THE LUNAR MAGNETIC VARIATIONS AT TOOLANGI

By H. F. Petersons,* D. E. Winch,* and L. Slaucitajs*†

[*Manuscript received March* 9, 1965]

Summary

The lunar semi-diurnal variations in H, D, and Z at Toolangi observatory have been analysed for the 4-yr period 1952–55 of the sunspot minimum.

Three methods of analysis of the lunar semi-diurnal component of D have been compared. It was found that the Chapman–Miller method was the most suitable and that the initial removal of the solar daily variation used in earlier methods is not essential.

I. Introduction

The lunar diurnal variation, L, of the Earth's magnetic field is of special interest owing to the relatively simple and regular distribution of the lunar tide which produces it. The significance of lunar influences on geophysical parameters in general can be seen in the recent works of Bradley (1962), Adderley (1963), Bartels (1963), Bigg (1963*a*, 1963*b*), and Bowen (1963). Knowledge of the behaviour of L, especially of a world-wide nature, is therefore likely to be of value even though its small magnitude, and the fact that its period is close to that of the solar transient daily variations, makes its calculation laborious.

Data available from the Toolangi (37° 32′ S., 145° 28′ E.) observatory were analysed. Furthermore, a comparison of three methods of computation, of which one was the Chapman–Miller method, was undertaken in order to determine the most suitable method. The results obtained were then arranged according to the seasons to obtain the seasonal variation in the amplitude and phase of L. Comparison of the amplitude C_2 of the lunar semidiurnal term from Tables 1, 2, and 4 shows that they are not significantly different, the differences between the amplitudes calculated by any one method and either of the other two being within the probable error range of any one of them.

II. Procedure

(*a*) *First Method*

The first method described here is that used by Chapman (1914) in his calculation of L for Pavlovsk and Pola (1857–1903).

The published geomagnetic data usually consist of hourly mean values of the three magnetic elements H, D, and Z, arranged in columns according to the hour, from 1 to 24, and in rows according to the day of the month, from the first of the month to the end of the month. The solar daily variation, S, is assumed to be independent of the lunar daily variation, so that in one month (or more correctly 29·5306 days) the lunar disturbance characteristic of any lunar hour affects in turn each group

* Department of Applied Mathematics, University of Sydney.

† On leave from Observatorio Astronomico, Universidad Nacional de La Plata, Argentina.

of 28–31 solar hours in like degree. This also assumes that the variation of L from day to day is small; the statistical fluctuations could, however, be considerable (Bartels and Johnston 1940; Schneider 1941).

If from each hourly value is subtracted the solar daily variation for that hour, the remainder will be due to causes not regularly solar, that is, to irregular disturbance, secular variation, and any lunar action. Where work can be carried out using computers, the computation of the mean hourly values for each day of the month centred at that day is not very difficult.

The hourly mean values minus their monthly mean hourly values are next arranged according to lunar time, and it is assumed that, if a sufficiently large number of days are taken, the irregular disturbance will average out, even though on individual days the magnitude of the disturbance may be greater than the lunar variation.

The length of the lunar day is on the average 24 hr 50 min, and it usually does not vary from this by more than half an hour. It is therefore convenient to treat the lunar day as being of duration 25 solar hours. Commencement of the lunar day is taken at the lower local transit of the Moon (lunar hour 0). The times for the upper and lower transit of the Moon at Greenwich can be obtained from the "Nautical Almanac", and interpolation to any other longitude will yield the local mean time of transit. These times do not as a rule fall at an exact solar hour, and the nearest solar hour is used. The difference for this hour (that is, the hourly mean value minus the monthly mean hourly value) and the differences for the 24 succeeding hours are arranged in a row on a lunar sheet, and represent the lunar variations together with accidental variations.

A further hourly difference is added at the end of the row to correct for the non-cyclic variation. The solar times which are adopted as most nearly corresponding to the time of lower lunar transit generally differ from one day to the next by 25 hr, so that the last entry on the row for one lunar day is the first in the row for the next day. When, however, the interval is 24 hr, further 2-hourly differences are added at the end of the row, and these will be the first two on the next row.

