{-3}+\omega^{-1})\,\mathrm{d}\omega$$
$$= -(c/2H)(\tfrac{1}{2}\omega_e^{-2}+\ln\omega_e^{-1}-\tfrac{1}{2}).$$

Hence

$$\tau(\nu_0) = \frac{\zeta N_0^2}{T_0^{3/2}\nu_0^2}\frac{c}{2H}\left(\tfrac{1}{2}(1+\delta)^2+\ln(1+\delta)-\tfrac{1}{2}\right). \tag{28}$$

The results of equation (28) are plotted in Figure 3. In this case absorption is only a moderately strong function of the redshift.

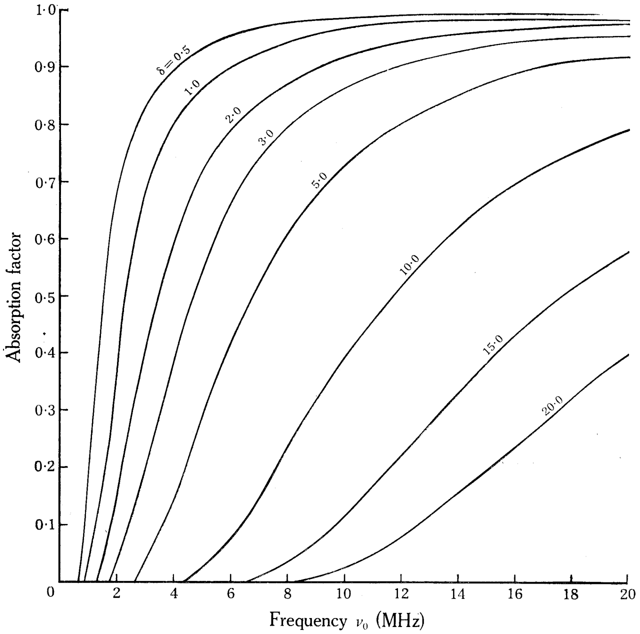

Fig. 3.—Dirac universe; absorption factor for various values of the redshift δ. $N_0 = 5\times10^{-5}$ cm^{-3}, $T_0 = 2\times10^4$ °K, $H = 100$ km sec^{-1} Mpc^{-1}.

VI. Absorption in Milne Universe

This model has the specifications

$$R = R_0(t/t_0), \qquad k = 1, \qquad H = 1/t_0. \tag{29}$$

As before, we find

$$D = (c/2H)(\omega^{-2}-1),$$

so that

$$\int_{D_e}^{0}\omega^{-1}\,\mathrm{d}D = (c/H)\int_{\omega_e}^{1}\omega^{-4}\,\mathrm{d}\omega$$

$$= \tfrac{1}{3}(c/H)(\omega_e^{-3}-1)$$

and

$$\tau(\nu_0) = \frac{\zeta N_0^2}{T^{3/2}\nu_0^2}\frac{c}{H}\left((1+\delta)^3-1\right). \tag{30}$$

Figure 4 shows absorption in a Milne universe to be a very strong function of the redshift.

VII. Discussion

In order to compare the different cosmological models, the absorption factor has been plotted in Figure 5 as a function of frequency for $\delta = 2 \cdot 5$ and for each type of universe.

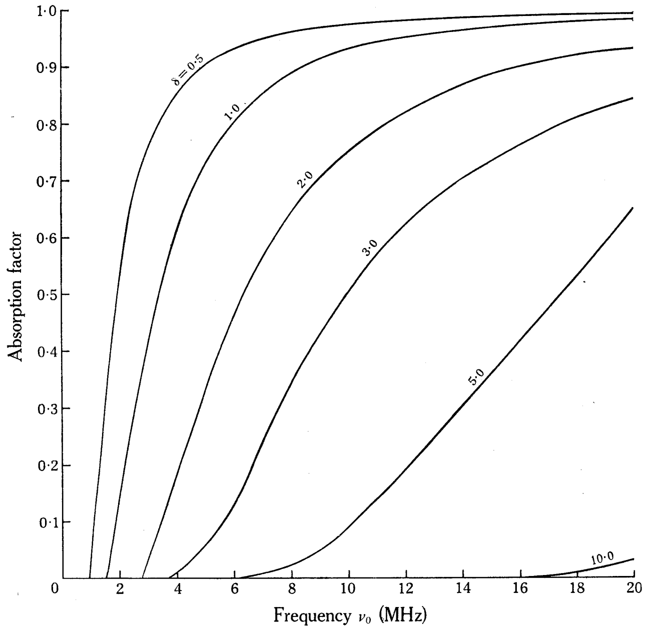

Fig. 4.—Milne universe; absorption factor for various values of the redshift δ. $N_0 = 5 \times 10^{-5}$ cm^{-3}, $T_0 = 2 \times 10^4$ °K, $H = 100$ km sec^{-1} Mpc^{-1}.