All the lunar days for one calendar month are written on the one lunar sheet, and a few hours are taken each from the preceding and following month to make all the rows on the lunar sheet complete. Totals and means of all the columns are then formed, and the resulting hourly means are then corrected for non-cyclic variation. These then give the mean lunar variation for the month but will usually include considerable accidental error due to magnetic disturbance. It is, therefore, necessary to combine the results from many months before a well-determined lunar daily variation can be expected to be obtained.

A computer program was written for an IBM 1620 machine, which calculated the lunar variation by the above method. Small modifications, which it was assumed would not significantly affect the results, were introduced to simplify the program and to make it possible for a single program to do the complete computation.

The first modification was to correct each solar day for the non-cyclic variation in order to remove any linear trends (e.g. non-cyclic variation) in the data. This was done using Chapman's formula for midnight local time, which is magnetically the quietest period, where the non-cyclic variation (NCV) in G.M.T. is given by

$$\mathrm{NCV} = \tfrac{1}{2}\{(14\tfrac{1}{2})+(13\tfrac{1}{2})-(-10\tfrac{1}{2})-(-9\tfrac{1}{2})\},$$

in which the numbers in the round brackets refer to the hourly means centred at the epochs indicated. Hourly values n ($n = 1$–24) were then corrected by adding

$$(n-0\cdot 5)\times(\mathrm{NCV})$$

to the original hourly values, and finally these were adjusted to have zero mean.

The second modification was to take exactly 30 solar days, starting with January 1, and organize them in five rows of 25 and one of 24, in order to make the deviation from the mean lunar day (taken as 24 hr 50 min) not more than half an hour. It was found that 30 solar days could be exactly arranged in 29 lunar days, which simplified the computer program considerably and is a good approximation to one complete lunation.

Means of the 25 columns were formed, and these were taken to represent the mean lunar daily variation for the month of January. The 11 succeeding lots of 30 days were taken to give values of L for the corresponding remaining months. This left 5 or 6 days (depending on whether the year was not or was a leap year) at the end of the year untreated, for treatment of the next year was started on January 1. This was done so that the lunations could be more easily classified according to the seasons.

(*b*) *Second Method*

The second method is the same as the first, except that the solar daily variation is not subtracted from the data. It is assumed that the arrangement of the data according to lunar time over 29 lunar days (approximately 1 complete lunation) is sufficient to average out the solar daily variation.

To show why the greater part of S is eliminated by arrangements of 30 solar days according to lunar time, consider data y_j such that

$$y_j = \alpha_1\cos\{(2\pi j/24)+\epsilon_1\} + \alpha_2\cos\{(2\pi j/24\cdot 84)+\epsilon_2\},$$

that is, where y_j contains the first harmonics of S and L respectively.

Let $\theta_1 = 2\pi/24$, and $\theta_2 = 2\pi/24\cdot 84$, then the lunar Fourier coefficient (in complex variable notation) from $N+1$ solar hourly values is given by

$$\begin{aligned}\sum_{j=0}^{N} y_j \exp \mathrm{i}\theta_2 j &= \sum_{j=0}^{N} \tfrac{1}{2}\alpha_1\Big(\exp \mathrm{i}\{j(\theta_1+\theta_2)+\epsilon_1\} + \exp[-\mathrm{i}\{j(\theta_1-\theta_2)+\epsilon_1\}]\Big)\\ &\quad + \sum_{j=0}^{N} \tfrac{1}{2}\alpha_2\Big(\exp \mathrm{i}(2j\theta_2+\epsilon_2) + \exp(-\mathrm{i}\epsilon_2)\Big)\\ &= \tfrac{1}{2}\alpha_2 \exp \mathrm{i}\epsilon_2 \frac{1-\exp 2\mathrm{i}\theta_2(N+1)}{1-\exp 2\mathrm{i}\theta_2} + \tfrac{1}{2}\alpha_2\exp\{-\mathrm{i}\epsilon_2(N+1)\}\\ &\quad + \tfrac{1}{2}\alpha_1 \exp \mathrm{i}\epsilon_1 \frac{1-\exp \mathrm{i}(\theta_1+\theta_2)(N+1)}{1-\exp \mathrm{i}(\theta_1+\theta_2)}\\ &\quad + \tfrac{1}{2}\alpha_1 \exp(-\mathrm{i}\epsilon_1) \frac{1-\exp\{-\mathrm{i}(\theta_1-\theta_2)(N+1)\}}{1-\exp\{-\mathrm{i}(\theta_1-\theta_2)\}}.\end{aligned}$$

If k is the number of lunar days used, then $N+1 = k \times 24 \cdot 84$. Now, to eliminate the contribution of S one must eliminate the terms containing α_1 without eliminating α_2. This is the case when $(0 \cdot 842/24)k = 1, 2, 3, \ldots$, that is,

$$k = 28 \cdot 50,\ 57 \cdot 00,\ 85 \cdot 50, \ldots \text{ (lunar days)}$$

$$= 29 \cdot 50,\ 59 \cdot 00,\ 88 \cdot 50, \ldots \text{ (solar days)},$$

so that S should be completely eliminated (if it remains constant in amplitude from day to day) over periods that are multiples of $29 \cdot 50$ solar days. This is only approximately true if 30 solar days are taken, but the approximation appears to be a good one, as shown by the result of Figure 1.

A similar proof holds for the other harmonics of S and L.

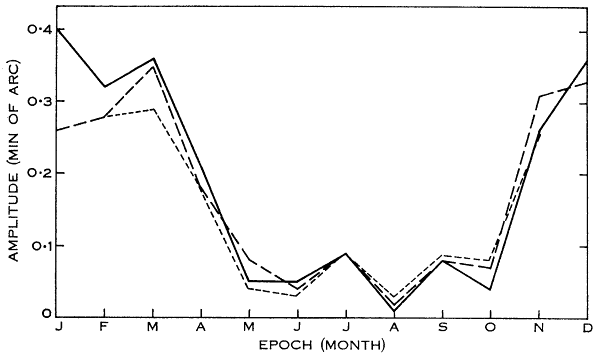

Fig. 1.—Amplitude of lunar semi-diurnal variation of declination for Toolangi, 1952–55. - - - - First method; — — — second method; ——— third method.

(c) *Third Method*

The third method employed, the Chapman–Miller method, was developed by Chapman and Miller (1940), and later Tschu (1949) described the computations required in the practical applications of it. The geomagnetic data at Toolangi were analysed following Tschu, and the probable errors were calculated as outlined by him and later by Chapman (1952). The method is briefly as follows.

Daily sequences of hourly values are arranged in 12 groups ($r = 0$–11) according to the ages as assigned to each day from "Geophysikalische Mond-Tafeln" by Bartels and Fanselau (1937). Thus the group r contains days for which the age of the Moon is r and $r+12$. To each daily sequence the hour from the preceding daily sequence is then added, so that allowance for the non-cyclic variations can be made. The number of daily sequences for the group r is N_r, and $\Sigma N_r = N$ (N = total number of daily sequences used).

For each group r, the hourly values are summed and the 12 group-sum sequences $g_{s,r}$ ($s = 0$–S, $r = 0$–11) are obtained; $S = 24$, 12, or 8 depending on whether 1-hourly, 2-hourly, or 3-hourly data are used. (This S is not to be confused with the S used above to denote solar daily variation.) These group-sum sequences are then harmonically analysed, giving

$$A_{p,r} = \sum_{s=0}^{S} g_{s,r}\cos(2\pi sp/S) + \tfrac{1}{2}(g_{S,r}-g_{0,r}),$$

$$B_{p,r} = \sum_{s=0}^{S} g_{s,r}\sin(2\pi sp/S) + \tfrac{1}{2}(g_{S,r}-g_{0,r})\cot(\pi p/S).$$

Here the last terms in the expressions for $A_{p,r}$ and $B_{p,r}$ correct the data for non-cyclic variation. The value of p determines which harmonic is to be analysed. For the calculation of the lunar semi-diurnal variation, $p = 2$.