It can be seen that the effects of free-free absorption are greatest in a Milne universe and least in a steady-state universe. Absorption increases rapidly once it has set in and all models have an optical depth greater than $1 \cdot 0$ at 4 MHz when $\delta = 2 \cdot 5$. The largest redshift measured is $2 \cdot 118$ for the source 1116+12 (Schmidt 1966) and presumably even greater redshifts will be found, a limit being set by experimental technique. Many discrete sources that have not been optically identified may have very large (at present undetectable) redshifts and the intergalactic medium will produce a most significant effect on their low frequency radio emission. For example, in the Einstein–de Sitter universe, with assumed values for N_0, T_0, and H, the radiation from a source with a redshift of 10 will have an optical depth equal to unity at approximately 20 MHz.

The results obtained in this paper are all rather tentative as they depend on uncertain parameters such as temperature, electron temperature, and the Hubble constant. Until the values of these parameters have been decided by independent methods, no real conclusions can be drawn as to a particular world model.

The low frequency spectra of discrete sources will therefore be affected by intergalactic absorption to a degree depending on the parameters mentioned above and the particular world model that may apply. Indeed, once these (at present) unknown parameters have been determined, observations of radio sources at low frequencies may provide a method for differentiating between the possible world

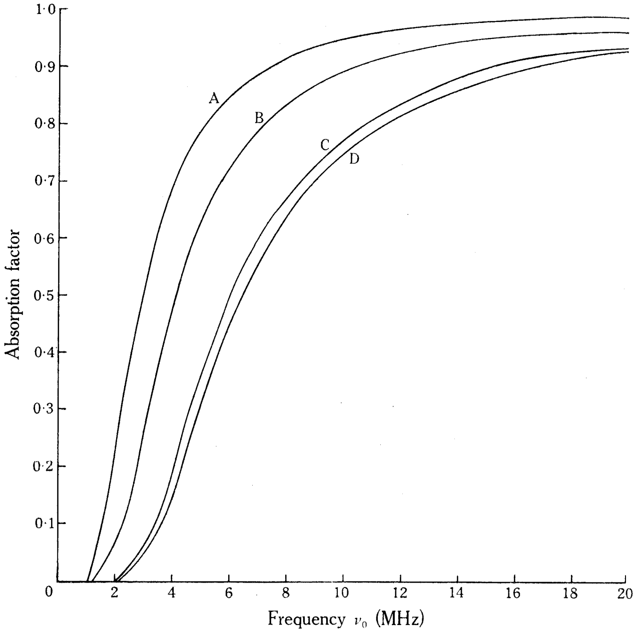

Fig. 5.—Absorption factor for $\delta = 2\cdot5$ in each world model. Curve A, steady-state; B, Dirac; C, Einstein–de Sitter; D, Milne. $N_0 = 5\times10^{-5}$ cm^{-3}, $T_0 = 2\times10^{4}$ °K, $H = 100$ km sec^{-1} Mpc^{-1}.

models. The preceding results have shown that it cannot be assumed that the low frequency cutoff in the spectra of discrete sources is due solely to synchrotron self absorption or thermal absorption in the galaxy or in the source itself. Surveys of discrete sources have been made at 38 MHz (Williams, Kenderdine, and Baldwin 1966), 26·3 MHz (Erickson and Cronyn 1965), and in the range 20–40 MHz (Bazelyan *et al.* 1965). Only a few flux measurements of discrete sources have been made at frequencies less than 20 MHz and until the observations are more reliable it does not seem worth while attempting to fit a theoretical curve to the measurements.

VIII. Acknowledgment

The financial assistance of a Commonwealth Post-graduate Scholarship is gratefully acknowledged.

IX. References

Bazelyan, L. L., Braude, S. Ya., Waisberg, V. V., Krymkin, V. V., Men, A. V., and Sodin, L. G. (1965).—*Astr. Zh.* **42**, 618.

Erickson, W. C., and Cronyn, W. M. (1965).—*Astrophys. J.* **142**, 1156.

Field, G. B., and Henry, R. C. (1964).—*Astrophys. J.* **140**, 1002.

Gold, T., and Hoyle, F. (1959).—Symp. IAU No. 9 (Paris 1958). p. 583.

Gould, R. J., and Burbidge, G. R. (1963).—*Astrophys. J.* **138**, 969.

Gould, R. J., and Ramsay, W. (1966).—*Astrophys. J.* **144**, 587.

Schmidt, M. (1966).—*Astrophys. J.* **144**, 443.

Sciama, D. R. (1964).—*Nature, Lond.* **204**, 767.

Williams, P. J. S., Kenderdine, S., and Baldwin, J. E. (1966).—*Mem. R. astr. Soc.* **68**, 163.