A secondary harmonic analysis now gives

$$N_{1,A} = \sum_{r=0}^{11} N_r\cos(2\pi r/R), \qquad N_{1,B} = \sum_{r=0}^{11} N_r\sin(2\pi r/R),$$

$$A_{p,A} = \sum_{r=0}^{11} A_{p,r}\cos(2\pi r/R), \qquad A_{p,B} = \sum_{r=0}^{11} A_{p,r}\sin(2\pi r/R),$$

$$B_{p,A} = \sum_{r=0}^{11} B_{p,r}\cos(2\pi r/R), \qquad B_{p,B} = \sum_{r=0}^{11} B_{p,r}\sin(2\pi r/R).$$

where $R = 12$, the number of lunar age groups.

Finally,

$$U_p = (A_{p,A}-B_{p,B}) - N^{-1}(A_{p,N}N_{1,A} - B_{p,N}N_{1,B}),$$

$$V_p = (B_{p,A}+A_{p,B}) - N^{-1}(B_{p,N}N_{1,A} + A_{p,N}N_{1,B}),$$

from which the lunar amplitude L_2 and phase λ_2 are given by the equations

$$L_2\sin\lambda_2 = U_2/Kd_{mpS}, \qquad L_2\cos\lambda_2 = V_2/Kd_{mpS},$$

$$K = 0\cdot4943\, NS[1-\{(N_{1,A})^2+(N_{1,B})^2\}N^{-2}],$$

$$d_{mpS} = (1/S)\sin\pi(m-p)\,[\cot\{\pi(m-p)/S\} + \cot(\pi p/S)],$$

$$m = 2(1-M^{-1}),$$

where M ($= 29\cdot5306$) is the number of mean solar days in the mean synodic period of the Moon.

A correction δ to the phase must now be carried out if the data are not tabulated for Greenwich mean time and at the latitude of Greenwich, given by

$$\delta = 2L/M - 15mH' + m(L-L') \quad \text{degrees},$$

where L denotes the longitude from Greenwich (measured positive if westward and negative if eastward) of the station to which the data refer. L' is similarly the meridian to which the data are tabulated, and H' is the initial solar hour used for the calculation.

(d) *Probable Errors*

(i) *First Method*.—The probable error in the amplitude of the lunar semi-diurnal variation L_2 was obtained using the simple probability theory given by Guest (1961). Here the variance of the Fourier coefficients a_n and b_n, and hence of $c_n = (a_n^2+b_n^2)^{\frac{1}{2}}$, is given by $2\sigma^2/r$, where σ is the standard deviation of an observation, r is the number of data points, and c_n is the amplitude of the nth harmonic. The probable error for c_n is therefore taken to be $2 \cdot 06(2\sigma^2/r)^{\frac{1}{2}}$.

(ii) *Second Method*.—The theory used to calculate the probable error in the values of L obtained by using the second method was the one developed by Miller (1934). This theory also served as the basis of the determination by Tschu (1949) of the probable error in L using the Chapman–Miller method.

Let $\mathbf{R}_m$ ($m = 1, \ldots, n$) denote a set of vectors representing the set of lunar semi-diurnal harmonic components. These vectors can be considered as consisting of two parts, namely, a vector $\mathbf{L}_m$ which is the mean value of the $\mathbf{R}_m$'s, and the vector $\mathbf{A}_m$ which represents the errors in the lunar variation. The vector $\mathbf{L}_m$ is assumed to represent the true lunar variation of the set of $\mathbf{R}_m$ vectors.

Further, let the vector $\mathbf{L}_m$ have a constant amplitude L and a known variable phase angle l_m, and let $\mathbf{A}_m$ have an amplitude A'_m and phase $(l_m+\theta_m)$, where θ_m is independent of l_m. Consider vectors $\mathbf{R}'_m$, $\mathbf{L}$, and $\mathbf{A}'_m$ which are equal to $\mathbf{R}_m$, $\mathbf{L}_m$, and $\mathbf{A}_m$ except that they are reduced in phase by l_m. Therefore,

$$R_m = R'_m, \qquad A_m = A'_m,$$

and

$$\mathbf{R}'_m = \mathbf{L}+\mathbf{A}'_m,$$

where $\mathbf{L}$ is a constant vector with zero phase.

Next, assume that the n vectors $\mathbf{A}'_m$ have a Gaussian distribution, so that the distribution function for any one of the vectors is given by

$$(1/2\pi\sigma^2)\exp(-r^2/2\sigma^2)\, r\,d\theta\,dr,$$

where r is the amplitude and θ the phase angle, so that the probable error r_0 in the amplitude A_m is

$$\frac{1}{2\pi\sigma^2}\int_0^{r_0}\int_0^{2\pi}\exp(-r^2/2\sigma^2)\, r\,d\theta\,dr = \tfrac{1}{2},$$

that is,

$$1-\exp(-r_0^2/2\sigma^2) = \tfrac{1}{2},$$

and therefore $r_0 = 0 \cdot 8326\sqrt{2}\,\sigma$.

To calculate σ, first the r_i were found, where

$$r_i = \{(a_i-\bar{a})^2+(b_i-\bar{b})^2\}^{\frac{1}{2}},$$

and where the a_i and b_i are the harmonic coefficients of the lunar semi-diurnal variation, and $\bar{a}$ and $\bar{b}$ are the corresponding mean values. Then the variance is given by

$$\sigma^2 = (1/n)\sum_i r_i^2-\bar{r}^2,$$

where $\bar{r} = (1/n)\sum\limits_i r_i$.

The value of σ then enables the r_0 to be calculated.

TABLE 1

HARMONIC COMPONENTS* OF THE LUNAR SEMI-DIURNAL VARIATION OF DECLINATION D, FROM THE FIRST METHOD

Values are monthly means, February–November, for the period 1952–55

	C_2 (tenths of min of arc)	ϵ_2 (degrees)	Probable Error in C_2 (same units)
Feb.:	2·75	159	1·84
Mar.:	2·88	108	1·60
Apr.:	1·83	145	1·39
May:	0·40	345	2·02
June:	0·34	204	0·31
July:	0·89	35	0·73
Aug.:	0·29	236	0·41
Sept.:	0·89	245	0·58
Oct.:	0·67	218	0·63
Nov.:	2·46	222	0·70

* Expressed in the form $C_2 \sin(2\tau+\epsilon_2)$, where τ = mean lunar time in angular measure reckoned from lower lunar transit at Toolangi.

TABLE 2

HARMONIC COMPONENTS* OF THE LUNAR SEMI-DIURNAL VARIATION OF DECLINATION D, FROM THE SECOND METHOD

Values are monthly means for the period 1952–55

	C_2 (tenths of min of arc)	ϵ_2 (degrees)	Probable Error in C_2 (same units)
Jan.:	2·62	284	1·24
Feb.:	2·77	291	0·74
Mar.:	3·46	336	1·06
Apr.:	1·76	307	0·57
May:	0·82	166	0·87
June:	0·42	242	0·58
July:	0·85	45	0·38
Aug.:	0·18	220	0·65
Sept.:	0·81	213	0·57
Oct.:	0·72	236	0·40
Nov.:	3·14	229	1·09
Dec.:	3·31	271	0·85

* Expressed in the form $C_2 \sin(2\tau+\epsilon_2)$, where τ = mean lunar time in angular measure reckoned from lower lunar transit at Toolangi.

Table 3

HARMONIC COMPONENTS* OF THE LUNAR SEMI-DIURNAL VARIATION OF HORIZONTAL INTENSITY H, FROM THE THIRD METHOD

Values are monthly, seasonal, and yearly means respectively for the period 1952–55

	C_2 (gammas)	ϵ_2 (degrees)	Probable Error in C_2 (same units)
Jan.:	0·89	64	0·65
Feb.:	0·90	19	0·64
Mar.:	0·87	44	0·69
Apr.:	0·10	20	0·80
May:	0·04	307	0·82
June:	0·57	301	0·57
July:	0·20	255	0·44
Aug.:	0·64	346	0·53
Sept.:	0·42	319	0·89
Oct.:	0·46	349	0·50
Nov.:	1·31	352	0·89
Dec.:	2·23	24	0·63
Summer:	1·23	21	0·36
Equinox:	0·38	10	0·37
Winter:	0·31	316	0·30
Year:	0·59	10	0·20

* Expressed in the form $C_2 \sin(2\tau+\epsilon_2)$, where τ = mean lunar time in angular measure reckoned from lower lunar transit at Toolangi.

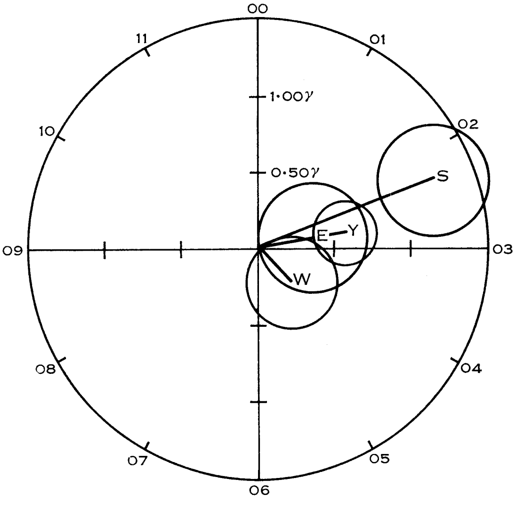

Fig. 2.—Harmonic dial for second Fourier component of lunar daily variation of H at Toolangi, 1952–55. S, summer; E, equinox; W, winter; Y, year.

TABLE 4

HARMONIC COMPONENTS* OF THE LUNAR SEMI-DIURNAL VARIATION OF DECLINATION D, FROM THE THIRD METHOD

Values are monthly, seasonal, and yearly means respectively for the period 1952–55

	C_2 (tenths of min of arc)	ϵ_2 (degrees)	Probable Error in C_2 (same units)
Jan.:	4·02	82	3·41
Feb.:	3·19	140	1·27
Mar.:	3·59	352	1·99
Apr.:	2·06	155	1·28
May:	0·52	149	0·95
June:	0·49	80	0·63
July:	0·94	60	0·66
Aug.:	0·11	114	1·15
Sept.:	0·81	49	1·28
Oct.:	0·40	118	1·11
Nov.:	2·63	86	1·07
Dec.:	3·59	126	1·61
Summer:	3·04	108	1·03
Equinox:	1·31	165	0·73
Winter:	0·42	87	0·44
Year:	1·42	121	0·45

* Expressed in the form $C_2 \sin(2\tau + \epsilon_2)$, where τ = mean lunar time in angular measure reckoned from lower lunar transit at Toolangi.

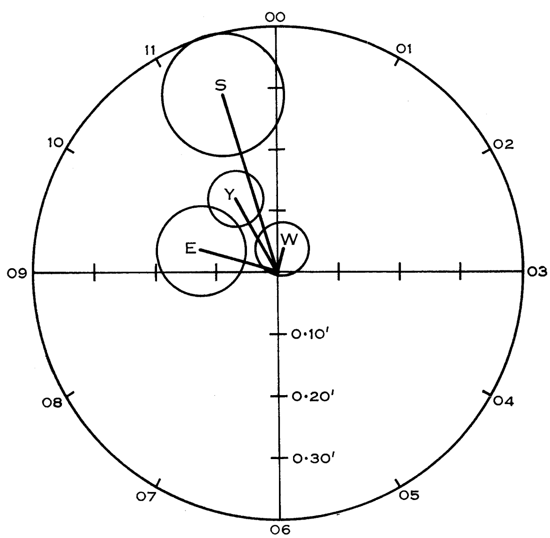

Fig. 3.—Harmonic dial for second Fourier component of lunar daily variation of D at Toolangi, 1952–55. S, summer; E, equinox; W, winter; Y, year.

TABLE 5

HARMONIC COMPONENTS* OF THE LUNAR SEMI-DIURNAL VARIATION OF VERTICAL INTENSITY Z, FROM THE THIRD METHOD

Values are monthly, seasonal, and yearly means respectively for the period 1952–55

	C_2 (gammas)	ϵ_2 (degrees)	Probable Error in C_2 (same units)
Jan.:	1·18	74	0·29
Feb.:	1·11	79	0·37
Mar.:	0·81	228	0·40
Apr.:	0·55	242	0·45
May:	0·69	245	0·32
June:	0·68	244	0·31
July:	0·64	72	0·21
Aug.:	0·83	240	0·24
Sept.:	0·66	219	0·35
Oct.:	0·48	225	0·35
Nov.:	0·91	239	0·45
Dec.:	1·32	79	0·34
Summer:	0·69	84	0·19
Equinox:	0·62	228	0·20
Winter:	0·39	239	0·14
Year:	0·18	192	0·10

* Expressed in the form $C_2 \sin(2\tau + \epsilon_2)$, where τ = mean lunar time in angular measure reckoned from lower lunar transit at Toolangi.

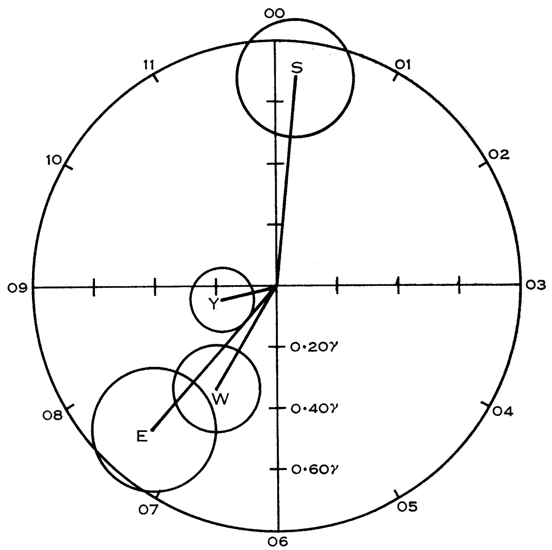

Fig. 4.—Harmonic dial for second Fourier component of Z at Toolangi, 1952–55. S, summer; E, equinox; W, winter; Y, year.

III. Results and Discussion

Figure 1 and Tables 1, 2, and 4 show the great difference in amplitude of the semi-diurnal variation of declination between the summer and winter months, over the 4-yr period 1952–55. It also shows how nearly equivalent are the values of the amplitude of the semi-diurnal variation obtained from the three methods of computation of L, in spite of the fact that the probable errors are so large. The probable errors for the three methods were all themselves of the same order.

Table 6

AMPLITUDE AND PHASE OF LUNAR SEMI-DIURNAL VARIATION OBTAINED AT BOTH NORTHERN AND SOUTHERN STATIONS ON LATITUDES NEAR THAT OF TOOLANGI

Units of C_2 and ϵ_2 are the same as in Tables 3 and 4

		Val-Joyeux	Pola	Amberley, N.Z.
Geographical latitude		48°·3 N.	44°·9 N.	43°·2 S.
Geomagnetic latitude		51°·3 N.	45°·1 N.	47°·7 S.
Variation of H	C_2	—	0·79	1·09
	ϵ_2	—	39	339
Variation of D	C_2	1·3	0·9	2·4
	ϵ_2	9	68	29
Worker		Rougerie (1950)	Chapman (1914)	Bullen and Cummack (1954)

Figures 2, 3, and 4, which illustrate the results of Tables 3, 4, and 5, are harmonic dials showing the seasonal variation in H, D, and Z, respectively, as computed by the Chapman–Miller method. The error circles have also been included. Where the radius of the error circle is greater than the amplitude of L, little significance can be given to the value of L. The time of maximum amplitude can be easily read from the harmonic dials.

Table 6 has been included to enable a comparison to be made with some of the values obtained by other workers at stations of latitude similar to that of Toolangi.

Figure 1 also shows that the initial inclusion or removal of the solar daily variation from the data does not significantly alter the value of the amplitude of the second harmonic of the lunar daily variation.

IV. Acknowledgment

We wish to acknowledge the helpful criticism of our work by Dr. O. Schneider of Buenos Aires, Argentina.

V. References

Adderley, E. E. (1963).—The influence of the Moon on atmospheric ozone. *J. Geophys. Res.* **68**: 1405.

Bartels, J. (1963).—Statistische Hintergründe für geophysikalische Synchronisierungs-Versuche und Kritik an behaupteten Mond-Einflüssen auf die erdmagnetische Aktivität. *Nachr. Akad. Wiss. Göttingen* (Math. Phys. Klasse) **23**: 1.

Bartels, J., and Fanselau, G. (1937).—Geophysikalische Mond-Tafeln 1850–1975. *Z. Geophys.* **13**: 311.

Bartels, J., and Johnston, H. F. (1940).—Geomagnetic tides in horizontal intensity at Huancayo. *Terr. Magn. Atmos. Elect.* **45**: 269.

Bigg, E. K. (1963*a*).—Influence of the Moon on geomagnetic disturbance. *J. Geophys. Res.* **68**: 1409.

Bigg, E. K. (1963*b*).—Lunar and planetary influence on geomagnetic disturbance. *J. Geophys. Res.* **68**: 4099.

Bowen, E. C. (1963).—A lunar effect on the incoming meteor rate. *J. Geophys. Res.* **68**: 1401.

Bradley, D. H., *et al.* (1962).—Lunar synodic period and widespread precipitation. *Science, N.Y.* **137**: 748.

Bullen, J. M., and Cummack, C. H. (1954).—The lunar diurnal variations of the Earth's magnetic field for all elements at Amberley, N.Z., based on five years' observations. *N.Z. Jl Sci. Technol.* B **35**(5): 371.

Chapman, S. (1914).—*L* at Pavlovsk and Pola. *Phil. Trans. R. Soc.* A **214**: 295.

Chapman, S. (1952).—The calculation of the probable error of determinations of lunar daily harmonic component variations in geophysical data: a correction. *Aust. J. Scient. Res.* A **5**: 218.

Chapman, S., and Miller, J. C. P. (1940).—The statistical determination of lunar daily variations in geomagnetic and meteorological elements. *Mon. Not. R. Astr. Soc. Geophys. Suppl.* **4**: 649.

Guest, P. (1961).—"Numerical Methods of Curve Fitting." (Cambridge Univ. Press.)

Miller, J. C. P. (1934).—On a special case in the determination of probable errors. *Mon. Not. R. Astr. Soc.* **94**: 860. (Suppl.)

Rougerie, P. (1950).—Variation diurne lunaire de la déclinaison magnétique et de la composante verticale du champ terrestre au Val-Joyeux. *Annls Géophys.* **6**: 300.

Schneider, O. (1941).—The variability of lunar magnetic variation. *Terr. Magn. Atmos. Elect.* **46**: 283.

Tschu, K. K. (1949).—On the practical determination of lunar and luni-solar daily variations in certain geophysical data. *Aust. J. Scient. Res.* A **2**: 1.